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Senior School Certificate Examination

March 2016

Marking Scheme — Mathematics 65(B)

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.
2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.
3. Alternative methods are accepted. Proportional marks are to be awarded.
4. In question (s) on differential equations, constant of integration has to be written.
5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.
6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
7. Separate Marking Scheme for all the three sets has been given.
8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65(B)
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. PQ intersects YZ plane where x-coordinate is zero

$\frac{1}{2}$

$$\begin{array}{c} \bullet \text{-----} \bullet \text{-----} \bullet \\ P(-2, 5, 9) \quad k : 1 \quad Q(3, -2, 4) \end{array}$$

$$\Rightarrow 0 = 3k - 2 \Rightarrow k = \frac{2}{3} \text{ or } 2 : 3$$

$\frac{1}{2}$

2. $\vec{a} + \vec{b} + \vec{c} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\frac{1}{2}$

A vector of magnitude 7 units in the direction of $\vec{a} + \vec{b} + \vec{c}$ is

$$\frac{7(3\hat{i} + 2\hat{j} + 6\hat{k})}{\sqrt{9 + 4 + 36}} \text{ i.e. } 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$\frac{1}{2}$

3. $AB = I \Rightarrow |A| |B| = 1$

$\frac{1}{2}$

$$|B| = \frac{1}{3}$$

$\frac{1}{2}$

4. $|2A| = 2^2 |A|$

$\frac{1}{2}$

$$\Rightarrow k = 4$$

$\frac{1}{2}$

5. Projection of $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$\frac{1}{2}$

$$\frac{(7\hat{i} + \hat{j} - 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})}{|2\hat{i} + 6\hat{j} + 3\hat{k}|} \text{ i.e. } \frac{8}{7}$$

$\frac{1}{2}$

6. $|\text{adj } A| = |A|^{n-1}$

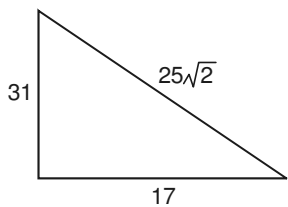
$\frac{1}{2}$

$$4^2 = 4^{n-1} \Rightarrow n = 3$$

$\frac{1}{2}$

SECTION B

7.
$$\text{LHS} = 2 \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{1}{7} \quad 1\frac{1}{2}$$



$$= \tan^{-1} \frac{\frac{4}{3} + \frac{1}{7}}{1 - \frac{4}{21}} = \tan^{-1} \frac{31/21}{17/21} = \tan^{-1} \frac{31}{17} = \text{RHS} \quad 1\frac{1}{2}$$

$$\text{RHS} = \cos^{-1} \frac{17}{25\sqrt{2}} = \tan^{-1} \frac{31}{17} \quad 1$$

OR

The given equation can be written as

$$2 \tan^{-1} \frac{1-x}{1+x} = \tan^{-1} x \quad 1$$

$$2 \tan^{-1} \frac{1-x}{1+x} = \tan^{-1} \frac{2(1-x)}{1+x} = \tan^{-1} x \quad 1\frac{1}{2}$$

$$\Rightarrow \frac{1-x^2}{2x} = x \Rightarrow 3x^2 = 1 \text{ or } x = \frac{1}{\sqrt{3}} \quad 1\frac{1}{2}$$

	Cutting section	Tailoring & packing section	Delhi	Mumbai		Delhi	Mumbai	
8.	Pant	$\begin{pmatrix} 7 & 3 \end{pmatrix}$	$\begin{pmatrix} 50 & 42 \end{pmatrix}$		=	$\begin{pmatrix} 440 & 429 \end{pmatrix}$		$\frac{1}{2} + \frac{1}{2} + 1$
	Shirt	$\begin{pmatrix} 3 & 2 \end{pmatrix}$	$\begin{pmatrix} 30 & 45 \end{pmatrix}$			$\begin{pmatrix} 210 & 216 \end{pmatrix}$		

