Secondary School Examination

March -2010

Marking Scheme--- Mathematics (Delhi) 30/1/1, 30/1/2, 30/1/3

General Instructions

1. The Marking Scheme provides general guidelines to reduce subjectivity and maintain uniformity among large number of examiners involved in the marking. The answers given in the marking scheme are the best suggested answers.

2. Marking is to be done as per the instructions provided in the marking scheme. (It should not be done according to one's own interpretation or any other consideration.)Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. Some of the questions may relate to higher order thinking ability. These questions will be indicated to you separately by a star mark. These questions are to be evaluated carefully and the students' understanding / analytical ability may be judged.

5. The Head-Examiners have to go through the first five answer-scripts evaluated by each evaluator to ensure that the evaluation has been done as per instructions given in the marking scheme. The remaining answer scripts meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

6. If a question is attempted twice and the candidate has not crossed any answer, only first attempt is to be evaluated. Write 'EXTRA' with second attempt.

7. A full scale of marks 0 to 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.

8. Separate Marking Scheme for all the three sets has been given.

QUESTION PAPER CODE 30/1/1

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. Terminating	2. $x^2 - 6x + 4$	3. $d = 2a$	
4. $1:9$	5. 5cm	6. $\frac{1}{3}$	$1 \times 10 = 10$m
7. $p = 3$	8. (3.5)	9. 48cm^2	
10. $\frac{3}{26}$			

SECTION - B

11. $p(x) = x^3 - 4x^2 - 3x + 12$

$\sqrt{3}$ and $-\sqrt{3}$ are zeroes of $p(x)$ $\Rightarrow$ (x^2-3) is a factor of $p(x)$ ½ m

$$\left(x^3 - 3x - 4x^2 + 12\right) \div \left(x^2 - 3\right) = x - 4$$ 1m

$\Rightarrow$ $x = 4$ is the third zero of $p(x)$ ½ m

12. For a pair of linear equations to have infinitely

many solutions : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ½ m

$$\therefore \overset{\text{(i)}}{\frac{2}{k-1}} = \overset{\text{(ii)}}{\frac{3}{k+2}} = \overset{\text{(iii)}}{\frac{7}{3k}}$$

From (i) and (ii) getting $k = 7$ 1 m

$k = 7$ satisfies (ii) and (iii) and (i) and (iii) also ½ m

$\therefore$ $k = 7$

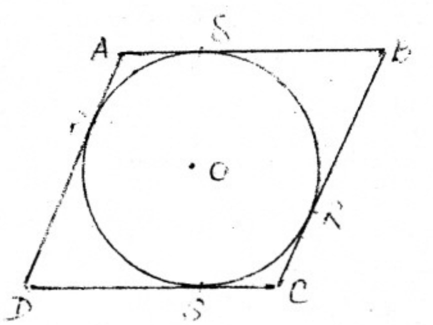
13. Here $a = 2$, $\ell = 29$ and $s_n = 155$ 1 m

$$\therefore\ 155 = \frac{n}{2}\,[2+29] \Rightarrow n = 10$$

Also, $29 = 2 + (10-1)\,d \Rightarrow d = 3$ 1 m

$\therefore$ Common difference = 3

14.



As parallelogram ABCD circumscribes a circle with centre O

$\therefore AB + CD = BC + AD$

$[\because AQ = AP, BQ = BR, CR = CS, PD = DS]$ ½ m

As ABCD is a parallelogram $\Rightarrow AB = DC$

and $BC = AD$

$2AB = 2AD$ or $AB = AD$ 1 m

$\therefore$ ABCD is a rhombus (As $AB = BC = CD = AD$) ½ m

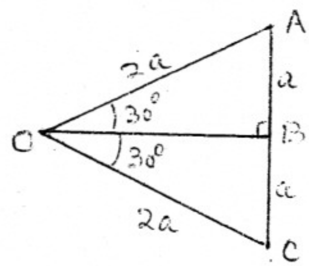
15. $\sec(90^\circ - \theta) = \operatorname{cosec}\theta$, $\tan(90^\circ - \theta) = \cot\theta$, $\cos 65^\circ = \sin 25$ 1 m

and $\tan 63^\circ = \cot 27^\circ$

$\therefore$ Given expression becomes

$$\frac{(\operatorname{cosec}^2\theta - \cot^2\theta) + (\cos^2 25^\circ + \sin^2 25^\circ)}{3\tan 27^\circ \cot 27^\circ} = \frac{1+1}{3} = \frac{2}{3}$$ 1 m

OR



correct Fig. ½ m

Draw rt. Δ OBA, in which $\angle BOA = 30^\circ$

Take $OA = 2a$. Replicate Δ OCB on ½ m

the other side of OB $\Rightarrow \angle AOC = 60^\circ$

and $OC = 2a$

$\therefore\ \Delta$ AOC is equilateral Δ and $AB = a$

$$\operatorname{cosec} 30^\circ = \frac{OA}{AB} = \frac{2a}{a} = 2$$

$$\therefore\ \operatorname{cosec} 30^\circ = 2$$

1m

SECTION -C

16. Let $2-3\sqrt{5} = x$, where x is a rational number ½ m

$\therefore\ 2-x = 3\sqrt{5}$ or $\frac{2-x}{3} = \sqrt{5}$ ....(i) ½ m

As x is a rational number, so is $\frac{2-x}{3}$ 1 m

$\therefore$ LHS of (i) is rational but RHS of (i) is irrational

$\therefore$ Our supposition that x is rational is wrong

$\Rightarrow\ 2-3\sqrt{5}$ is irrational 1 m

17. Let the fraction be $\frac{x}{y},\ y \neq 0$

$\therefore$ According to the question $x+y=2y-3$ 1 m

or $x=y-3$....................................(i)

