Strictly Confidential — (For Internal and Restricted Use Only)

Senior School Certificate Examination

March — 2015

Marking Scheme — Mathematics 65/1/RU, 65/2/RU, 65/3/RU

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggestive answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 65/1/RU

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$ ½ m

$= 0$ ½ m

2. order 2, degree 1 (any one correct) ½ m

sum = 3 ½ m

3. $\frac{dx}{dy} + \frac{2y}{1+y^2} . x = \cot y$ ½ m

Integrating factor = $e^{\log(1+y^2)}$ or $(1+y^2)$ ½ m

4. $\left|2\hat{a}+\hat{b}+\hat{c}\right|^2 = (2\hat{a})^2 + (\hat{b})^2 + (\hat{c})^2 + 2\left(2\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot 2\hat{a}\right)$ ½ m

$\therefore \left|2\hat{a}+\hat{b}+\hat{c}\right| = \sqrt{6}$ ½ m

5. $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\hat{i}+\hat{j}$ ½ m

unit vector is $-\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$ ½ m

6. $\frac{x - 3/5}{1/5} = \frac{y + 7/15}{1/15} = \frac{z - 3/10}{-1/10}$ ½ m

Direction cosines are $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$ or $\frac{-6}{7}, \frac{-2}{7}, \frac{3}{7}$ ½ m

SECTION - B

7. $$\begin{pmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{pmatrix} \begin{pmatrix} 50 \\ 20 \\ 40 \end{pmatrix} = \begin{pmatrix} 30000 \\ 23000 \\ 39000 \end{pmatrix}$$ 2 m

cost incurred respectively for three villages is Rs. 30,000, Rs. 23,000, Rs. 39,000 1 m

One value : Women welfare or Any other relevant value 1 m

8. $$\tan^{-1}\left(\frac{x+1+x-1}{1-(x+1)(x-1)}\right) = \tan^{-1}\left(\frac{8}{31}\right)$$ 2 m

$$\Rightarrow \quad \frac{2x}{2-x^2} = \frac{8}{31} \qquad \therefore \quad 4x^2 + 31x - 8 = 0$$ 1 m

$$\therefore \quad x = \frac{1}{4}, \quad -8 \text{ (Rejected)}$$ 1 m

OR

$$\text{L.H.S.} = \tan^{-1}\left(\frac{x-y}{1+xy}\right) + \tan^{-1}\left(\frac{y-z}{1+yz}\right) + \tan^{-1}\left(\frac{z-x}{1+zx}\right)$$ 2 m

$$\left.\begin{aligned} &= \tan^{-1}x - \tan^{-1}y + \tan^{-1}y - \tan^{-1}z + \tan^{-1}z - \tan^{-1}x \\ &= 0 = \text{RHS} \end{aligned}\right\}$$ 2 m

9. $$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Taking a, b & c common from C_1, C_2 and C_3 1 m

$$= 2\,abc \begin{vmatrix} a+c & c & a+c \\ a+b & b & a \\ b+c & b+c & c \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$ and taking 2 common from C_1 1 m

$$= 2\,abc \begin{vmatrix} a+c & c & 0 \\ a+b & b & -b \\ b+c & b+c & -b \end{vmatrix} \quad C_3 \to C_3 - C_1$$ 1 m

$$= 2\,abc \begin{vmatrix} a+c & c & 0 \\ a-c & -c & 0 \\ b+c & b+c & -b \end{vmatrix} \quad R_2 \to R_2 - R_3$$ ½ m

Expand by C_3, $= 2\,abc\,(-b)\,(-ac - c^2 - ac + c^2) = 4a^2\,b^2\,c^2$ ½ m

10. $$\text{Adj } A = \begin{pmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{pmatrix}; \; |A| = 27$$ 2+1 m

$$A.\,\text{Adj } A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{pmatrix} = 27 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = |A|\, I_3$$ 1 m

11. $f(x) = |x-1| + |x+1|$

$$L\,f'(-1) = \lim_{x \to (-1)^-} \frac{\{-(x-1)-(x+1)\}-2}{x-(-1)} = \lim_{x \to (-1)^-} \frac{-2\,(x+1)}{x+1} = -2$$ 1 m

$$R\,f'(-1) = \lim_{x \to (-1)^+} \frac{\{-(x-1)+(x+1)\}-2}{x-(-1)} = \lim_{x \to (-1)^+} \frac{0}{x+1} = 0$$ 1 m

$-2 \neq 0 \;\therefore\; f(x)$ is not differentiable at $x = -1$

$$L\,f'(1) = \lim_{x \to 1^-} \frac{\{-(x-1)+(x+1)\}-2}{x-1} = \lim_{x \to 1^-} \frac{0}{x-1} = 0$$ 1 m

$$R\,f'(1) = \lim_{x \to 1^+} \frac{\{x-1+x+1\}-2}{x-1} = \lim_{x \to 1^+} \frac{2\,(x-1)}{x-1} = 2$$ 1 m

$0 \neq 2 \;\therefore\; f(x)$ is not differentiable at $x = 1$

12. $y = e^{m\sin^{-1}x}$, differentiate w.r.t. "x", we get $\frac{dy}{dx} = \frac{m\, e^{m\sin^{-1}x}}{\sqrt{1-x^2}}$ 1½ m

$\Rightarrow \sqrt{1-x^2}\ \frac{dy}{dx} = my$, Differentiate again w.r.t. "x"

$\Rightarrow \sqrt{1-x^2}\ \frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\ \frac{dy}{dx} = m\frac{dy}{dx}$ 1½ m

$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} - x\ \frac{dy}{dx} = m\left(\sqrt{1-x^2}\ \frac{dy}{dx}\right) = m\,(my)$ ½ m

$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} - x\ \frac{dy}{dx} - m^2y = 0$ ½ m

13. $f(x) = \sqrt{x^2+1},\ g(x) = \frac{x+1}{x^2+1},\ h(x) = 2x-3$

Differentiating w.r.t. "x", we get

$f'(x) = \frac{x}{\sqrt{x^2+1}},\ g'(x) = \frac{1-2x-x^2}{\left(x^2+1\right)^2},\ h'(x) = 2$ 1+1½+1 m

$\therefore\ f'\left(h'\left(g'(x)\right)\right) = \frac{2}{\sqrt{5}}$ ½ m

14. $\int(3-2x)\sqrt{2+x-x^2}\ dx = 2\int\sqrt{\left(\frac{3}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}\ dx + \int(1-2x)\sqrt{2+x-x^2}\ dx$ 2 m

$= 2\cdot\left\{\frac{x-\frac{1}{2}}{2}\sqrt{2+x-x^2}+\frac{9}{8}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{3}{2}}\right)\right\} + \frac{2}{3}\left(2+x-x^2\right)^{3/2}+c$ 2 m

or $\left(\frac{2x-1}{2}\sqrt{2+x-x^2}+\frac{9}{4}\sin^{-1}\left(\frac{2x-1}{3}\right)+\frac{2}{3}\left(2+x-x^2\right)^{3/2}+c\right)$

OR

$$\int \frac{x^2+x+1}{(x^2+1)(x+2)}\,dx = \frac{1}{5}\int \frac{2x+1}{x^2+1}\,dx + \frac{3}{5}\int \frac{1}{x+2}\,dx$$ 2 m

$$= \frac{1}{5}\int \frac{2x}{x^2+1}\,dx + \frac{1}{5}\int \frac{1}{x^2+1}\,dx + \frac{3}{5}\int \frac{1}{x+2}\,dx$$ ½ m

$$= \frac{1}{5}\log\left|x^2+1\right| + \frac{1}{5}\tan^{-1}x + \frac{3}{5}\log\left|x+2\right| + c$$ 1½ m