Shirt = Less costly in Delhi

Pant = Less costly in Mumbai

Value = Taking care of weaker section

OR

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

using $C_1 \rightarrow C_1 + C_2 + C_3$ and taking $(a + b + c)$ common

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

using $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = (a+b+c)(ab+bc+ca-a^2-b^2-c^2)$$

$$= (a+b+c) \left[-\frac{1}{2} \left[(a-b)^2 + (b-c)^2 + (c-a)^2 \right] \right] \left. \vphantom{\begin{matrix} \\ \\ \\ \end{matrix}} \right\} \text{ as } a, b, c \text{ are positive and unequal } \Rightarrow \Delta \text{ is negative}$$

$$9. \quad y = (\tan^{-1} x)^2 \Rightarrow \frac{dy}{dx} = \frac{2 \tan^{-1} x}{(1+x^2)} \text{ or } (1+x^2) \frac{dy}{dx} = 2 \tan^{-1} x \quad 1 + \frac{1}{2}$$

Diff. again we get

$$(1+x^2) \frac{d^2y}{dx^2} + 2x \cdot \frac{dy}{dx} = \frac{2}{1+x^2} \quad 1+1$$

$$(1+x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2) \frac{dy}{dx} = 2 \quad \frac{1}{2}$$

OR

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{2 \sin^2 2x}{(2x)^2} \cdot \frac{1}{\frac{1}{4}} = 8 \lim_{x \rightarrow 0^-} \left(\frac{\sin 2x}{2x} \right)^2 = 8 \quad 1\frac{1}{2}$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\sqrt{x} [\sqrt{16 + \sqrt{x}} + 4]}{(\sqrt{16 + \sqrt{x}})^2 - 16} = \lim_{x \rightarrow 0^+} \frac{\cancel{\sqrt{x}} [\sqrt{16 + \sqrt{x}} + 4]}{\cancel{16} + \cancel{\sqrt{x}} - \cancel{16}} = 8 \quad 1\frac{1}{2}$$

$$f(0) = a \Rightarrow a = 8 \quad 1$$

10. $y = u + v$, where $u = x^{\sin x - \cos x}$, $v = \frac{x^2 - 1}{x^2 + 1} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad 1\frac{1}{2}$

$$\log u = (\sin x - \cos x) \log x \Rightarrow \frac{du}{dx} = x^{\sin x - \cos x} \left[\frac{1}{x} (\sin x - \cos x) + (\cos x + \sin x) \log x \right] \quad 1\frac{1}{2}$$

$$\frac{dv}{dx} = \frac{(x^2 + 1) 2x - (x^2 - 1) 2x}{(x^2 + 1)^2} = \frac{4x}{(x^2 + 1)^2} \quad 1$$

$$\therefore \frac{dy}{dx} = x^{\sin x - \cos x} \left[\frac{\sin x - \cos x}{x} + (\cos x + \sin x) \log x \right] + \frac{4x}{(x^2 + 1)^2} \quad 1$$

11. $f(x) = \cos \left(2x + \frac{\pi}{4} \right) \Rightarrow f'(x) = -2 \sin \left(2x + \frac{\pi}{4} \right) \quad 2$

$$\left. \begin{aligned} \text{Given that } \frac{3\pi}{8} < x < \frac{5\pi}{8} &\Rightarrow \frac{3\pi}{4} < 2x < \frac{5\pi}{4} \\ &\Rightarrow \pi < \left(2x + \frac{\pi}{4} \right) < \frac{3\pi}{2} \\ &\Rightarrow \left(2x + \frac{\pi}{4} \right) \text{ lies in III Quadrant} \Rightarrow \sin \left(2x + \frac{\pi}{4} \right) < 0 \end{aligned} \right\} \quad 1\frac{1}{2}$$

$$\Rightarrow f'(x) > 0 \Rightarrow f(x) \text{ is increasing in the given interval} \quad 1\frac{1}{2}$$