Also, $\frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2x-y=1$..(ii) 1 m

From (i) and (ii), $x=4, y=7$

$\therefore$ Required fraction $=\frac{4}{7}$ 1 m

OR

$\frac{4}{x}+3y=8$ .... (i), $\frac{6}{x}-4y=-5$ ....(ii)

(i) $\times$ 3 $\Rightarrow$ $\frac{12}{x}+9y=24$ and (ii) $\times$ 2 $\Rightarrow$ $\frac{12}{x}-8y= -10$ 1½ m

Solving to get $y=2$ and $x=2$ 1½ m

18. $S_{10} = -150$ and $S_{20} = -(150+550) = -700$ ½ m

$\therefore\ -150 = 5(2a+9d)$ and $-700 = 10(2a+19d)$ 1m

$\Rightarrow\ 2a + 9d = -30$ and $2a + 19d = -70$

$\Rightarrow\ d = -4$ and $a = 3$ 1m

$\therefore$ A . P is $3, -1, -5, \ldots\ldots\ldots\ldots$ ½ m

19. $AB^2 = AC^2 + BC^2$ ....(i) and $AD^2 = AC^2 + DC^2$

$= AC^2 + \frac{BC^2}{4} \left(\because\ DC = \frac{1}{2} BC\right)$ 1+½ m

or $4\,AD^2 = 4\,AC^2 + BC^2$ ....(ii) ½ m

From (i) and (ii), $AB^2 = AC^2 + 4\,AD^2 - 4\,AC^2$

or $AB^2 = 4\,AD^2 - 3\,AC^2$ 1 m

20. The given expression can be written as

$$\frac{\frac{\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}} + \frac{1}{\frac{\sin A}{\cos A}\left(1 - \frac{\sin A}{\cos A}\right)}$$ 1 m

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$$ ½ m

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A(\sin A - \cos A)} = \frac{\sin^2 A + \cos^2 A + \sin A \cos A}{\sin A \cos A}$$ 1 m

$$= \tan A + \cot A + 1$$ ½ m

OR

$$\text{LHS} = \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$ ½ m

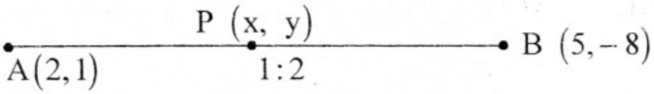
$$= \frac{1-\sin^2 A}{\sin A} \cdot \frac{1-\cos^2 A}{\cos A} = \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} = \sin A \cos A$$ 1½ m

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \frac{1}{\dfrac{\sin^2 A}{\sin A \cos A} + \dfrac{\cos^2 A}{\sin A \cos A}} = \frac{1}{\tan A + \cot A} = \text{RHS}$$ 1 m

21. Correct construction of triangle Δ ABC — 1 m

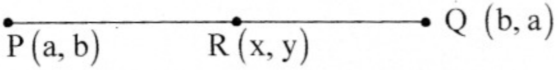
Correct construction of triangle similar to Δ ABC — 2m

22. $\dfrac{AP}{AB} = \dfrac{1}{3} \Rightarrow \dfrac{AP}{PB} = \dfrac{1}{2}$ — ½ m



$x = \dfrac{4+5}{3} = 3,\quad y = \dfrac{-8+2}{3} = -2 \Rightarrow P(3, -2)$ — 1+½ =1½ m

P lies on $2x - y + k = 0 \Rightarrow 6 + 2 + k = 0 \Rightarrow k = -8$ — 1 m

23. 

If P, R, Q are collinear, ar (Δ PRQ) = 0 — ½ m

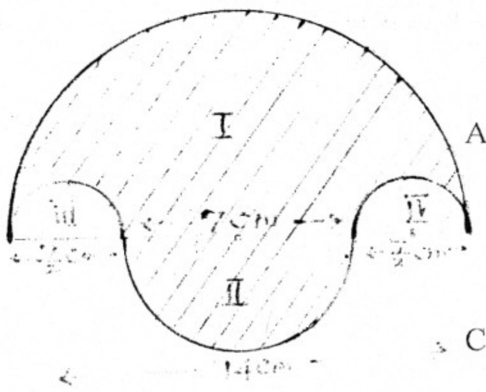
$\therefore\ a(y-a) + x(a-b) + b(b-y) = 0$ — 1 m

or $ay - a^2 + ax - bx + b^2 - by = 0$

or, $(a-b)(x+y) = a^2 - b^2$ — 1 m

$\Rightarrow x + y = a + b$ — ½ m

24.