15. $$\int_0^{\pi/4} \frac{1}{\cos^3 x\sqrt{2\sin 2x}}\,dx = \int_0^{\pi/4} \frac{1}{\cos^4 x\, 2\sqrt{\tan x}}\,dx$$ 1 m

$$= \int_0^{\pi/4} \frac{(1+\tan^2 x)}{2\sqrt{\tan x}}\sec^2 x\,dx$$ 1 m

$$= \frac{1}{2}\int_0^1 \frac{1+t^2}{\sqrt{t}}\,dt$$ Taking, tan x = t; 1 m

$$= \frac{1}{2}\left[2\sqrt{t} + \frac{2}{5}t^{5/2}\right]_0^1$$ ½ m

$$= \frac{1}{2}\left[2+\frac{2}{5}\right] = \frac{6}{5}$$ ½ m

16. $$\int \log x \cdot \frac{1}{(x+1)^2}\,dx = \log x \cdot \frac{-1}{x+1} + \int \frac{1}{x}\cdot\frac{1}{x+1}\,dx$$ 2 m

$$= \frac{-\log x}{x+1} + \int \frac{1}{x}\,dx - \int \frac{1}{x+1}\,dx$$ 1 m

$$= \frac{-\log x}{x+1} + \log x - \log(x+1) + c$$ 1 m

or $$\frac{-\log x}{x+1} + \log\left(\frac{x}{x+1}\right) + c$$

17. $\vec{a}-\vec{b} = -\hat{i}+\hat{j}+\hat{k}$; $\vec{c}-\vec{b} = \hat{i}-5\hat{j}-5\hat{k}$ 1½ m

$$\left(\vec{a}-\vec{b}\right)\times\left(\vec{c}-\vec{b}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix} = -4\hat{j} + 4\hat{k}$$ 1½ m

$\therefore$ Unit vector perpendicular to both of the vectors $= -\dfrac{\hat{j}}{\sqrt{2}} + \dfrac{\hat{k}}{\sqrt{2}}$ 1 m

18. let the equation of line passing through $(1, 2, -4)$ be

$\vec{r} = \hat{i}+2\hat{j}-4\hat{k} + \lambda\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$ 1 m

Since the line is perpendicular to the two given lines $\therefore$

$\therefore$ $3a-16\,b+7\,c=0$

$3a+8\,b-5\,c=0$ 1½ m

Solving we get, $\dfrac{a}{24} = \dfrac{b}{36} = \dfrac{c}{72}$ or $\dfrac{a}{2} = \dfrac{b}{3} = \dfrac{c}{6}$ 1 m

$\therefore$ Equation of line is : $\vec{r} = \hat{i}+2\hat{j}-4\hat{k} + \lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$ ½ m

OR

Equation of plane is : $\begin{vmatrix} x+1 & y-2 & z \\ 2+1 & 2-2 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$ 3 m

Solving we get, $x+2y+3z-3=0$ 1 m

19. Let $x =$ No. of spades in three cards drawn

x	:	0	1	2	3	1 m
P(x)	:	${}^3C_0\left(\frac{3}{4}\right)^3 = \frac{27}{64}$	${}^3C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 = \frac{27}{64}$	${}^3C_2\left(\frac{1}{4}\right)^2\frac{3}{4} = \frac{9}{64}$	${}^3C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^0 = \frac{1}{64}$	2 m
x . P(x)	:	0	$\frac{27}{64}$	$\frac{18}{64}$	$\frac{3}{64}$	½ m

Mean $= \sum x\cdot P(x) = \dfrac{48}{64} = \dfrac{3}{4}$ ½ m

OR

let p = probability of success ; q = Probability of failure

then, $9\ P(x = 4) = P(x = 2)$

$\Rightarrow \quad 9 \cdot {}^6C_4\, p^4 \cdot q^2 = {}^6C_2 \cdot p^2 \cdot q^4$ 2 m

$\Rightarrow \quad 9p^2 = q^2 \quad \therefore \quad q = 3p$ 1 m

Also, $p + q = 1 \Rightarrow p + 3p = 1 \quad \therefore \quad p = \frac{1}{4}$ 1 m

SECTION - C

20. $f: R_+ \to [-9, \infty)\ ;\ f(x) = 5x^2 + 6x - 9\ ;\ f^{-1}(y) = \frac{\sqrt{54+5y} - 3}{5}$

$fof^{-1}(y) = 5\left\{\frac{\sqrt{54+5y} - 3}{5}\right\}^2 + 6\left\{\frac{\sqrt{54+5y} - 3}{5}\right\} - 9 = y$ 3 m

$f^{-1}of\ (x) = \frac{\sqrt{54 + 5\left(5x^2 + 6x - 9\right)} - 3}{5} = x$ 2½ m

Hence 'f' is invertible with $f^{-1}(y) = \frac{\sqrt{54+5y} - 3}{5}$ ½ m

OR

(i) commutative : let $x, y \in R - \{-1\}$ then

$x * y = x + y + xy = y + x + yx = y * x \quad \therefore$ * is commutative 1½ m

(ii) Associative : let $x, y, z \in R - \{-1\}$ then

$x * (y * z) = x * (y + z + yz) = x + (y + z + yz) + x\,(y + z + yz)$

$= x + y + z + xy + yz + zx + xyz$ 1½ m

$(x * y) * z = (x + y + xy) * z = (x + y + xy) + z + (x + y + xy) \cdot z$

$= x + y + z + xy + yz + zx + xyz$ 1 m

$x * (y * z) = (x * y) * z \quad \therefore$ * is Associative

(iii) Identity Element : let $e \in R - \{-1\}$ such that $a * e = e * a = a \ \forall \ a \in R - \{-1\}$ ½ m

$\therefore \quad a + e + ae = a \quad \Rightarrow \quad e = 0$ ½ m

(iv) Inverse : let $a * b = b * a = e = 0$; $a, b \in R - \{-1\}$ ½ m

$\Rightarrow \quad a + b + ab = 0 \quad \therefore \quad b = \frac{-a}{1+a} \quad \text{or} \quad a^{-1} = \frac{-a}{1+a}$ ½ m

21. Solving the two curves to get the points of intersection $(\pm 3\sqrt{p}, 8)$ 1½ m

m_1 = slope of tangent to first curve $= \frac{-2x}{9p}$ 1½ m

m_2 = slope of tangent to second curve $= \frac{2x}{p}$ 1½ m

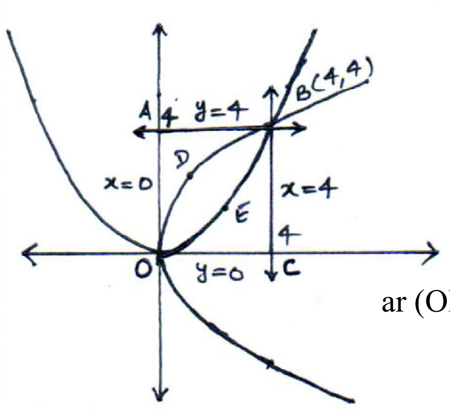
curves cut at right angle iff $\frac{-2x}{9p} \times \frac{2x}{p} = -1$ ½ m

$\Leftrightarrow \quad 9p^2 = 4x^2$ (Put $x = \pm 3\sqrt{p}$)

$\Leftrightarrow \quad 9p^2 = 4(9p)$

$\therefore \quad p = 0 \ ; \ p = 4$ 1 m

22.



correct figure 1½ m

$\text{ar (ABDOA)} = \frac{1}{4}\int_0^4 y^2 dy = \left.\frac{y^3}{12}\right]_0^4 = \frac{16}{3}$ ......(i) 1½ m

$\text{ar (OEBDO)} = \int_0^4 2\sqrt{x}\, dx - \int_0^4 \frac{x^2}{4} dx = \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$