12. $I = \int [\sqrt{\cot x} + \sqrt{\tan x}] dx = \int \sqrt{\tan x} (1 + \cot x) dx$

$$\text{Let } \tan x = t^2 \Rightarrow \sec^2 x dx = 2t dt \Rightarrow dx = \frac{2t}{1+t^4} dt \quad 1$$

$$\therefore I = \int t \left(1 + \frac{1}{t^2} \right) \times \frac{2t}{1+t^4} dt = 2 \int \frac{\left(1 + \frac{1}{t^2} \right)}{\left(1 - \frac{1}{t} \right)^2 + 2} dt \quad 1\frac{1}{2}$$

$$= \frac{2}{\sqrt{2}} \tan^{-1} \left(\frac{t - \frac{1}{t}}{\sqrt{2}} \right) + C = \sqrt{2} \tan^{-1} \left(\frac{t^2 - 1}{\sqrt{2}t} \right) + C = \sqrt{2} \tan^{-1} \left(\frac{\tan x - 1}{\sqrt{2} \tan x} \right) + C \quad 1\frac{1}{2}$$

13. Let $x^2 = t$, Given expression is $\frac{(t+1)(t+2)}{(t+3)(t+4)} = 1 - \frac{4t+10}{(t+3)(t+4)}$ 1

$$\frac{4t+10}{(t+3)(t+4)} \equiv \frac{A}{t+3} + \frac{B}{t+4}, \text{ getting } A = -2, B = 6 \quad 1$$

$$\therefore I = \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx = \int \left(1 - \frac{-2}{(x^2+3)} + \frac{6}{x^2+4} \right) dx \quad 1$$

$$= x + \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - 3 \tan^{-1} \frac{x}{2} + C \quad 1$$

14. $I = \int_{-1}^{\frac{3}{2}} |x \sin(\pi x)| dx$

$$f(x) = |x \sin \pi x| = \begin{cases} x \sin \pi x, & \text{for } -1 \leq x \leq 1 \\ -x \sin \pi x, & \text{for } 1 \leq x \leq \frac{3}{2} \end{cases}$$

$$\therefore I = \int_{-1}^1 x \sin \pi x dx - \int_1^{\frac{3}{2}} x \sin \pi x dx \quad 1\frac{1}{2}$$

$$= \left[\frac{-x \cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2} \right]_{-1}^1 - \left[\frac{-x \cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2} \right]_1^{\frac{3}{2}} \quad (1+1)$$

$$= \frac{2}{\pi} + \frac{1}{\pi^2} + \frac{1}{\pi} = \frac{3}{\pi} + \frac{1}{\pi^2} \quad 1$$

$$I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx = \pi \int_0^{\pi} \frac{\sin x dx}{1 + \cos^2 x} - I \quad 1 + \frac{1}{2}$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\sin x dx}{1 + \cos^2 x} \Rightarrow I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx, \text{ let } \cos x = t, -\sin x dx = dt \quad 1$$

$$x = 0, t = 1, x = \pi, t = -1\frac{1}{2}$$

$$\therefore I = -\frac{\pi}{2} \int_1^{-1\frac{1}{2}} \frac{dt}{1+t^2} = \pi \int_0^1 \frac{dt}{1+t^2} = \pi \left[\tan^{-1} t \right]_0^1 = \frac{\pi^2}{4} \quad 1$$

15. The given diff. eqn., on simplification, can be written as

$$\frac{dy}{dx} = \frac{\left(\frac{y}{x}\right) \cos\left(\frac{y}{x}\right) + \frac{y^2}{x^2} \sin\left(\frac{y}{x}\right)}{\frac{y}{x} \sin\left(\frac{y}{x}\right) - \cos\left(\frac{y}{x}\right)} \quad \dots(i) \quad 1\frac{1}{2}$$

Taking $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$ 1\frac{1}{2}

(i) becomes $v + x \frac{dv}{dx} = \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v}$ or $x \frac{dv}{dx} = \frac{2v \cos v}{v \sin v - \cos v}$ 1

or $\int \frac{v \sin v - \cos v}{v \cos v} dv = 2 \int \frac{dx}{x}$ or $\int \tan v dv - \int \frac{dv}{v} = 2 \log |x| + C$ 1

$$\left. \begin{aligned} \Rightarrow \log \sec v - \log |v| &= 2 \log |x| + \log C \text{ or } \log \left(\frac{\sec v}{vx^2} \right) = \log C \\ \Rightarrow \frac{\sec \left(\frac{y}{x} \right)}{xy} &= C \\ \text{or } \sec \left(\frac{y}{x} \right) &= C xy \end{aligned} \right\} 1$$