Area of semi-circle I $= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7\text{cm}$

$= 77 \text{ cm}^2$ ½ m

Area of semi-circle II $= \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$

$= \frac{77}{4} \text{ cm}^2$ ½ m

$\therefore$ Combined area of semi-circles III and IV

$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \text{ cm}^2 = \frac{77}{8} \text{ cm}^2$ 1m

$\therefore$ Area of shaded region $= 77\left[\frac{1}{4} - \frac{1}{8} + 1\right] \text{ cm}^2$

$= \frac{693}{8} \text{ cm}^2 \text{ or } 86.625 \text{ cm}^2$ 1m

OR

$AB^2 = AC^2 + BC^2$ (as $\angle ACB = 90^\circ$)

$= 24^2 + 10^2 = 26^2 \quad \Rightarrow \quad AB = 26\text{cm}$ ½ m

$\therefore$ Area of semi-circle ACBOA $= \left(\frac{1}{2} \times 3.14 \times 13 \times 13\right) \text{cm}^2$ 1m

$\therefore$ Area of Δ ACB $= \left(\frac{1}{2} \times 24 \times 10\right) \text{cm} = 120 \text{ cm}^2$ ½ m

$\therefore$ Area of shaded region $= \left[\left(\frac{1}{2} \times 3.14 \times 13 \times 13\right) - 120\right] \text{ cm}^2$

$= (265.33 - 120) \text{ or } 145.33 \text{ cm}^2$ 1m

25. Total number of cards = 18 ½ m

(i) Prime numbers less than 15 are 3, 5, 7, 11, 13 – Five in number ½ m

$\therefore$ P (a prime no. less than 15) $= \frac{5}{18}$ 1m

(ii) numbers divisible by 3 and 5 is only 15 (one in number) ½ m

$\therefore$ P (a no. divisible by 3 and 5) $= \frac{1}{18}$ ½ m

SECTION D

26. Let the three consecutive numbers be $x, x+1, x+2$ 1 m

According to the question

$$x^2 + (x+1)(x+2) = 46$$

or $2x^2 + 3x - 44 = 0 \Rightarrow 2x^2 + 11x - 8x - 44 = 0$ 2 m

$\Rightarrow (x-4)(2x+11) = 0$ 1 m

As x is positive $\Rightarrow x = 4 \left(x = \frac{-11}{2} \text{ rejected} \right)$ 1 m

$\therefore$ The numbers are 4, 5, 6 1 m

OR

Let the two numbers be x, y where $x > y$

$\left.\begin{array}{l} \therefore \ x^2 - y^2 = 88 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(i) \\ \text{Also, } x = 2y - 5 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii) \end{array}\right\}$ 2 m

From (i) and (ii), $(2y-5)^2 - y^2 = 88$

$\Rightarrow 3y^2 - 20y - 63 = 0$ 1 m

or $3y^2 - 27y + 7y - 63 = 0 \Rightarrow (3y+7)(y-9) = 0$

$\Rightarrow y = 9, -7/3$ 1 m

$x = 2y - 5 = 13$ or $x = -\frac{29}{3}$ 1 m

$\therefore$ The numbers are 13, 9 (Rejecting $x = \frac{-29}{3}$

and $y = -7/3$) 1 m

27. Correctly stated Given, To Prove, Construction and correct Figure $\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)$ 2 m

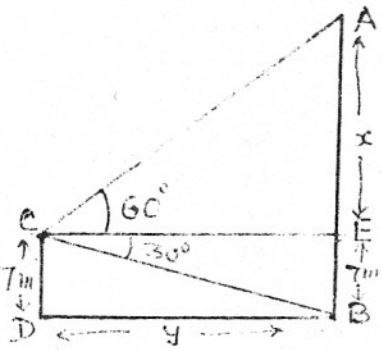
Correct Proof 2 m

Let ABC and PQR be two similar triangles ½ m

$$\frac{\text{ar }(ABC)}{\text{ar }(PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} = 1 \Rightarrow \begin{array}{l} AB = PQ, \\ BC = QR, AC = PR \end{array}$$ 1½ m

$\therefore \Delta ABC \cong \Delta PQR$ (by SSS) ½ m

28. Let CD be the building and AB, the tower



Correct Figure 1m

Writing trigonometric equations

(i) $\frac{7}{y} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow y = 7\sqrt{3}$

$= 12.124$ m 1½ m

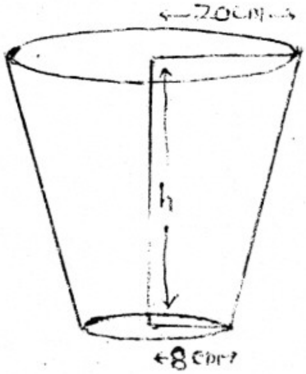
(ii) $\frac{x}{y} = \tan 60^\circ = \sqrt{3}$ 1½ m

or $\frac{x}{7\sqrt{3}} = \sqrt{3} \Rightarrow x = 21$ 1 m

$\therefore$ Height of tower $= (21 + 7)$ m or 28 m 1 m

29. Volume of bucket $= 10459\frac{3}{7}$ cm^3

$= \frac{73216}{7}$ cm^3



$$\therefore \quad \frac{73216}{7} = \frac{1}{3} \times \frac{22}{7} \times h \left[20^2 + 8^2 + 20 \times 8\right] \quad 1m$$

$$\therefore \ h = \frac{73216}{7} \times \frac{21}{22} \times \frac{1}{624} = 16 \text{ cm} \quad 1m$$

