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Senior School Certificate Examination

March — 2015

Marking Scheme — Mathematics 65(B)

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

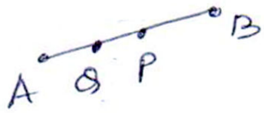
QUESTION PAPER CODE 65(B)

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. P.V. of $P = \dfrac{13\vec{a} - 6\vec{b}}{5} \left(\because \dfrac{AP}{PB} = \dfrac{3}{2}\right)$ ½ m



P.V. of $Q = \dfrac{23}{10}\vec{a} + \dfrac{9}{10}\vec{b} \left(\because \dfrac{AQ}{QP} = \dfrac{1}{1}\right)$ ½ m

2. $\vec{a} \cdot \vec{b} = 0$ as $\vec{a} \perp \vec{b}$ ½ m

$2\lambda - 3\lambda - 5 = 0$

$\Rightarrow \lambda = -5$ ½ m

3. D.R. of normal to plane 3 , 4 , 2 ½ m

Also point (3, 4, 2) lies on plane

$3x + 4y + 2z + d = 0$

$\Rightarrow \quad d = -29$

So cartesian Equation of plane is

$3x + 4y + 2z - 29 = 0$ ½ m

4. $A = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 5 & 6 & 7 \end{vmatrix}$

$a_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = 3$ 1 m

5. Order = 2 ½ m

or Degree = 1

So A + B = 3 ½ m

6. $y = ax + x^2$

$y_1 = a + 2x$

$y_1 - 2x = a$ ½ m

So $y = (y_1 - 2x)\,x + x^2$

$\Rightarrow \quad xy_1 = y + x^2$ ½ m

SECTION - B

7. Total Expenditure incurred for villages x, y, z

are

$$\begin{bmatrix} 200 & 400 & 200 \\ 350 & 600 & 300 \\ 225 & 375 & 150 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \\ 15 \end{bmatrix} = \begin{bmatrix} 7000 \\ 11,000 \\ 6375 \end{bmatrix}$$ 2 m

So Expenditure on village x = ₹ 7000
So Expenditure on village y = ₹ 11,000
So Expenditure on village z = ₹ 6375 } 1 m

Value : Sensitization about hygehic habits ..... or Any other relevant value 1 m

8. $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$ 1 m

$A^2 - \lambda A + \mu I = 0$

$\Rightarrow \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} 2\lambda & -\lambda \\ -\lambda & 2\lambda \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 1 m

$\Rightarrow \left.\begin{aligned} 5 - 2\lambda + \mu &= 0 \\ -4 + \lambda &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} \lambda &= 4 \\ \mu &= 3 \end{aligned}$ 1 m 1 m

OR

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix} = 1$$ 1 m

$c_{11} = 7 \quad c_{21} = -3 \quad c_{31} = -3$
$c_{12} = -1 \quad c_{22} = 1 \quad c_{32} = 0$
$c_{13} = -1 \quad c_{23} = 0 \quad c_{33} = 1$

$$\text{Adj } A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$ 1½ m

$$A \cdot (\text{adj } A) = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$ ½ m

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{.................................... (i)}$$

Since $|A| = 1$

$$\text{So } |A| \; I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{.................................... (ii)}$$ ½ m

from (i) & (ii)

$A \cdot (\text{adj } A) = |A| I$ ½ m

9. $$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$$

$$R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 - 3R_1$$

$$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix}$$ 3 m

$$= a \begin{vmatrix} a & 2a+b \\ 3a & 7a+3b \end{vmatrix}$$

$$= a^2 \begin{vmatrix} 1 & 2a+b \\ 3 & 7a+3b \end{vmatrix}$$

$$= a^2 \left(7a + 3b - 6a - 3b\right)$$

$$= a^3$$ 1 m

10. $$I = \int_0^{\pi/4} \log\left(1 + \tan x\right) dx$$

$$= \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right) dx$$ 1 m

$$= \int_0^{\pi/4} \log\left(1 + \frac{1 - \tan x}{1 + \tan x}\right) dx$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1 + \tan x}\right) dx$$ 1 m