16. I.F. = $e^{\int \cot x \, dx} = \sin x$ 1

The solution is $y \cdot \sin x = \int 4x \operatorname{cosec} x \sin x \, dx$

$$y \sin x = \int 4x \, dx = 2x^2 + C \quad \left. \vphantom{y \sin x} \right\} 1 \frac{1}{2}$$

$$\left. \begin{aligned} \text{when } x &= \frac{\pi}{2}, y = 0 \\ \Rightarrow 0 &= 2 \frac{\pi^2}{4} + C \Rightarrow C = -\frac{\pi^2}{2} \end{aligned} \right\} 1$$

$$\therefore \text{ The solution is } y \sin x = 2x^2 - \frac{\pi^2}{2} \quad \frac{1}{2}$$

17. It is given that $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 2$


$$|\vec{a} + \vec{b} + \vec{c}| = 0 \Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 0 \quad 1$$

$$\text{or } |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0 \quad 1 \frac{1}{2}$$

$$\text{or } 9 + 16 + 4 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0 \quad 1$$

$$\text{or } \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{29}{2} \quad \frac{1}{2}$$

18.

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$$


P(1, 0, 0)

Any general point on line l is $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$ 1

Let this be point Q

d.r's of PQ are $2\lambda, -3\lambda - 1, 8\lambda - 10$ 1 \frac{1}{2}

$$PQ \perp l \Rightarrow 2(2\lambda) - 3(-3\lambda - 1) + 8(8\lambda - 10) = 0 \Rightarrow 77\lambda = 77 \text{ or } \lambda = 1 \quad 1$$

\therefore Point Q is $(3, -4, -2)$ 1 \frac{1}{2}

$$\text{and distance } PQ = \sqrt{4 + (-4)^2 + (-2)^2} = 2\sqrt{6} \quad 1$$

19. let A be the event that the number on drawn card is odd and B be the event that the number on drawn card is > 7 $\frac{1}{2}$

$$S = \{1, 2, 3, \dots, 12\}, A = \{1, 3, 5, 7, 9, 11\}, B = \{8, 9, 10, 11, 12\} \therefore A \cap B = \{9, 11\}$$

$$\therefore P(B) = \frac{5}{12}; P(A \cap B) = \frac{1}{6} \quad \frac{1}{2}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{5} \quad 1$$

SECTION C

20. Let $x_1, x_2 \in \mathbb{R} - \{3\}$ such that $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3} \Rightarrow x_1 x_2 - 2x_2 + \cancel{3} - 3x_1 = x_1 x_2 - 2x_1 - 3x_2 + \cancel{3} \Rightarrow x_1 = x_2$$

$$\therefore f: A \rightarrow B \text{ is one-one} \quad \frac{1}{2}$$

Let $y \in \mathbb{R} - \{1\}$ such that $f(x) = y$

$$\frac{x - 2}{x - 3} = y \Rightarrow x = \frac{2 - 3y}{1 - y} \in A, x \neq 3 \quad 1$$

Corresponding to every $y \in B$, there exists $\frac{2 - 3y}{1 - y} \in A$, so that $\frac{2 - 3y}{1 - y} = x \Rightarrow f$ is onto 1

$$\therefore f^{-1}(x) = \frac{2 - 3x}{1 - x} \quad 1$$

OR

Commutativity: let $(a, b), (c, d) \in A$, then

$$(a, b) * (c, d) = (a + c, b + d) \text{ and } (c, d) * (a, b) = (c + a, d + b)$$