$$\ell^2 = h^2 + \left(r_2 - r_1\right)^2 = 16^2 + (20-8)^2$$

$$= 400$$

$$\Rightarrow \ell = 20 \text{ cm} \quad 1m$$

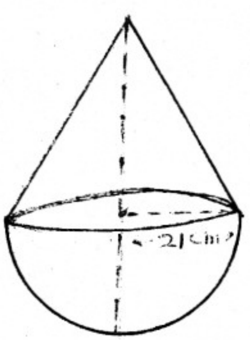
Total surface area of metal sheet used

$$\left.\begin{aligned} &= \frac{22}{7} \times 20 \times (20+8) + \frac{22}{7}(8)^2 \text{ cm}^2 \\ &= \left(1760 + \frac{1408}{7}\right) \text{ cm}^2 \end{aligned}\right\} \quad 2\text{ m}$$

$$\therefore \ \text{Cost of metal sheet} = \text{Rs}\left(1760 + \frac{1408}{7}\right)\frac{14}{10}$$

$$= \text{Rs } 2745.60 \quad 1\text{ m}$$

OR



$$\text{Volume of hemisphere} = \frac{2}{3} \times \frac{22}{7} \times (21)^3 \text{ cm}^3 \quad 1\text{ m}$$

$$\therefore \quad \text{Volume of cone} = \left(\frac{2}{3} \cdot \frac{2}{3} \ \frac{22}{7} \times 21 \times 21 \times 21\right) \text{ cm}^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times (21)^2 \times h \quad 1\tfrac{1}{2}\text{ m}$$

$$\Rightarrow h = \frac{2 \times 2 \times \cancel{22} \times 7 \times \cancel{21}}{\cancel{22} \times \cancel{21}} = 28 \text{ cm} \quad 1\text{ m}$$

$$\therefore \quad \ell^2 = h^2 + r^2 = 28^2 + 21^2 = 1225 = (35)^2$$

$$\Rightarrow \ell = 35 \text{ cm} \quad 1\text{ m}$$

Surface area of toy = $2\pi r^2 + \pi r \ell$

$$= \left(2 \times \frac{22}{7} \times 21 \times 21 + \frac{22}{7} \times 21 \times 35\right) \text{ cm}^2$$ 1½ m

$$= 5082 \text{ cm}^2$$

30.

Classes	0–10	10–20	20–30	30–40	40–50	50–60	60–70
class marks (x_i)	5	15	25	35	45	55	65
$d_i = \frac{x_i - 35}{10}$	–3	–2	–1	0	1	2	3
f_i	4	4	7	10	12	8	5
$f_i d_i$	–12	–8	–7	0	12	16	15

$\sum f_i = 50, \ \sum f_i d_i = 16$ 1 m

(i) $\bar{x} = 35 + \frac{16}{50} \times 10 = 38.2$ 1 m

(ii) Modal Class = 40 – 50 ½ m

Mode $= 40 + \frac{12-10}{24-18} \times 10 = 43.33$ 1½ m

(iii) Median Class = 30 – 40 ½ m

Median $= 30 + \frac{\frac{50}{2} - 15}{10} \times 10 = 40.00$ 1½ m

Note : If a candidate finds any two of the measures of central tendency correctly and finds the third correctly using Empirical formula, give full credit

QUESTION PAPER CODE 30/1/2

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1.	1 : 9	2.	p = 3	3 .	(3, 5)	
4.	48 cm^2	5.	$\frac{3}{26}$	6.	$\frac{1}{2}$	1 × 10 = 10 m
7.	5 cm	8.	2a	9 .	$x^2 - 6x + 4$	
10.	Terminating					

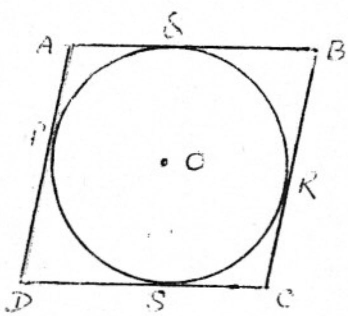
SECTION - B

11. As $\sqrt{3}$ and $-\sqrt{3}$ are zeroes of p(x) $\Rightarrow$ (x^2–3) is a factor of p(x) ½ m

$$\left(x^3 - 5x - 3x + 15\right) \div \left(x^2 - 3\right) = x - 5$$ 1m

$\therefore$ 5 is the third zero of the polynomial ½ m

12.



As parallelogram ABCD circumscribes a circle

with centre O

$\therefore$ AB + CD = BC + AD

[$\because$ AQ = AP, BQ = BR, CR = CS, PD = DS] ½ m

As ABCD is a parallelogram $\Rightarrow$ AB = DC

and BC = AD

2AB = 2AD or AB = AD 1 m

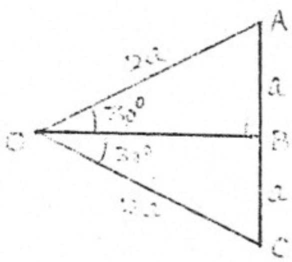
$\therefore$ ABCD is a rhombus (As AB = BC = CD = AD) ½ m

13. $\sec(90^\circ - \theta) = \operatorname{cosec}\theta$, $\tan(90^\circ - \theta) = \cot\theta$, $\cos 65^\circ = \sin 25^\circ$ and $\tan 63^\circ = \cot 27^\circ$ 1 m

$\therefore$ Given expression becomes

$$\frac{(\operatorname{cosec}^2\theta - \cot^2\theta) + (\cos^2 25^\circ + \sin^2 25^\circ)}{3\tan 27^\circ \cot 27^\circ} = \frac{1+1}{3} = \frac{2}{3}$$ 1 m