$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$ ....(ii) 1½ m

$\text{ar (OEBCO)} = \frac{1}{4}\int_0^4 x^2 dx = \left.\frac{x^3}{12}\right]_0^4 = \frac{16}{3}$ ....(iii) 1½ m

From (i), (ii) and (iii) we get ar (ABDOA) = ar (OEBDO) = ar (OEBCO)

23. $\frac{dy}{dx} = \frac{y^2}{xy - x^2} \Rightarrow \frac{dy}{dx} = \frac{\left(y/x\right)^2}{\frac{y}{x} - 1}$, Hence the differential equation is homogeneous 1 m

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get $v + x\frac{dv}{dx} = \frac{v^2}{v-1}$ 1+1 m

$\therefore\ x\frac{dv}{dx} = \frac{v^2}{v-1} - v = \frac{v}{v-1}$ 1 m

$\int \frac{v-1}{v}\, dv = \int \frac{1}{x}\, dx \Rightarrow v - \log v = \log x + c$ 1 m

$\therefore\ \frac{y}{x} - \log\frac{y}{x} = \log x + c \quad \left(\text{or, } \frac{y}{x} = \log y + c\right)$ 1 m

OR

Given differential equation can be written as $\frac{dx}{dy} + \frac{1}{1+y^2}\, x = \frac{\tan^{-1}y}{1+y^2}$ 1 m

Integrating factor $= e^{\tan^{-1}y}$ and solution is : $x\, e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2}\, dy$ 1+1½ m

$x\, e^{\tan^{-1}y} = \int te^t\, dt = te^t - e^t + c = e^{\tan^{-1}y}\,(\tan^{-1}y - 1) + c$ (where $\tan^{-1}y = t$) 1½ m

$x = 1, y = 0 \Rightarrow c = 2 \ \therefore\ x \cdot e^{\tan^{-1}y} = e^{\tan^{-1}y}\,(\tan^{-1}y - 1) + 2$ 1 m

or $x = \tan^{-1}y - 1 + 2\, e^{-\tan^{-1}y}$

24. Equation of line through A and B is $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$ (say) 2 m

General point on the line is $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$ 1 m

If this is the point of intersection with plane $2x + y + z = 7$

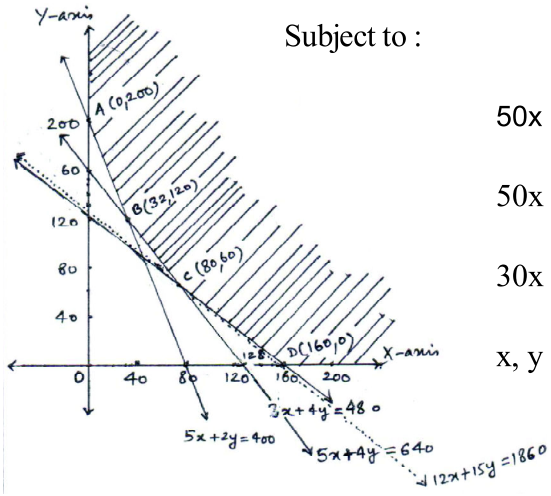
then, $2(-\lambda + 3) + \lambda - 4 + 6\lambda - 5 = 7 \Rightarrow \lambda = 2$ 1 m

$\therefore$ Point of intersection is $(1, -2, 7)$ 1 m

Required distance = $\sqrt{(3-1)^2 + (4+2)^2 + (4-7)^2} = 7$ 1 m

25. Let the two factories I and II be in operation for x and y days respectively to produce the order with the minimum cost then, the LPP is :

Minimise cost : $z = 12000\,x + 15000\,y$ 1 m



Subject to :

$50x + 40y \geq 6400$ or $5x + 4y \geq 640$

$50x + 20y \geq 4000$ or $5x + 2y \geq 400$ 2 m

$30x + 40y \geq 4800$ or $3x + 4y \geq 480$

$x, y \geq 0$

correct graph 2 m

Vertices are A (0, 200) ; B (32, 120)

C (80, 60) ; D (160, 0) ½ m

z (A) = Rs. 30,00,000; z (B) = Rs. 21,84,000;

z (C) = Rs. 18,60,000 (Min.); z (D) = Rs. 19,20,000;

On plotting $z < 1860000$

or $12x + 15y < 1860$, we get no point common to the feasible region

$\therefore$ Factory I operates for 80 days

Factory II operates for 60 days ½ m

26. E_1 : Bolt is manufactured by machine A

E_2 : Bolt is manufactured by machine B

E_3 : Bolt is manufactured by machine C

A : Bolt is defective

$$P(E_1) = \frac{30}{100};\ P(E_2) = \frac{50}{100};\ P(E_3) = \frac{20}{100};$$

$$P(A/E_1) = \frac{3}{100};\ P(A/E_2) = \frac{4}{100};\ P(A/E_3) = \frac{1}{100}$$ 3 m

$$P(E_2/A) = \frac{\frac{50}{100} \times \frac{4}{100}}{\frac{30}{100} \times \frac{3}{100} + \frac{50}{100} \times \frac{4}{100} + \frac{20}{100} \times \frac{1}{100}} = \frac{200}{90 + 200 + 20} = \frac{20}{31}$$ 2 m

$$P(\overline{E}_2/A) = 1 - P(E_2/A) = \frac{11}{31}$$ 1 m

QUESTION PAPER CODE 65/2/RU

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\left|2\hat{a}+\hat{b}+\hat{c}\right|^2 = (2\hat{a})^2+(\hat{b})^2+(\hat{c})^2+2\left(2\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot 2\hat{a}\right)$ ½ m

$\therefore \left|2\hat{a}+\hat{b}+\hat{c}\right| = \sqrt{6}$ ½ m

2. $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & 1 & 0\end{vmatrix} = -\hat{i}+\hat{j}$ ½ m

unit vector is $-\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{\sqrt{2}}$ ½ m

3. $\frac{x-3/5}{1/5} = \frac{y+7/15}{1/15} = \frac{z-3/10}{-1/10}$ ½ m

Direction cosines are $\frac{6}{7},\frac{2}{7},\frac{-3}{7}$ or $\frac{-6}{7},\frac{-2}{7},\frac{3}{7}$ ½ m

4. $\Delta = \begin{vmatrix}x+y+z & x+y+z & x+y+z\\ z & x & y\\ -3 & -3 & -3\end{vmatrix}$ ½ m

$= 0$ ½ m

5. order 2, degree 1 (any one correct) ½ m

sum = 3 ½ m

6. $\frac{dx}{dy}+\frac{2y}{1+y^2}.x = \cot y$ ½ m

Integrating factor = $e^{\log\left(1+y^2\right)}$ or $\left(1+y^2\right)$ ½ m

SECTION - B

7. $y = e^{m\sin^{-1}x}$, differentiate w.r.t. "x", we get $\frac{dy}{dx} = \frac{m\, e^{m\sin^{-1}x}}{\sqrt{1-x^2}}$ 1½ m

$\Rightarrow \sqrt{1-x^2}\ \frac{dy}{dx} = my$, Differentiate again w.r.t. "x"

$\Rightarrow \sqrt{1-x^2}\ \frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\ \frac{dy}{dx} = m\ \frac{dy}{dx}$ 1½ m

$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} - x\ \frac{dy}{dx} = m\left(\sqrt{1-x^2}\ \frac{dy}{dx}\right) = m\,(my)$ ½ m

$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} - x\ \frac{dy}{dx} - m^2y = 0$ ½ m

8. $f(x) = \sqrt{x^2+1}$, $g(x) = \frac{x+1}{x^2+1}$, $h(x) = 2x-3$

Differentiating w.r.t. "x", we get

$f'(x) = \frac{x}{\sqrt{x^2+1}}$, $g'(x) = \frac{1-2x-x^2}{\left(x^2+1\right)^2}$, $h'(x) = 2$ 1+1½+1 m