$$= \int_0^{\pi/4} \left(\log 2 - \log\left(1 + \tan x\right)\right) dx$$

$$I = \int_0^{\pi/4} \log 2 \, dx - I$$ 1 m

$$2I = \frac{\pi}{4} \log 2$$

or $I = \frac{\pi}{8} \log 2$ 1 m

11. $\int \frac{\sqrt{1 - \sin x}}{1 + \cos x} \cdot e^{-x/2} dx \; ; \; 0 \le x \le \frac{\pi}{2}$

$$= \int \frac{-\sin \frac{x}{2} + \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \cdot e^{-x/2} dx$$ 1 m

$$= \frac{1}{2} \int \left(-\sec \frac{x}{2} \tan \frac{x}{2} + \sec \frac{x}{2} \right) e^{-x/2} dx$$ 1 m

Put $-\frac{x}{2} = t$

$\Rightarrow \frac{-1}{2} dx = dt$

$$= -\int (\sec t + \sec t \tan t) \, e^t dt$$

$$= -e^t \sec t + c$$ 1 m

$$= -e^{-x/2} \sec \left(\frac{-x}{2} \right) + c$$

$$= -e^{-x/2} \sec \left(\frac{x}{2} \right) + c$$ 1 m

OR

$$\int \frac{x^2 + 1}{x^2 - 5x + 6} \, dx$$

$$= \int \left(1 + \frac{5x-5}{x^2-5x+6}\right) dx = \int \left(1 + \frac{5x-5}{(x-2)(x-3)}\right) dx$$ 1 m

$$= \int dx + \int \frac{-5}{x-2}\, dx + \int \frac{10}{x-3}\, dx$$ 1½ m

$$= x - 5\log|x-2| + 10\log|x-3| + c$$ 1½ m

12. S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Event A : No. on card is 'more than 5' 1 m

A = {6, 7, 8, 9, 10}

Event B : Even no. on card

B = {2, 4, 6, 8, 10}

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$ 1 m

$$= \frac{3/10}{5/10} = 3/5$$ 2 m

13. Given $|\vec{a}| = |\vec{b}|$

$\cos\theta = \cos 60^\circ = \frac{1}{2}$, θ angle between $\vec{a}$ & $\vec{b}$ 1 m

$$\vec{a} \cdot \vec{b} = \frac{1}{2}$$

Use $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$$\Rightarrow \frac{1}{2} = \frac{\frac{1}{2}}{|\vec{a}||\vec{b}|} = \frac{\frac{1}{2}}{|\vec{a}|^2}$$

$\Rightarrow \; |\vec{a}|^2 = 1$

$\Rightarrow \; |\vec{a}| = |\vec{b}| = 1$ 2 m

14. $\vec{r}_1 = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$

$\vec{r}_2 = 2\hat{i} + \hat{j} - \hat{k} + \mu\ \left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$

S.D. between $\vec{r}_1$ & $\vec{r}_2$ $= \left| \dfrac{\vec{b} - \vec{a} \cdot \vec{c} \times \vec{d}}{|\vec{c} \times \vec{d}|} \right|$ 1 m

$\left(\vec{b} - \vec{a}\right) \cdot \left(\vec{c} \times \vec{d}\right) = \begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$ 1 m

$= 10$

$\vec{c} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$

$= 3\hat{i} - \hat{j} - 7\hat{k}$

$|\vec{c} \times \vec{d}| = \sqrt{59}$ 1 m

Hence S.D. $= \left| \dfrac{10}{\sqrt{59}} \right| = \dfrac{10}{\sqrt{59}}$ units 1 m

15. $y = \cot^{-1}\left(\sqrt{\cos x}\right) - \tan^{-1}\left(\sqrt{\cos x}\right)$