OR



correct Fig. ½ m

Draw rt. Δ OBA, in which $\angle BOA = 30^\circ$. Take OA = 2a. Replicate Δ OCB on the other side of OB $\Rightarrow$ $\angle AOC = 60^\circ$ ½ m

and OC = 2a

$\therefore$ Δ AOC is equilateral Δ and AB = a

$$\operatorname{cosec} 30^\circ = \frac{OA}{AB} = \frac{2a}{a} = 2$$ 1m

$\therefore$ $\operatorname{cosec} 30^\circ = 2$

14. Here a = 2, $\ell = 29$ and $s_n = 155$ 1 m

$\therefore$ $155 = \frac{n}{2}[2+29] \Rightarrow n = 10$

Also, $29 = 2 + (10-1)\,d \Rightarrow d = 3$ 1 m

$\therefore$ Common difference = 3

15. For a pair of linear equations to have infinitely many solutions : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ½ m

$$\therefore \underset{\text{(i)}}{\frac{2}{k-1}} = \underset{\text{(ii)}}{\frac{3}{k+2}} = \underset{\text{(iii)}}{\frac{7}{3k}}$$ 1 m

From (i) and (ii) getting k = 7

k = 7 satisfies (ii) and (iii) and (i) and (iii) also ½ m

$\therefore \quad k = 7$

SECTION -C

16. Let $4 - 5\sqrt{2} = x$, where x is a rational number

$\therefore \; 5\sqrt{2} = 4 - x \;\; \Rightarrow \sqrt{2} = \dfrac{4-x}{5}$ ....(i) 1 m

As x is a rational number, so is $\dfrac{4-x}{5}$ 1 m

$\therefore$ From (i) $\sqrt{2}$ is also rational which is a contradiction

$\therefore$ x is not rational $\Rightarrow 4 - 5\sqrt{2}$ is irrational 1 m

17. Total number of cards = 18 ½ m

(i) Prime numbers less than 15 are 3, 5, 7, 11, 13 – Five in number ½ m

$\therefore$ P (a prime no. less than 15) $= \dfrac{5}{18}$ 1m

(ii) numbers divisible by 3 and 5 is only 15 (one in number) ½ m

$\therefore$ P (a no. divisible by 3 and 5) $= \dfrac{1}{18}$ ½ m

18. $AB^2 = AC^2 + BC^2$ ....(i) and $AD^2 = AC^2 + DC^2$

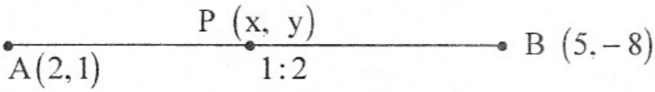
$$= AC^2 + \frac{BC^2}{4} \left(\because DC = \frac{1}{2} BC\right)$$ 1+½ m

or $4\,AD^2 = 4\,AC^2 + BC^2$ .... (ii) ½ m

From (i) and (ii), $AB^2 = AC^2 + 4\,AD^2 - 4\,AC^2$

or $AB^2 = 4\,AD^2 - 3\,AC^2$ 1 m

19. $\dfrac{AP}{AB} = \dfrac{1}{3} \Rightarrow \dfrac{AP}{PB} = \dfrac{1}{2}$ ½ m



$x = \dfrac{4+5}{3} = 3,\quad y = \dfrac{-8+2}{3} = -2 \Rightarrow P(3,-2)$ 1+½ =1½ m

P lies on $2x - y + k = 0 \Rightarrow 6 + 2 + k = 0 \Rightarrow k = -8$ 1 m

20. The given expression can be written as

$$\frac{\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}} + \frac{1}{\frac{\sin A}{\cos A}\left(1-\frac{\sin A}{\cos A}\right)}$$ 1 m

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$$ ½ m

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A(\sin A - \cos A)} = \frac{\sin^2 A + \cos^2 A + \sin A \cos A}{\sin A \cos A}$$ 1 m

$$= \tan A + \cot A + 1$$ ½ m

OR

$$\text{LHS} = \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$ ½ m

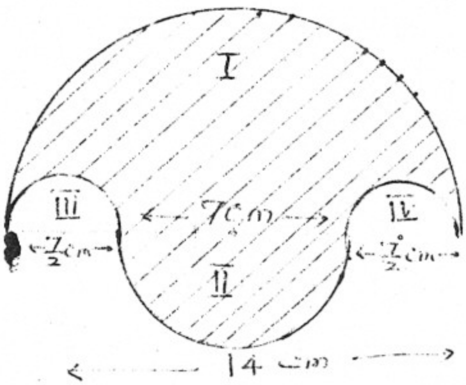
$$= \frac{1-\sin^2 A}{\sin A} \cdot \frac{1-\cos^2 A}{\cos A} = \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} = \sin A \cos A$$ 1½ m

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \frac{1}{\frac{\sin^2 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A}} = \frac{1}{\tan A + \cot A} = \text{RHS}$$ 1 m

21. Correct construction of Δ ABC 1 m

Correct construction of triangle similar to Δ ABC 2 m

22.