$\therefore\ f'\left(h'\left(g'(x)\right)\right) = \frac{2}{\sqrt{5}}$ ½ m

9. $\int (3-2x)\sqrt{2+x-x^2}\ dx = 2\int\sqrt{\left(\frac{3}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2}\ dx + \int (1-2x)\sqrt{2+x-x^2}\ dx$ 2 m

$= 2\cdot\left\{\frac{x-\frac{1}{2}}{2}\sqrt{2+x-x^2} + \frac{9}{8}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{3}{2}}\right)\right\} + \frac{2}{3}\left(2+x-x^2\right)^{3/2} + c$ 2 m

or $\left(\frac{2x-1}{2}\sqrt{2+x-x^2} + \frac{9}{4}\sin^{-1}\left(\frac{2x-1}{3}\right) + \frac{2}{3}\left(2+x-x^2\right)^{3/2} + c\right)$

OR

$$\int \frac{x^2+x+1}{(x^2+1)(x+2)}\,dx = \frac{1}{5}\int \frac{2x+1}{x^2+1}\,dx + \frac{3}{5}\int \frac{1}{x+2}\,dx$$ 2 m

$$= \frac{1}{5}\int \frac{2x}{x^2+1}\,dx + \frac{1}{5}\int \frac{1}{x^2+1}\,dx + \frac{3}{5}\int \frac{1}{x+2}\,dx$$ ½ m

$$= \frac{1}{5}\log\left|x^2+1\right| + \frac{1}{5}\tan^{-1}x + \frac{3}{5}\log\left|x+2\right| + c$$ 1½ m

10. $$\begin{pmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{pmatrix}\begin{pmatrix} 50 \\ 20 \\ 40 \end{pmatrix} = \begin{pmatrix} 30000 \\ 23000 \\ 39000 \end{pmatrix}$$ 2 m

cost incurred respectively for three villages is Rs. 30,000, Rs. 23,000, Rs. 39,000 1 m

One value : Women welfare or Any other relevant value 1 m

11. $$\tan^{-1}\left(\frac{x+1+x-1}{1-(x+1)(x-1)}\right) = \tan^{-1}\left(\frac{8}{31}\right)$$ 2 m

$$\Rightarrow \quad \frac{2x}{2-x^2} = \frac{8}{31} \qquad \therefore \quad 4x^2+31x-8=0$$ 1 m

$$\therefore \quad x = \frac{1}{4}, \quad -8 \text{ (Rejected)}$$ 1 m

OR

$$\text{L.H.S.} = \tan^{-1}\left(\frac{x-y}{1+xy}\right) + \tan^{-1}\left(\frac{y-z}{1+yz}\right) + \tan^{-1}\left(\frac{z-x}{1+zx}\right)$$ 2 m

$$\left.\begin{aligned} &= \tan^{-1}x - \tan^{-1}y + \tan^{-1}y - \tan^{-1}z + \tan^{-1}z - \tan^{-1}x \\ &= 0 = \text{RHS} \end{aligned}\right\}$$ 2 m

12. $$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Taking a, b & c common from C_1, C_2 and C_3 1 m

$$= 2\,abc \begin{vmatrix} a+c & c & a+c \\ a+b & b & a \\ b+c & b+c & c \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$ and taking 2 common from C_1 1 m

$$= 2\,abc \begin{vmatrix} a+c & c & 0 \\ a+b & b & -b \\ b+c & b+c & -b \end{vmatrix} \quad C_3 \to C_3 - C_1$$ 1 m

$$= 2\,abc \begin{vmatrix} a+c & c & 0 \\ a-c & -c & 0 \\ b+c & b+c & -b \end{vmatrix} \quad R_2 \to R_2 - R_3$$ ½ m

Expand by C_3, $= 2\,abc\,(-b)\,(-ac - c^2 - ac + c^2) = 4a^2\,b^2\,c^2$ ½ m

13. $$\text{Adj } A = \begin{pmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{pmatrix};\ |A| = 27$$ 2+1 m

$$A.\ \text{Adj } A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{pmatrix} = 27 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = |A|\ I_3$$ 1 m

14. $f(x) = |x-1| + |x+1|$

$$L\,f'(-1) = \lim_{x \to (-1)^-} \frac{\{-(x-1)-(x+1)\}-2}{x-(-1)} = \lim_{x \to (-1)^-} \frac{-2\,(x+1)}{x+1} = -2$$ 1 m

$$R\ f'(-1) = \lim_{x \to (-1)^+} \frac{\{-(x-1)+(x+1)\}-2}{x-(-1)} = \lim_{x \to (-1)^+} \frac{0}{x+1} = 0 \qquad 1\text{ m}$$

$-2 \neq 0 \quad \therefore$ f (x) is not differentiable at $x = -1$

$$L\ f'(1) = \lim_{x \to 1^-} \frac{\{-(x-1)+(x+1)\}-2}{x-1} = \lim_{x \to 1^-} \frac{0}{x-1} = 0 \qquad 1\text{ m}$$

$$R\ f'(1) = \lim_{x \to 1^+} \frac{\{x-1+x+1\}-2}{x-1} = \lim_{x \to 1^+} \frac{2(x-1)}{x-1} = 2 \qquad 1\text{ m}$$

$0 \neq 2 \quad \therefore$ f (x) is not differentiable at $x = 1$

15. let the equation of line passing through $(1, 2, -4)$ be

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(a\hat{i} + b\hat{j} + c\hat{k}\right)$ 1 m

Since the line is perpendicular to the two given lines $\therefore$

$\therefore \quad 3a - 16\,b + 7\,c = 0$

$3a + 8\,b - 5\,c = 0$ 1½ m

Solving we get, $\frac{a}{24} = \frac{b}{36} = \frac{c}{72}$ or $\frac{a}{2} = \frac{b}{3} = \frac{c}{6}$ 1 m

$\therefore$ Equation of line is : $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$ ½ m

OR

Equation of plane is : $\begin{vmatrix} x+1 & y-2 & z \\ 2+1 & 2-2 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$ 3 m

Solving we get, $x + 2y + 3z - 3 = 0$ 1 m

16. Let x = No. of spades in three cards drawn

x :	0	1	2	3	1 m
P(x) :	${}^3C_0 \left(\frac{3}{4}\right)^3 = \frac{27}{64}$	${}^3C_1 \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 = \frac{27}{64}$	${}^3C_2 \left(\frac{1}{4}\right)^2 \frac{3}{4} = \frac{9}{64}$	${}^3C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^0 = \frac{1}{64}$	2 m
x . P(x) :	0	$\frac{27}{64}$	$\frac{18}{64}$	$\frac{3}{64}$	½ m

Mean $= \sum x \cdot P(x) = \frac{48}{64} = \frac{3}{4}$ ½ m

OR

let p = probability of success ; q = Probability of failure

then, $9\,P(x=4) = P(x=2)$

$\Rightarrow \quad 9 \cdot {}^6C_4\, p^4 \cdot q^2 = {}^6C_2 \cdot p^2 \cdot q^4$ 2 m

$\Rightarrow \quad 9p^2 = q^2 \quad \therefore \quad q = 3p$ 1 m

Also, $p + q = 1 \Rightarrow p + 3p = 1 \quad \therefore \quad p = \frac{1}{4}$ 1 m

17. $\int_0^{\pi/4} \frac{1}{\cos^3 x \sqrt{2 \sin 2x}}\, dx = \int_0^{\pi/4} \frac{1}{\cos^4 x\, 2\sqrt{\tan x}}\, dx$ 1 m