$y = \dfrac{\pi}{2} - 2\tan^{-1}\left(\sqrt{\cos x}\right) \;\; \because \left(\cot^{-1}x + \tan^{-1}x = \dfrac{\pi}{2}\right)$ 1 m

or $y - \dfrac{\pi}{2} = -2\tan^{-1}\left(\sqrt{\cos x}\right)$

or $\frac{\pi}{2} - y = \cos^{-1}\left(\frac{1-\cos x}{1+\cos x}\right) \quad \left(\because\ 2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)$ 1 m

or $\cos\left(\frac{\pi}{2} - y\right) = \frac{2\sin^2 x/2}{2\cos^2 x/2}$ 1 m

or $\sin y = \tan^2\left(\frac{x}{2}\right)$ 1 m

Hence proved

OR

$$\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$$

$$\tan^{-1}\left(\frac{\frac{x+1}{x-1} + \frac{x-1}{x}}{1 - \left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)}\right) = \tan^{-1}(-7)$$ 1 m

or $\tan^{-1}\left(\frac{2x^2+1-x}{1-x}\right) = \tan^{-1}(-7)$ 1 m

or $2x^2 + 1 - x = -7(1-x)$ ½ m

or $2x^2 - 8x + 8 = 0$

or $(x-2)^2 = 0$

$\Rightarrow$ $x = 2$ 1 m

since $x = 2$ does not satisfy the given equation.

Hence no solution ½ m

16. $y = \left(3\cot^{-1}x\right)^2$

$y_1 = 2\left(3\cot^{-1}x\right)\left(\dfrac{-3}{1+x^2}\right)$

$= -18\,\dfrac{\cot^{-1}x}{1+x^2}$ 2 m

or $y_1\left(1+x^2\right) = -18\cot^{-1}x$

or $y_2\left(1+x^2\right) + 2\,xy_1 = \dfrac{18}{1+x^2}$ 1 m

or $y_2\left(1+x^2\right)^2 + 2\,x\left(1+x^2\right)y_1 = 18$ 1 m

OR

$f(x) = \;|\,x-3\,|, \quad x \in R$
$f(x) = \;x-3, \quad x \geq 3$
$\quad = -(x-3), \quad x < 3$

To show continuity

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^-} f(x) = f(3)$ 1 m

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3} x-3 = 0$

$\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3} -(x-3) = 0$

$f(3) = 3-3 = 0$

So f(x) is continuous at $x = 3$ 1 m

For derivability at $x = 3$ need to show that

R.H.D = LHD

In this case

$$\text{R.H.D}(3) = \lim_{h\to 0} \frac{h}{h} = 1$$

$$\text{L.H.D}(3) = \lim_{h\to 0} \frac{h}{-h} = -1$$ 1 m

So func is not differentiable at $x = 3$ 1 m

17. $y = \left(x + \frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

or $y = e^{x \log\left(x+\frac{1}{x}\right)} + e^{\left(1+\frac{1}{x}\right)\log x}$ 1 m

$$\frac{dy}{dx} = e^{x \log\left(x+\frac{1}{x}\right)} \left[\log\left(1+\frac{1}{x}\right) + \frac{x\left(1-\frac{1}{x^2}\right)}{1+\frac{1}{x}} \right]$$

$$+ \; e^{\left(1+\frac{1}{x}\right)\log x} \left[\left(\frac{-1}{x^2}\right)\log + \left(1+\frac{1}{x}\right)\left(\frac{1}{x}\right) \right]$$

$$= \left(x + \frac{1}{x}\right)^x \left[\log\left(x + \frac{1}{x}\right) + \frac{x^2-1}{x^2+1} \right]$$ 1½+1½ m

$$+ \; (x)^{\left(1+\frac{1}{x}\right)} \left[\frac{x^2+1-\log x}{x^2} \right]$$

18. $y = (x-2)^2$

$$\frac{dy}{dx} = 2(x-2)$$ 1 m

Let (x_1, y_1) be the point of contact

$$\left.\frac{dy}{dx}\right|_{(x_1, y_1)} = 2(x_1 - 2)$$

Slope of chord = $m = \frac{4-0}{4-2} = 2$

$2(x_1 - 2) = 2$

$\Rightarrow \quad x_1 = 3$

since (x_1, y_1) lies on curve $y = (x-2)^2$

So $\quad y_1 = (3-2)^2 = 1$

So point of contact is (3, 1) 2 m

Also, equation of tangent is

$y - 1 = 2(x - 3)$

or $\quad y - 2x + 5 = 0$ 1 m

19. $I = \int (6x+5)\sqrt{6+x-x^2}\,dx$

$6x + 5 = A(1-2x) + B$

$\Rightarrow \quad A = -3, \quad B = 8$ 1 m

So, $I = -3\int (1-2x)\sqrt{6+x-x^2}\,dx + 8\int\sqrt{6+x-x^2}\,dx$

$= -2\left(6+x-x^2\right)^{3/2} + 8\int\sqrt{\left(\frac{5}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2}\,dx$ 1 m