Area of semi-circle I $= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7\text{cm}$

$= 77 \text{ cm}^2$ ½ m

Area of semi-circle II $= \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$

$= \frac{77}{4} \text{ cm}^2$ ½ m

Combined area of semi-circles III and IV

$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \text{ cm}^2 = \frac{77}{8} \text{ cm}^2$ 1m

$\therefore$ Area of shaded region $= 77\left[\frac{1}{4} - \frac{1}{8} + 1\right] \text{ cm}^2$

$= \frac{693}{8} \text{ cm}^2$ or 86.625 cm^2 1m

OR

$AB^2 = AC^2 + BC^2$ (as $\angle ACB = 90^\circ$)

$= 24^2 + 10^2 = 26^2 \Rightarrow AB = 26\text{cm}$ ½ m

$\therefore$ Area of semi-circle ACBOA $= \left(\frac{1}{2} \times 3.14 \times 13 \times 13\right) \text{cm}^2$ 1m

$\therefore$ Area of Δ ACB $= \left(\frac{1}{2} \times 24 \times 10\right) \text{cm} = 120 \text{ cm}^2$ ½ m

$\therefore$ Area of shaded region $= \left[\left(\frac{1}{2} \times 3.14 \times 13 \times 13\right) - 120\right] \text{cm}^2$

$= (265.33 - 120)$ or 145.33 cm^2 1m

23. For P (a, b + c), Q (b, c + a), R (c, a + b) to be collinear ½ m

area (Δ PQR), = 0

area of Δ PQR

$= a\,[c + \cancel{a} - \cancel{a} - b] + b\,[a + \cancel{b} - \cancel{b} - c] + c\,[b + \cancel{c} - \cancel{c} - a]$ 1 m

$= \cancel{ac} - \cancel{ab} + \cancel{ab} - \cancel{bc} + \cancel{bc} - \cancel{ac} = 0$ 1 m

$\therefore$ P, Q and R are collinear ½ m

24. $S_{10} = -150$ and $S_{20} = -(150+550) = -700$ ½ m

$\therefore\ -150 = 5\,(2a + 9d)$ and $-700 = 10\,(2a + 19d)$ 1m

$\Rightarrow\ 2a + 9d = -30$ and $2a + 19d = -70$

1m

$\Rightarrow\ d = -4$ and $a = 3$

$\therefore$ A . P is 3, –1, – 5, ... ½ m

25. Let the fraction be $\frac{x}{y},\ y \neq 0$

$\therefore$ According to the question $x + y = 2y - 3$ 1 m

or $x = y - 3$......................................(i)

Also, $\frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2x - y = 1$...(ii) 1 m

From (i) and (ii), $x = 4,\ y = 7$

$\therefore$ Required fraction $= \frac{4}{7}$ 1 m

OR

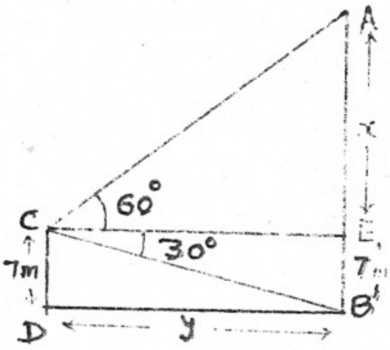
$\frac{4}{x} + 3y = 8$ .... (i), $\frac{6}{x} - 4y = -5$ ....(ii)

(i) $\times$ 3 $\Rightarrow\ \frac{12}{x} + 9y = 24$ and (ii) $\times$ 2 $\Rightarrow\ \frac{12}{x} - 8y = -10$ 1½ m

Solving to get y = 2 and x = 2 1½ m

SECTION D

26. Let CD be the building and AB, the tower



Correct Figure — 1m

Writing trigonometric equations

(i) $\frac{7}{y} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow y = 7\sqrt{3}$

$= 12.124 \text{ m}$ — $1\frac{1}{2}$ m

(ii) $\frac{x}{y} = \tan 60^\circ = \sqrt{3}.$ — $1\frac{1}{2}$ m

or $\frac{x}{7\sqrt{3}} = \sqrt{3} \Rightarrow x = 21$ — 1 m

$\therefore$ Height of tower $= (21 + 7)$ m or 28 m — 1 m

27. Correctly stated Given, To Prove, Construction and correct Figure $\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)$ — 2 m

Correct Proof — 2 m

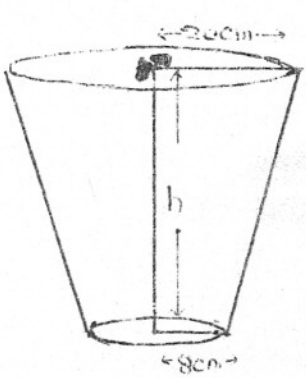
PA = PB, AL = LM and BN = MN......................(i) — 1 m

PA = PL + AL = PL + LM

PB = PN + BN = PN + MN ....(ii)

Form (i) and (ii) PL + LM = PN + MN — ½ + ½ m

28.