$= \int_0^{\pi/4} \frac{(1 + \tan^2 x)}{2\sqrt{\tan x}} \sec^2 x\, dx$ 1 m

$= \frac{1}{2} \int_0^1 \frac{1 + t^2}{\sqrt{t}}\, dt$ Taking, $\tan x = t$; 1 m

$= \frac{1}{2} \left[2\sqrt{t} + \frac{2}{5} t^{5/2} \right]_0^1$ ½ m

$= \frac{1}{2} \left[2 + \frac{2}{5} \right] = \frac{6}{5}$ ½ m

18. $\int \log x \cdot \frac{1}{(x+1)^2}\, dx = \log x \cdot \frac{-1}{x+1} + \int \frac{1}{x} \cdot \frac{1}{x+1}\, dx$ 2 m

$= \frac{-\log x}{x+1} + \int \frac{1}{x}\, dx - \int \frac{1}{x+1}\, dx$ 1 m

$$= \frac{-\log x}{x+1} + \log x - \log(x+1) + c$$ 1 m

$$\text{or} \quad \frac{-\log x}{x+1} + \log\left(\frac{x}{x+1}\right) + c$$

19. $\vec{a} - \vec{b} = -\hat{i} + \hat{j} + \hat{k}\ ;\ \vec{c} - \vec{b} = \hat{i} - 5\hat{j} - 5\hat{k}$ 1½ m

$$\left(\vec{a} - \vec{b}\right) \times \left(\vec{c} - \vec{b}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix} = -4\hat{j} + 4\hat{k}$$ 1½ m

$\therefore$ Unit vector perpendicular to both of the vectors $= -\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}$ 1 m

SECTION - C

20. $\frac{dy}{dx} = \frac{y^2}{xy - x^2} \Rightarrow \frac{dy}{dx} = \frac{\left(y/x\right)^2}{\frac{y}{x} - 1}$, Hence the differential equation is homogeneous 1 m

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get $v + x\frac{dv}{dx} = \frac{v^2}{v-1}$ 1+1 m

$$\therefore\ x\frac{dv}{dx} = \frac{v^2}{v-1} - v = \frac{v}{v-1}$$ 1 m

$$\int \frac{v-1}{v}\,dv = \int \frac{1}{x}\,dx \Rightarrow v - \log v = \log x + c$$ 1 m

$$\therefore\ \frac{y}{x} - \log\frac{y}{x} = \log x + c \quad \left(\text{or, } \frac{y}{x} = \log y + c\right)$$ 1 m

OR

Given differential equation can be written as $\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{\tan^{-1}y}{1+y^2}$ 1 m

Integrating factor $= e^{\tan^{-1}y}$ and solution is : $x\,e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2}\,dy$ 1+1½ m

$x\,e^{\tan^{-1}y} = \int te^t\,dt = te^t - e^t + c = e^{\tan^{-1}y}(\tan^{-1}y - 1) + c$ (where $\tan^{-1}y = t$) 1½ m

$x = 1, y = 0 \Rightarrow c = 2 \therefore x \cdot e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + 2$ 1 m

or $x = \tan^{-1}y - 1 + 2\,e^{-\tan^{-1}y}$

21. Equation of line through A and B is $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$ (say) 2 m

General point on the line is $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$ 1 m

If this is the point of intersection with plane $2x + y + z = 7$

then, $2(-\lambda + 3) + \lambda - 4 + 6\lambda - 5 = 7 \Rightarrow \lambda = 2$ 1 m

$\therefore$ Point of intersection is $(1, -2, 7)$ 1 m

Required distance $= \sqrt{(3-1)^2 + (4+2)^2 + (4-7)^2} = 7$ 1 m

22. $f: R_+ \to [-9, \infty)$; $f(x) = 5x^2 + 6x - 9$; $f^{-1}(y) = \frac{\sqrt{54+5y} - 3}{5}$

$fof^{-1}(y) = 5\left\{\frac{\sqrt{54+5y} - 3}{5}\right\}^2 + 6\left\{\frac{\sqrt{54+5y} - 3}{5}\right\} - 9 = y$ 3 m

$f^{-1}o\,f(x) = \frac{\sqrt{54 + 5(5x^2 + 6x - 9)} - 3}{5} = x$ 2½ m

Hence 'f' is invertible with $f^{-1}(y) = \frac{\sqrt{54+5y} - 3}{5}$ ½ m

OR

(i) commutative : let $x, y \in R - \{-1\}$ then

$x * y = x + y + xy = y + x + yx = y * x \quad \therefore \; *$ is commutative 1½ m

(ii) Associative : let $x, y, z \in R - \{-1\}$ then

$x * (y * z) = x * (y + z + yz) = x + (y + z + yz) + x\,(y + z + yz)$

$= x + y + z + xy + yz + zx + xyz$ 1½ m

$(x * y) * z = (x + y + xy) * z = (x + y + xy) + z + (x + y + xy) \cdot z$

$= x + y + z + xy + yz + zx + xyz$ 1 m

$x * (y * z) = (x * y) * z \quad \therefore \; *$ is Associative

(iii) Identity Element : let $e \in R - \{-1\}$ such that $a * e = e * a = a \; \forall \; a \in R - \{-1\}$ ½ m

$\therefore \quad a + e + ae = a \quad \Rightarrow \quad e = 0$ ½ m

(iv) Inverse : let $a * b = b * a = e = 0 \,;\; a, b \in R - \{-1\}$ ½ m

$\Rightarrow \quad a + b + ab = 0 \quad \therefore \quad b = \dfrac{-a}{1+a} \quad \text{or} \quad a^{-1} = \dfrac{-a}{1+a}$ ½ m

23. Solving the two curves to get the points of intersection $\left(\pm 3\sqrt{p}, 8\right)$ 1½ m

m_1 = slope of tangent to first curve $= \dfrac{-2x}{9p}$ 1½ m

m_2 = slope of tangent to second curve $= \dfrac{2x}{p}$ 1½ m

curves cut at right angle iff $\dfrac{-2x}{9p} \times \dfrac{2x}{p} = -1$ ½ m

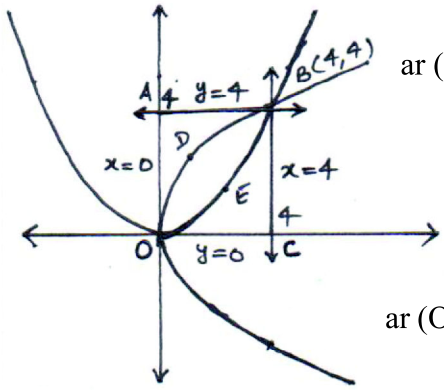
$\Leftrightarrow \quad 9p^2 = 4x^2 \; (\text{Put } x = \pm 3\sqrt{p})$

$\Leftrightarrow \quad 9p^2 = 4\,(9\,p)$

$\therefore \quad p = 0 \,;\; p = 4$ 1 m

24. correct figure 1½ m

ar (ABDOA) $= \dfrac{1}{4} \displaystyle\int_0^4 y^2 dy = \left.\dfrac{y^3}{12}\right]_0^4 = \dfrac{16}{3}$ ......(i) 1½ m



$$\text{ar (OEBDO)} = \int_0^4 2\sqrt{x}\, dx - \int_0^4 \frac{x^2}{4} dx = \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \quad \text{.........(ii)} \qquad 1\tfrac{1}{2}\text{ m}$$

$$\text{ar (OEBCO)} = \frac{1}{4}\int_0^4 x^2\, dx = \left.\frac{x^3}{12}\right]_0^4 = \frac{16}{3} \quad \text{................(iii)} \qquad 1\tfrac{1}{2}\text{ m}$$

From (i), (ii) and (iii) we get ar (ABDOA) = ar (OEBDO) = ar (OEBCO)