$= -2\left(6+x-x^2\right)^{3/2} + \frac{8}{4}\left((2x-1)\sqrt{6+x-x^2} + \frac{25}{2}\sin^{-1}\left(\frac{2x-1}{5}\right)\right)$ 1 m

$= -2\left(6+x-x^2\right)^{3/2} + 2\left((2x-1)\sqrt{6+x-x^2} + \frac{25}{2}\sin^{-1}\left(\frac{2x-1}{5}\right)\right) + c$ 1 m

SECTION - C

20. $f(x) = 3x + 2, \quad f: R \to R$

$g(x) = \frac{x}{x^2+1}, \quad g: R \to R$

(i) $\text{fog}(x) = f(g(x)), \quad \text{fog}: R \to R$

$$= f\left(\frac{x}{x^2+1}\right)$$

$$= 3\left(\frac{x}{x^2+1}\right)+2$$

$$= \frac{2x^2+3x+2}{x^2+1}$$ 2 m

(ii) $\text{fof}(x) = f(f(x)), \quad \text{fof}: R \to R$

$$= f(3x+2)$$

$$= 3(3x+2)+2$$

$$= 9x+8$$ 2 m

(iii) $\text{gog}(x) = g(g(x)), \quad \text{gog}: R \to R$

$$= g\left(\frac{x}{x^2+1}\right)$$

$$= \frac{\frac{x}{x^2+1}}{\left(\frac{x}{x^2+1}\right)^2+1} = \frac{x\left(x^2+1\right)}{3x^2+1+x^4}$$

$$= \frac{x\left(x^2+1\right)}{x^4+3x^2+1}$$ 2 m

OR

$f: A \times B \to B \times A$ s.t.

$f(a, b) = (b, a)$

To show f is one – one

Let (a, b,) & (c, d) be any arbitrany element in $A \times B$ s.t.

$a \neq c, \quad a, c \in A$

$b \neq d, \quad b, d \in B$

then $f(a, b) = (b, a)$

$f(c, d) = (d, c)$

$(b, a) \neq (d, c) \quad (\because\ b \neq d, a \neq c)$

$\Rightarrow\ f(a, b) \neq f(c, d)$

$\Rightarrow$ f is one – one .... (i) 2 m

f is onto

$\forall\ a \in A,\ b \in B,$

$(b, a) \in B \times A$

$\Rightarrow\ (a, b) \in A \times B$

So f is onto .... (ii) 1 m

Hence, from (i) & (ii)

f is bijective function 1 m

21. Area $= \ = \int_0^{ae} y\,dx$

$= 2\int_0^{ae} \frac{b}{a}\sqrt{a^2 - x^2}\,dx$ 1 m

$= \frac{2b}{a}\int_0^{ae} \sqrt{a^2 - x^2}\,dx$ 1 m

$= \frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^{ae}$ 2 m

$$= \frac{2b}{a}\left[\frac{ae}{2}\sqrt{a^2(1-e^2)} + \frac{a^2}{2}\sin^{-1}e\right] - 0$$ 1 m

$$= b\left[eb + a\sin^{-1}e\right]$$

or $b^2e + ab\sin^{-1}e$ 1 m

22. $$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$$

or $\frac{dy}{dx} = \frac{y}{x} + \sec\left(\frac{y}{x}\right)$ 1 m

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 m

$$x\frac{dv}{dx} = \sec v$$

$\cos v\, dv = \frac{dx}{x}$ 1 m

$$\Rightarrow \int \cos v\, dv = \int \frac{dx}{x}$$

or $\sin v = \log|x| + c$ 1 m

when $y = \frac{\pi}{4},\ x = 1$

$\frac{1}{\sqrt{2}} = \log 1 + c$ 1 m

$$\Rightarrow c = \frac{1}{\sqrt{2}}$$

Particular solution is $\sin\left(\frac{y}{x}\right) = \log|x| + \frac{1}{\sqrt{2}}$ 1 m