for all $a, b, c, d \in \mathbb{R}$, $a + c = c + a$ and $b + d = d + b$

$$\therefore (a, b) * (c, d) = (c, d) * (a, b)$$

$$\therefore * \text{ is commutative on } A \quad \frac{1}{2}$$

Associativity: For any $(a, b), (c, d), (e, f) \in A$

$$\begin{aligned} \{(a, b) * (c, d)\} * (e, f) &= (a + c, b + d) * (e, f) \\ &= (a + c + e, b + d + f) \end{aligned}$$

$$\text{Similarly } (a, b) * \{(c, d) * (e, f)\} = (a + c + e, b + d + f) \quad \frac{1}{2}$$

$$\therefore * \text{ is associative on } A$$

Let (x, y) be identity element in A, then $(a, b) * (x, y) = (a, b)$

$$\text{or } (a + x, b + y) = (a, b) \Rightarrow x = 0, y = 0 \in \mathbb{R}$$

$$\therefore (0, 0) \text{ is the identity element in } A \quad \frac{1}{2}$$

Let (l, m) be inverse of (a, b) in A $\Rightarrow (a, b) * (l, m) = (0, 0)$

$$\Rightarrow (a + l, b + m) = (0, 0) \Rightarrow l = -a, m = -b; \text{ which lies in } \mathbb{R} \therefore \text{ inverse is } (-a, -b) \quad \frac{1}{2}$$

21. $A = \begin{pmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{pmatrix}$, $|A| = 1(4) - 2(1) + 5(5) = 27 \neq 0 \Rightarrow A^{-1}$ exists

$$\text{adj } A = \begin{pmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{pmatrix}, \Rightarrow A^{-1} = \frac{1}{27} \begin{pmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{pmatrix} \quad 2\frac{1}{2}$$

The given system of equation can be written as $AX = B$

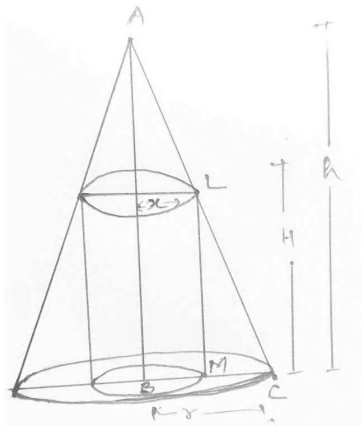
where A is given as above, $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $B = \begin{pmatrix} 10 \\ -2 \\ -11 \end{pmatrix}$ 1

$$\therefore X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} B = \frac{1}{27} \begin{pmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{pmatrix} \begin{pmatrix} 10 \\ -2 \\ -11 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} \quad 1$$

$$\therefore x = -1, y = -2, z = 3 \quad \frac{1}{2}$$

22.

Correct Figure



In ΔABC and ΔLMC , $\frac{H}{h} = \frac{r-x}{r} \Rightarrow H = \frac{h(r-x)}{r}$ 1

Let S the curved surface area of cylinder

$$S = 2\pi x H = 2\pi x \frac{h}{r} (r-x) \quad 1$$

$$S = \frac{2\pi h}{r} (xr - x^2) \Rightarrow S'(x) = \frac{2\pi h}{r} (r - 2x) \quad 1$$

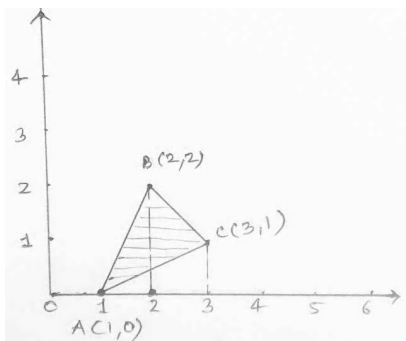
$$S'(x) = 0 \text{ gives } r = 2x \text{ or } x = \frac{r}{2} \quad 1$$

$$S''(x) = -\frac{4\pi h}{r} < 0 \quad 1$$

$$\therefore S \text{ is greatest at } x = \frac{r}{2}$$

23.