25. E_1 : Bolt is manufactured by machine A

E_2 : Bolt is manufactured by machine B

E_3 : Bolt is manufactured by machine C

A : Bolt is defective

$$P(E_1) = \frac{30}{100}; P(E_2) = \frac{50}{100}; P(E_3) = \frac{20}{100};$$

$$P(A/E_1) = \frac{3}{100}; P(A/E_2) = \frac{4}{100}; P(A/E_3) = \frac{1}{100} \qquad 3\text{ m}$$

$$P(E_2/A) = \frac{\frac{50}{100}\times\frac{4}{100}}{\frac{30}{100}\times\frac{3}{100}+\frac{50}{100}\times\frac{4}{100}+\frac{20}{100}\times\frac{1}{100}} = \frac{200}{90+200+20} = \frac{20}{31} \qquad 2\text{ m}$$

$$P(\overline{E}_2/A) = 1 - P(E_2/A) = \frac{11}{31} \qquad 1\text{ m}$$

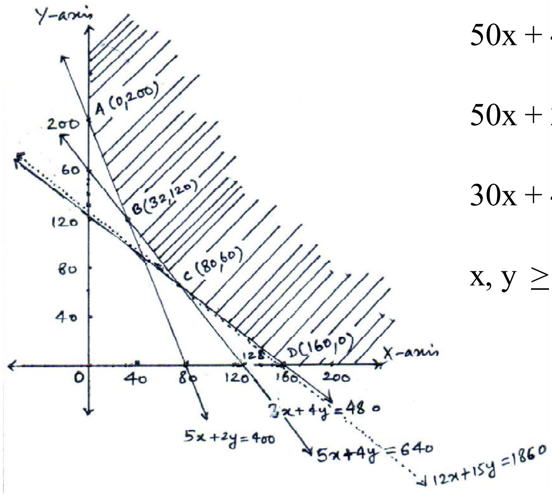
26. Let the two factories I and II be in operation for x and y

days respectively to produce the order with the minimum cost

then, the LPP is :

Minimise cost : z = 12000 x + 15000 y 1 m

Subject to :



$50x + 40y \geq 6400$ or $5x + 4y \geq 640$

$50x + 20y \geq 4000$ or $5x + 2y \geq 400$ 2 m

$30x + 40y \geq 4800$ or $3x + 4y \geq 480$

$x, y \geq 0$

correct graph 2 m

Vertices are A (0, 200) ; B (32, 120)

C (80, 60) ; D (160, 0) ½ m

z (A) = Rs. 30,00,000; z (B) = Rs. 21,84,000;

z (C) = Rs. 18,60,000 (Min.); z (D) = Rs. 19,20,000;

On plotting $z < 1860000$

or $12x + 15y < 1860$, we get no

point common to the feasible region

$\therefore$ Factory I operates for 80 days

Factory II operates for 60 days ½ m

QUESTION PAPER CODE 65/3/RU

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1.	$\frac{dx}{dy} + \frac{2y}{1+y^2} . x = \cot y$	½ m
	Integrating factor = $e^{\log(1+y^2)}$ or $(1+y^2)$	½ m
2.	$\Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$	½ m
	$= 0$	½ m
3.	order 2, degree 1 (any one correct)	½ m
	sum = 3	½ m
4.	$\frac{x - 3/5}{1/5} = \frac{y + 7/15}{1/15} = \frac{z - 3/10}{-1/10}$	½ m
	Direction cosines are $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$ or $\frac{-6}{7}, \frac{-2}{7}, \frac{3}{7}$	½ m
5.	$\left|2\hat{a} + \hat{b} + \hat{c}\right|^2 = (2\hat{a})^2 + (\hat{b})^2 + (\hat{c})^2 + 2\left(2\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot 2\hat{a}\right)$	½ m
	$\therefore \left|2\hat{a} + \hat{b} + \hat{c}\right| = \sqrt{6}$	½ m
6.	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\hat{i} + \hat{j}$	½ m
	unit vector is $-\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$	½ m

SECTION - B

7. $\vec{a}-\vec{b} = -\hat{i}+\hat{j}+\hat{k}$; $\vec{c}-\vec{b} = \hat{i}-5\hat{j}-5\hat{k}$ 1½ m

$$\left(\vec{a}-\vec{b}\right)\times\left(\vec{c}-\vec{b}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix} = -4\hat{j} + 4\hat{k}$$ 1½ m

$\therefore$ Unit vector perpendicular to both of the vectors $= -\dfrac{\hat{j}}{\sqrt{2}} + \dfrac{\hat{k}}{\sqrt{2}}$ 1 m

8. let the equation of line passing through $(1, 2, -4)$ be

$\vec{r} = \hat{i}+2\hat{j}-4\hat{k} + \lambda\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$ 1 m

Since the line is perpendicular to the two given lines $\therefore$

$\therefore$ $3a - 16b + 7c = 0$

$3a + 8b - 5c = 0$ 1½ m

Solving we get, $\dfrac{a}{24} = \dfrac{b}{36} = \dfrac{c}{72}$ or $\dfrac{a}{2} = \dfrac{b}{3} = \dfrac{c}{6}$ 1 m

$\therefore$ Equation of line is : $\vec{r} = \hat{i}+2\hat{j}-4\hat{k} + \lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$ ½ m

OR

Equation of plane is : $\begin{vmatrix} x+1 & y-2 & z \\ 2+1 & 2-2 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$ 3 m

Solving we get, $x + 2y + 3z - 3 = 0$ 1 m

9. Let x = No. of spades in three cards drawn

x :	0	1	2	3	1 m
P(x) :	$^3C_0\left(\frac{3}{4}\right)^3 = \frac{27}{64}$	$^3C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 = \frac{27}{64}$	$^3C_2\left(\frac{1}{4}\right)^2\frac{3}{4} = \frac{9}{64}$	$^3C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^0 = \frac{1}{64}$	2 m
x . P(x) :	0	$\frac{27}{64}$	$\frac{18}{64}$	$\frac{3}{64}$	½ m

$$\text{Mean} = \sum x \cdot P(x) = \frac{48}{64} = \frac{3}{4}$$ ½ m

OR

let p = probability of success ; q = Probability of failure

then, $9\,P(x=4) = P(x=2)$

$\Rightarrow \quad 9 \cdot {}^{6}C_{4}\, p^{4} \cdot q^{2} = {}^{6}C_{2} \cdot p^{2} \cdot q^{4}$ 2 m

$\Rightarrow \quad 9p^{2} = q^{2} \quad \therefore \quad q = 3p$ 1 m

Also, $p + q = 1 \Rightarrow p + 3p = 1 \quad \therefore \quad p = \frac{1}{4}$ 1 m

10. $y = e^{m\sin^{-1}x}$, differentiate w.r.t. "x", we get $\frac{dy}{dx} = \frac{m\,e^{m\sin^{-1}x}}{\sqrt{1-x^{2}}}$ 1½ m

$\Rightarrow \quad \sqrt{1-x^{2}}\,\frac{dy}{dx} = my$, Differentiate again w.r.t. "x"

$\Rightarrow \quad \sqrt{1-x^{2}}\,\frac{d^{2}y}{dx^{2}} - \frac{x}{\sqrt{1-x^{2}}}\,\frac{dy}{dx} = m\,\frac{dy}{dx}$ 1½ m

$\Rightarrow \quad (1-x^{2})\,\frac{d^{2}y}{dx^{2}} - x\,\frac{dy}{dx} = m\left(\sqrt{1-x^{2}}\,\frac{dy}{dx}\right) = m\,(my)$ ½ m

$\Rightarrow \quad (1-x^{2})\,\frac{d^{2}y}{dx^{2}} - x\,\frac{dy}{dx} - m^{2}y = 0$ ½ m

11. $f(x) = \sqrt{x^{2}+1}, \; g(x) = \frac{x+1}{x^{2}+1}, \; h(x) = 2x-3$

Differentiating w.r.t. "x", we get

$f'(x) = \frac{x}{\sqrt{x^{2}+1}}, \; g'(x) = \frac{1-2x-x^{2}}{(x^{2}+1)^{2}}, \; h'(x) = 2$ 1+1½+1 m