Volume of bucket $= 10459\frac{3}{7} \text{ cm}^3$

$= \frac{73216}{7} \text{ cm}^3$

$\therefore \frac{73216}{7} = \frac{1}{3} \times \frac{22}{7} \times h \left[20^2 + 8^2 + 20 \times 8\right]$ — 1m

$\therefore h = \frac{73216}{7} \times \frac{21}{22} \times \frac{1}{624} = 16 \text{ cm}$ — 1m

$$\ell^2 = h^2 + (r_2 - r_1)^2 = 16^2 + (20-8)^2$$

$$= 400$$

$$\Rightarrow \ell = 20 \text{ cm}$$ 1m

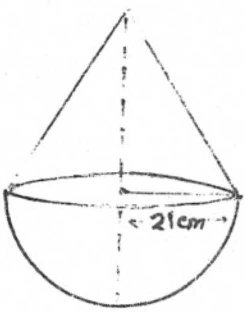
Total surface area of metal sheet used

$$= \frac{22}{7} \times 20 \times (20+8) + \frac{22}{7}(8)^2 \text{ cm}^2$$ 2 m

$$= \left(1760 + \frac{1408}{7}\right) \text{ cm}^2$$

$$\therefore \text{ Cost of metal sheet } = \text{Rs}\left(1760 + \frac{1408}{7}\right)\frac{14}{10}$$

$$= \text{Rs } 2745.60$$ 1 m



OR

$$\text{Volume of hemisphere } = \frac{2}{3} \times \frac{22}{7} \times (21)^3 \text{ cm}^3$$ 1 m

$$\therefore \text{ Volume of cone } = \left(\frac{2}{3} \cdot \frac{2}{3}\ \frac{22}{7} \times 21 \times 21 \times 21\right) \text{ cm}^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times (21)^2 \times h$$ 1½ m

$$\Rightarrow h = \frac{2 \times 2 \times 22 \times 7 \times 21}{22 \times 21} = 28 \text{ cm}$$ 1 m

$$\therefore \ell^2 = h^2 + r^2 = 28^2 + 21^2 = 1225 = (35)^2$$

$$\Rightarrow \ell = 35 \text{ cm}$$ 1 m

Surface area of toy $= 2\pi r^2 + \pi r \ell$

$$= \left(2 \times \frac{22}{7} \times 21 \times 21 + \frac{22}{7} \times 21 \times 35\right) \text{ cm}^2$$ 1½ m

$$= 5082 \text{ cm}^2$$

26. Let the three consecutive numbers be $x, x+1, x+2$ 1 m

According to the question

$$x^2 + (x+1)(x+2) = 46$$

or $2x^2 + 3x - 44 = 0 \Rightarrow 2x^2 + 11x - 8x - 44 = 0$ 2 m

$\Rightarrow (x-4)(2x+11) = 0$ 1 m

As x is positive $\Rightarrow x = 4 \left(x = \frac{-11}{2} \text{ rejected} \right)$ 1 m

$\therefore$ The numbers are 4, 5, 6 1 m

OR

Let the two numbers be x, y where $x > y$

$\therefore \; x^2 - y^2 = 88$ ....(i)

2 m

Also, $x = 2y - 5$ ....(ii)

From (i) and (ii), $(2y-5)^2 - y^2 = 88$

$\Rightarrow 3y^2 - 20y - 63 = 0$ 1 m

or $3y^2 - 27y + 7y - 63 = 0 \Rightarrow (3y+7)(y-9) = 0$

$\Rightarrow y = 9, -7/3$ 1 m

$x = 2y - 5 = 13$ or $x = -\frac{29}{3}$ 1 m

$\therefore$ The numbers are 13, 9 (Rejecting $x = \frac{-29}{3}$

and $y = {}^{-7}\!/_{3}$) 1 m

30.

Classes	0–10	10–20	20–30	30–40	40–50	50–60	60–70
class marks (x_i)	5	15	25	35	45	55	65
$d_i = \dfrac{x_i - 35}{10}$	–3	–2	–1	0	1	2	3
f_i	8	8	14	22	30	8	10
$f_i d_i$	–24	–16	–14	0	30	16	30

$\sum f_i = 100,\ \sum f_i d_i = 22$ 1 m

(i) $\bar{x} = 35 + \dfrac{22}{100} \times 10 = 37.2$ 1 m

(ii) Modal Class = 40 – 50 ½ m

$\therefore$ Mode $= 40 + \dfrac{30-22}{6-30} \times 10 = 42.66$ 1½ m

(iii) Median Class = 30 – 40 ½ m

Median $= 30 + \dfrac{50-30}{22} \times 10 = 39.1$ 1½ m

Note : If a candidate finds any two of the measures of central tendency correctly and finds third correctly using Empirical formula, give full credit

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. 2a   2. 1 : 9   3 . 5cm

4. $x^2 - 6x + 4$ 5. Terminating   6. $\frac{1}{2}$ 1×10

$= 10$ m

7. $\frac{3}{26}$ 8. 48 cm^2 9 . (3.5)

10. p = 3

SECTION - B

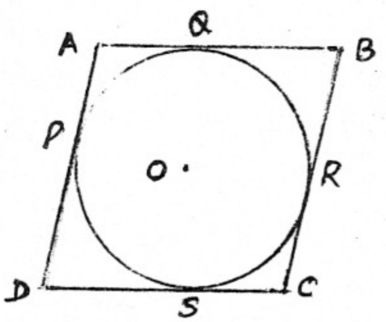
11. $\sqrt{5}$ and $-\sqrt{5}$ are zeroes of p(x) $= x^3 + 3x^2 - 5x - 15$

$\therefore$ $x^2 - 5$ is a factors of p(x) ½m

$$\left(x^3 - 3x^2 - 5x - 15\right) \div \left(x^2 - 5\right) = \left(x + 3\right)$$ 1m

$\therefore$ The third zero of p(x) is -3 ½m

12.