Correct Figure



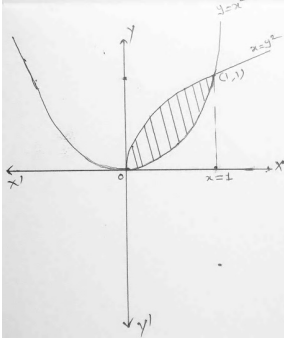
Equation of AB is $y = 2(x-1)$, equation of BC is $y = 4-x$,
equation of AC is $y = \frac{x-1}{2}$ 1

$$\therefore \text{Reqd area} = \int_1^2 2(x-1)dx + \int_2^3 (4-x)dx - \int_1^3 \frac{x-1}{2} dx \quad 1\frac{1}{2}$$

$$= 2 \left[\frac{x^2}{2} - x \right]_1^2 + \left[4x - \frac{x^2}{2} \right]_2^3 - \frac{1}{2} \left[\frac{x^2}{2} - x \right]_1^3 \quad 1$$

$$= 2 \times \frac{1}{2} + \left(4 - \frac{1}{2} \right) - \frac{1}{2} \left(\frac{3}{2} + \frac{1}{2} \right) = \frac{3}{2} \text{ sq. units} \quad 1$$

OR



Correct Figure

1

Points of intersection of two curves (0, 0) (1, 1)

 $1\frac{1}{2}$

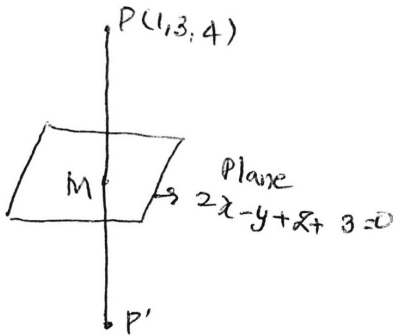
$$\therefore \text{Required area} = \int_0^1 (\sqrt{x} - x^2) dx$$

 $1\frac{1}{2}$

$$= \left[\frac{2}{3} x^{3/2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{3} \text{ sq. unit}$$

2

24.

Eqn. of any line through P \perp to given plane

$$\text{is } \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda (\text{say}) \quad \dots(i)$$

1

The coordinates of a general point on (i) is $(2\lambda + 1, -\lambda + 3, \lambda + 4)$

1

Let these be the coordinates P' (the image of P in the plane)

M bisects PP' \Rightarrow coordinates of M are

$$\left(\lambda + 1, \frac{-\lambda + 6}{2}, \frac{\lambda}{2} + 4 \right) \text{ or } \left(\lambda + 1, \frac{-\lambda}{2} + 3, \frac{\lambda}{2} + 4 \right)$$

 $1\frac{1}{2}$

M lies on the plane, so should satisfy its equation

$$\therefore 2(\lambda + 1) - \left(\frac{-\lambda + 6}{2} \right) + \left(\frac{\lambda}{2} + 4 \right) + 3 = 0$$

1

$$\Rightarrow \lambda = -2$$

Co-ordinate of P' is $(-3, 5, 2)$

$$\therefore \text{length } PP' = \sqrt{(1+3)^2 + (3-5)^2 + (4-2)^2} = \sqrt{24} \text{ or } 2\sqrt{6}$$

 $\frac{1}{2}$ 25. Let E_1 and E_2 be the events that the student resides in hostel and does not reside in the hostel respectively. Let A be the event that student gets A grade

1

$$\therefore P(E_1) = \frac{70}{100} = \frac{7}{10}, P(E_2) = \frac{3}{10}$$

 $\frac{1}{2} + \frac{1}{2}$

$$P(A/E_1) = \frac{4}{10}, P(A/E_2) = \frac{2}{10}$$

 $\frac{1}{2} + \frac{1}{2}$

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

1

$$= \frac{\frac{7}{10} \times \frac{4}{10}}{\frac{7}{10} \times \frac{4}{10} + \frac{3}{10} \times \frac{2}{10}} = \frac{14}{17}$$

26. Mathematical formulation of problem

To minimise $Z = 50x + 70y$

2

Subject to constraints

$$2x + y \geq 8, x + 2y \geq 10; x, y \geq 0$$

2 + 2