$\therefore \quad f'(h'(g'(x))) = \frac{2}{\sqrt{5}}$ ½ m

12. $\int (3-2x)\sqrt{2+x-x^2}\,dx = 2\int \sqrt{\left(\frac{3}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2}\,dx + \int (1-2x)\sqrt{2+x-x^2}\,dx$ 2 m

$= 2\cdot\left\{ \frac{x-\frac{1}{2}}{2}\sqrt{2+x-x^2} + \frac{9}{8}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{3}{2}}\right) \right\} + \frac{2}{3}\left(2+x-x^2\right)^{3/2} + c$ 2 m

or $\left(\frac{2x-1}{2}\sqrt{2+x-x^2} + \frac{9}{4}\sin^{-1}\left(\frac{2x-1}{3}\right) + \frac{2}{3}\left(2+x-x^2\right)^{3/2} + c\right)$

OR

$\int \frac{x^2+x+1}{\left(x^2+1\right)\left(x+2\right)}\,dx = \frac{1}{5}\int \frac{2x+1}{x^2+1}\,dx + \frac{3}{5}\int \frac{1}{x+2}\,dx$ 2 m

$= \frac{1}{5}\int \frac{2x}{x^2+1}\,dx + \frac{1}{5}\int \frac{1}{x^2+1}\,dx + \frac{3}{5}\int \frac{1}{x+2}\,dx$ ½ m

$= \frac{1}{5}\log\left|x^2+1\right| + \frac{1}{5}\tan^{-1}x + \frac{3}{5}\log\left|x+2\right| + c$ 1½ m

13. $\int_0^{\pi/4} \frac{1}{\cos^3 x\sqrt{2\sin 2x}}\,dx = \int_0^{\pi/4} \frac{1}{\cos^4 x\, 2\sqrt{\tan x}}\,dx$ 1 m

$= \int_0^{\pi/4} \frac{\left(1+\tan^2 x\right)}{2\sqrt{\tan x}}\sec^2 x\,dx$ 1 m

$= \frac{1}{2}\int_0^1 \frac{1+t^2}{\sqrt{t}}\,dt$ Taking, tan x = t; 1 m

$= \frac{1}{2}\left[2\sqrt{t} + \frac{2}{5}t^{5/2}\right]_0^1$ ½ m

$= \frac{1}{2}\left[2+\frac{2}{5}\right] = \frac{6}{5}$ ½ m

14. $\int \log x \cdot \frac{1}{(x+1)^2}\, dx = \log x \cdot \frac{-1}{x+1} + \int \frac{1}{x} \cdot \frac{1}{x+1}\, dx$ 2 m

$= \frac{-\log x}{x+1} + \int \frac{1}{x}\, dx - \int \frac{1}{x+1}\, dx$ 1 m

$= \frac{-\log x}{x+1} + \log x - \log(x+1) + c$ 1 m

or $\frac{-\log x}{x+1} + \log\left(\frac{x}{x+1}\right) + c$

15. $\begin{pmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{pmatrix} \begin{pmatrix} 50 \\ 20 \\ 40 \end{pmatrix} = \begin{pmatrix} 30000 \\ 23000 \\ 39000 \end{pmatrix}$ 2 m

cost incurred respectively for three villages is Rs. 30,000, Rs. 23,000, Rs. 39,000 1 m

One value : Women welfare or Any other relevant value 1 m

16. $\tan^{-1}\left(\frac{x+1+x-1}{1-(x+1)(x-1)}\right) = \tan^{-1}\left(\frac{8}{31}\right)$ 2 m

$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31} \quad \therefore \quad 4x^2 + 31x - 8 = 0$ 1 m

$\therefore \quad x = \frac{1}{4}, \quad -8$ (Rejected) 1 m

OR

L.H.S. = $\tan^{-1}\left(\frac{x-y}{1+xy}\right) + \tan^{-1}\left(\frac{y-z}{1+yz}\right) + \tan^{-1}\left(\frac{z-x}{1+zx}\right)$ 2 m

$= \tan^{-1}x - \tan^{-1}y + \tan^{-1}y - \tan^{-1}z + \tan^{-1}z - \tan^{-1}x$

$= 0 =$ RHS 2 m

17. $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

Taking a, b & c common from C_1, C_2 and C_3 1 m

$$= 2\,abc \begin{vmatrix} a+c & c & a+c \\ a+b & b & a \\ b+c & b+c & c \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$ and taking 2 common from C_1 1 m

$$= 2\,abc \begin{vmatrix} a+c & c & 0 \\ a+b & b & -b \\ b+c & b+c & -b \end{vmatrix} \quad C_3 \to C_3 - C_1$$ 1 m

$$= 2\,abc \begin{vmatrix} a+c & c & 0 \\ a-c & -c & 0 \\ b+c & b+c & -b \end{vmatrix} \quad R_2 \to R_2 - R_3$$ ½ m

Expand by C_3, $= 2\,abc\,(-b)\,(-ac - c^2 - ac + c^2) = 4a^2\,b^2\,c^2$ ½ m

18. $\text{Adj } A = \begin{pmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{pmatrix}$; $|A| = 27$ 2+1 m

$A.\,\text{Adj } A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{pmatrix} = 27 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = |A|\, I_3$ 1 m

19. $f(x) = |x-1| + |x+1|$

$L\,f'(-1) = \lim\limits_{x \to (-1)^-} \dfrac{\{-(x-1)-(x+1)\}-2}{x-(-1)} = \lim\limits_{x \to (-1)^-} \dfrac{-2\,(x+1)}{x+1} = -2$ 1 m

$$R\ f'(-1) = \lim_{x \to (-1)^+} \frac{\{-(x-1)+(x+1)\}-2}{x-(-1)} = \lim_{x \to (-1)^+} \frac{0}{x+1} = 0$$ 1 m

$-2 \neq 0 \quad \therefore\ f(x)$ is not differentiable at $x = -1$

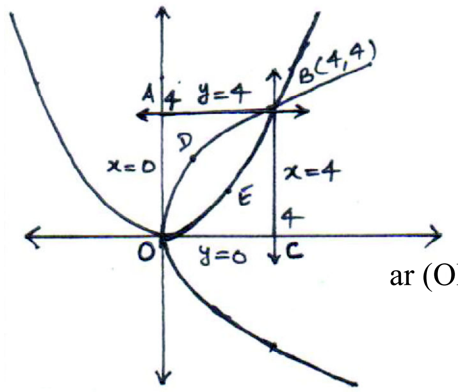
$$L\ f'(1) = \lim_{x \to 1^-} \frac{\{-(x-1)+(x+1)\}-2}{x-1} = \lim_{x \to 1^-} \frac{0}{x-1} = 0$$ 1 m

$$R\ f'(1) = \lim_{x \to 1^+} \frac{\{x-1+x+1\}-2}{x-1} = \lim_{x \to 1^+} \frac{2(x-1)}{x-1} = 2$$ 1 m

$0 \neq 2 \quad \therefore\ f(x)$ is not differentiable at $x = 1$

SECTION - C

20.



correct figure 1½ m

$$\text{ar (ABDOA)} = \frac{1}{4}\int_0^4 y^2 dy = \left.\frac{y^3}{12}\right]_0^4 = \frac{16}{3} \text{......(i)}$$ 1½ m

$$\text{ar (OEBDO)} = \int_0^4 2\sqrt{x}\, dx - \int_0^4 \frac{x^2}{4} dx = \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{.........(ii)}$$ 1½ m

$$\text{ar (OEBCO)} = \frac{1}{4}\int_0^4 x^2\, dx = \left.\frac{x^3}{12}\right]_0^4 = \frac{16}{3} \text{................(iii)}$$ 1½ m