OR

$$\frac{dy}{dx} - y = \cos x$$

I.F. $= e^{\int -dx} = e^{-x}$ $\left(\text{Here } P = -1, Q = \cos x \text{ and equation is in form } \frac{dy}{dx} + Py = Q(x)\right)$ 1 m

So general solution is

$y \cdot e^{-x} = \int e^{-x} \cos x \, dx + c$ .... (i) 1 m

consider

$$I = \int e^{-x} \cos x \, dx = -\cos x \, e^{-x} + \int \left(-\sin x \cdot e^{-x}\right) dx$$

$= -\cos x \cdot e^{-x} - \left[-\sin x \cdot e^{-x} + \int \cos x \, e^{-x} \, dx\right]$ 2 m

$$2I = \left(-\cos x + \sin x\right) e^{-x} + c$$

$I = \left(\frac{\sin x - \cos x}{2}\right) e^{-x} + c$ .... (ii) 1 m

From (i) & (ii), general solution of given D.E. is

$y \cdot e^{-x} = \left(\frac{\sin x - \cos x}{2}\right) e^{-x} + c$ 1 m

or $2y = \sin x - \cos x + ce^{x}$

23. Given planes are

$$2x + 2y - 3z - 7 = 0$$

and $2x + 5y + 3z - 9 = 0$

Equation of plane passing through intersection of two given planes is

$(2x + 2y - 3z - 7) + k\,(2x + 5y + 3z - 9) = 0$ 1½ m

or $(2 + 2k)\,x + (2 + 5k)\,y + (-3 + 3k) = -7 - 9\,k = 0$ 1 m

This plane passes through point (2, 1, 3)

So $(2 + 2k)\,(2) + (2 + 5k)\,(1) + (-3 + 3k)\,(3) - 7 - 9k = 0$

$-10 + 9k = 0$ 2 m

or $k = \frac{10}{9}$

So equation of plane is

$$\left(2 + 2\left(\frac{10}{9}\right)\right)x + \left(2 + \frac{5(10)}{9}\right)y + \left(-3 + \frac{3(10)}{9}\right)z - 7 - \frac{9(10)}{9} = 0$$

$38x + 68y + 3k - 153 = 0$ 1 m

Hence vec. equ. of plane passing through the intersection of plane is

$\vec{r} \cdot \left(38\hat{i} + 68\hat{j} + 3\hat{k}\right) = 153$ ½ m

24. E_1 : Ball from bag I

E_2 : Ball from bag II 1 m

E_3 : Drawing black ball

$P(E_1) = P(E_2) = \frac{1}{2}$

$P(B/E_1) = \frac{4}{7}, \quad P(B/E_2) = \frac{6}{11}$ 2 m

Prob. of ball drawn found to be black, drawn from bag II

$$P(E_2/B) = \frac{P(E_2) \cdot P(B/E_2)}{P(E_1) \cdot P(B/E_1) + P(E_2) \cdot P(B/E_2)}$$ 1 m

$$= \frac{\frac{1}{2}\left(\frac{6}{11}\right)}{\frac{1}{2}\left(\frac{4}{7}\right) + \frac{1}{2}\left(\frac{6}{11}\right)} = \frac{21}{43}$$ +1 m

25.

	Returns	Investment
Bond A	10%	x
Bond B	15%	y

L.P.P. is

objective func. $z = \frac{10}{100}x + \frac{15}{100}y = 0.1x + 0.15y$ 2 m

Subject to

$x + y \leq 50{,}000$ 1 m

$x \geq 15{,}000$ 1 m

$y \leq 20{,}000$ 1 m

$x, y \geq 0$ +1 m

26. Let the two numbers be x and y

$x + y = 16$

$f(x) = x^3 + y^3$

$= x^3 + (16 - x)^3$ 1½ m

$f'(x) = 3x^2 + 3(16 - x)^2(-1)$

$= 96x - 768$ 1½ m

$f'(x) = 0 \Rightarrow x = 8$ 1 m

So x = 8 may be point of maximum or minimum

consider $f''(x) = 96 > 0$ 1 m

$\Rightarrow$ x = 8 is point of minima

when x = 8, y = 8

So 8 and 8 are numbers such that their sum is 16 and sum of their cubes is minimum. 1 m