From (i), (ii) and (iii) we get ar (ABDOA) = ar (OEBDO) = ar (OEBCO)

21. $\dfrac{dy}{dx} = \dfrac{y^2}{xy - x^2} \Rightarrow \dfrac{dy}{dx} = \dfrac{\left(y/x\right)^2}{\dfrac{y}{x} - 1}$, Hence the differential equation is homogeneous 1 m

Put $y = vx$ and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$, we get $v + x\dfrac{dv}{dx} = \dfrac{v^2}{v-1}$ 1+1 m

$$\therefore\ x\ \frac{dv}{dx} = \frac{v^2}{v-1} - v = \frac{v}{v-1}$$ 1 m

$$\int \frac{v-1}{v}\,dv = \int \frac{1}{x}\,dx \ \Rightarrow\ v - \log v = \log x + c$$ 1 m

$$\therefore\ \frac{y}{x} - \log\frac{y}{x} = \log x + c \ \left(\text{or, } \frac{y}{x} = \log y + c\right)$$ 1 m

OR

Given differential equation can be written as $\frac{dx}{dy} + \frac{1}{1+y^2}\,x = \frac{\tan^{-1}y}{1+y^2}$ 1 m

Integrating factor $= e^{\tan^{-1}y}$ and solution is : $x\,e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2}\,dy$ 1+1½ m

$x\,e^{\tan^{-1}y} = \int te^t\,dt = te^t - e^t + c = e^{\tan^{-1}y}(\tan^{-1}y - 1) + c$ (where $\tan^{-1}y = t$) 1½ m

$x = 1, y = 0 \Rightarrow c = 2 \ \therefore\ x \cdot e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + 2$ 1 m

or $x = \tan^{-1}y - 1 + 2\,e^{-\tan^{-1}y}$

22. E_1 : Bolt is manufactured by machine A

E_2 : Bolt is manufactured by machine B

E_3 : Bolt is manufactured by machine C

A : Bolt is defective

$$P(E_1) = \frac{30}{100};\ P(E_2) = \frac{50}{100};\ P(E_3) = \frac{20}{100};$$

$$P(A/E_1) = \frac{3}{100};\ P(A/E_2) = \frac{4}{100};\ P(A/E_3) = \frac{1}{100}$$ 3 m

$$P(E_2/A) = \frac{\frac{50}{100}\times\frac{4}{100}}{\frac{30}{100}\times\frac{3}{100}+\frac{50}{100}\times\frac{4}{100}+\frac{20}{100}\times\frac{1}{100}} = \frac{200}{90+200+20} = \frac{20}{31}$$ 2 m

$$P(\overline{E}_2/A) = 1 - P(E_2/A) = \frac{11}{31}$$ 1 m

23. Equation of line through A and B is $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$ (say) 2 m

General point on the line is $(-\lambda+3, \lambda-4, 6\lambda-5)$ 1 m

If this is the point of intersection with plane $2x + y + z = 7$

then, $2(-\lambda+3)+\lambda-4+6\lambda-5=7 \Rightarrow \lambda=2$ 1 m

$\therefore$ Point of intersection is $(1, -2, 7)$ 1 m

Required distance = $\sqrt{(3-1)^2+(4+2)^2+(4-7)^2} = 7$ 1 m

24. Let the two factories I and II be in operation for x and y days respectively to produce the order with the minimum cost

then, the LPP is :

Minimise cost : $z = 12000x + 15000y$ 1 m

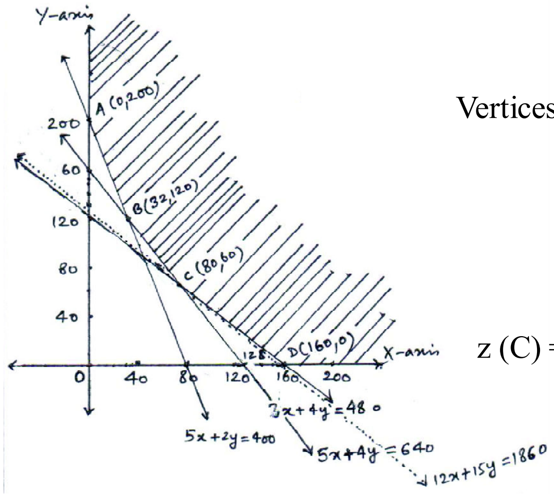
Subject to :

$50x + 40y \geq 6400$ or $5x + 4y \geq 640$

$50x + 20y \geq 4000$ or $5x + 2y \geq 400$ 2 m

$30x + 40y \geq 4800$ or $3x + 4y \geq 480$

$x, y \geq 0$



correct graph 2 m

Vertices are A (0, 200) ; B (32, 120)

C (80, 60) ; D (160, 0) ½ m

z (A) = Rs. 30,00,000; z (B) = Rs. 21,84,000;

z (C) = Rs. 18,60,000 (Min.); z (D) = Rs. 19,20,000;

On plotting $z < 1860000$

or $12x + 15y < 1860$, we get no

point common to the feasible region

$\therefore$ Factory I operates for 80 days

Factory II operates for 60 days ½ m

25. $f: R_+ \to [-9, \infty)$; $f(x) = 5x^2 + 6x - 9$; $f^{-1}(y) = \dfrac{\sqrt{54+5y} - 3}{5}$

$$fof^{-1}(y) = 5\left\{\frac{\sqrt{54+5y}-3}{5}\right\}^2 + 6\left\{\frac{\sqrt{54+5y}-3}{5}\right\} - 9 = y$$ 3 m

$$f^{-1}of(x) = \frac{\sqrt{54+5(5x^2+6x-9)}-3}{5} = x$$ 2½ m

Hence 'f' is invertible with $f^{-1}(y) = \dfrac{\sqrt{54+5y} - 3}{5}$ ½ m

OR

(i) commutative : let $x, y \in R - \{-1\}$ then

$x * y = x + y + xy = y + x + yx = y * x \;\; \therefore \; *$ is commutative 1½ m

(ii) Associative : let $x, y, z \in R - \{-1\}$ then

$x * (y * z) = x * (y + z + yz) = x + (y + z + yz) + x (y + z + yz)$

$= x + y + z + xy + yz + zx + xyz$ 1½ m

$(x * y) * z = (x + y + xy) * z = (x + y + xy) + z + (x + y + xy) \cdot z$

$= x + y + z + xy + yz + zx + xyz$ 1 m

$x * (y * z) = (x * y) * z \; \therefore \; *$ is Associative

(iii) Identity Element : let $e \in R - \{-1\}$ such that $a * e = e * a = a \; \forall \; a \in R - \{-1\}$ ½ m

$\therefore \; a + e + ae = a \Rightarrow e = 0$ ½ m

(iv) Inverse : let $a * b = b * a = e = 0$; $a, b \in R - \{-1\}$ ½ m

$\Rightarrow \quad a + b + ab = 0 \quad \therefore \quad b = \frac{-a}{1+a} \quad$ or $\quad a^{-1} = \frac{-a}{1+a}$ ½ m

26. Solving the two curves to get the points of intersection $(\pm 3\sqrt{p}, 8)$ 1½ m

m_1 = slope of tangent to first curve $= \frac{-2x}{9p}$ 1½ m

m_2 = slope of tangent to second curve $= \frac{2x}{p}$ 1½ m

curves cut at right angle iff $\frac{-2x}{9p} \times \frac{2x}{p} = -1$ ½ m

$\Leftrightarrow \; 9p^2 = 4x^2$ (Put $x = \pm 3\sqrt{p}$)

$\Leftrightarrow \; 9p^2 = 4\,(9\,p)$

$\therefore \; p = 0 \; ; \; p = 4$ 1 m