As parallelogram ABCD circumscribes a circle with centre O

$\therefore$ AB + CD = BC + AD

[$\because$ AQ = AP, BQ = BR, CR = CS, PD = DS] ½m

As ABCD is a parallelogram $\Rightarrow$ AB = DC

and BC = AD

2AB = 2AD or AB = AD 1 m

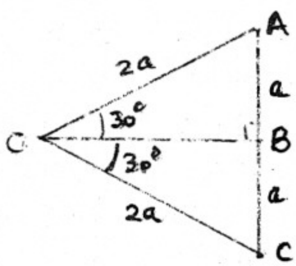
$\therefore$ ABCD is a rhombus (As AB = BC = CD = AD) ½m

13. $\sec(90^\circ - \theta) = \operatorname{cosec}\theta$, $\tan(90^\circ - \theta) = \cot\theta$, $\cos 65^\circ = \sin 25^\circ$ and $\tan 63^\circ = \cot 27^\circ$ — 1 m

$\therefore$ Given expression becomes

$$\frac{(\operatorname{cosec}^2\theta - \cot^2\theta) + (\cos^2 25^\circ + \sin^2 25^\circ)}{3\tan 27^\circ \cot 27^\circ} = \frac{1+1}{3} = \frac{2}{3}$$ 1 m

OR



correct Fig. — ½ m

Draw rt . Δ OBA, in which $\angle BOA = 30^\circ$, Take OA = 2a . Replicate Δ OCB on the other side of OB $\Rightarrow \angle AOC = 60^\circ$ — ½ m

and OC = 2a

$\therefore$ Δ AOC is equilateral Δ and AB = a

$$\operatorname{cosec} 30^\circ = \frac{OA}{AB} = \frac{2a}{a} = 2$$ 1m

$\therefore \operatorname{cosec} 30^\circ = 2$

14. For a pair of linear equations to have infinitely many solutions : $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ — ½ m

$$\therefore \overset{\text{(i)}}{\frac{2}{k-1}} = \overset{\text{(ii)}}{\frac{3}{k+2}} = \overset{\text{(iii)}}{\frac{7}{3k}}$$ 1 m

From (i) and (ii) getting k = 7

k = 7 satisfies (ii) and (iii) and (i) and (iii) also — ½ m

$\therefore$ k = 7

15. Here $a = 2,\ \ell = 29$ and $s_n = 155$ 1 m

$$\therefore\ 155 = \frac{n}{2}\left[2+29\right] \Rightarrow n = 10$$

Also, $29 = 2 + (10-1)\,d \Rightarrow d = 3$ 1 m

$\therefore$ Common difference = 3

SECTION -C

16. Let $2\sqrt{3} - 1 = x$, where x is a rational number ½ m

$$\therefore\ 2\sqrt{3} = x+1 \text{ or } \frac{x+1}{2} = \sqrt{3} \text{ ....(i)}$$ ½ m

As x is a rational number, so is $\frac{x+1}{2}$ also rational 1 m

$\therefore$ LHS of (i) is rational number

but $\sqrt{3}$ is irrational. which is a contradiction

$\therefore\ 2\sqrt{3}-1$ is an irrational number 1 m

17. $AB^2 = AC^2 + BC^2$ ....(i) and $AD^2 = AC^2 + DC^2$

$$= AC^2 + \frac{BC^2}{4}\left(\because\ DC = \frac{1}{2}BC\right)$$ 1+½ m

or $4\,AD^2 = 4\,AC^2 + BC^2$ ....(ii) ½ m

From (i) and (ii), $AB^2 = AC^2 + 4\,AD^2 - 4\,AC^2$

or $AB^2 = 4\,AD^2 - 3\,AC^2$ 1 m

18. The given expression can be written as

$$\frac{\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}} + \frac{1}{\frac{\sin A}{\cos A}\left(1-\frac{\sin A}{\cos A}\right)}$$ 1 m

$$= \frac{\sin^2 A}{\cos A\left(\sin A - \cos A\right)} + \frac{\cos^2 A}{\sin A\left(\cos A - \sin A\right)}$$ ½ m

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A\left(\sin A - \cos A\right)} = \frac{\sin^2 A + \cos^2 A + \sin A \cos A}{\sin A \cos A}$$ 1 m

$$= \tan A + \cot A + 1$$ ½ m

OR

$$\text{LHS} = \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$ ½ m

$$= \frac{1-\sin^2 A}{\sin A}\cdot\frac{1-\cos^2 A}{\cos A} = \frac{\cos^2 A}{\sin A}\cdot\frac{\sin^2 A}{\cos A} = \sin A \cos A$$ 1½ m

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \frac{1}{\frac{\sin^2 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A}} = \frac{1}{\tan A + \cot A} = \text{RHS}$$ 1 m

19. $S_{10} = -150$ and $S_{20} = -(150+550) = -700$ ½ m

$\therefore\ -150 = 5\,(2a + 9d)$ and $-700 = 10\,(2a + 19d)$ 1m

$\Rightarrow\ 2a + 9d = -30$ and $2a + 19d = -70$

1m

$\Rightarrow\ d = -4$ and $a = 3$

$\therefore$ A . P is $3, -1, -5, \ldots\ldots\ldots\ldots$ ½ m

20. Let the fraction be $\frac{x}{y},\ y \neq 0$

$\therefore$ According to the question $x + y = 2y - 3$ 1 m or $x = y - 3$ ....(i)

Also, $\frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2x - y = 1$ ....(ii) 1 m

From (i) and (ii), $x = 4, y = 7$

$\therefore$ Required fraction $= \frac{4}{7}$ 1 m

OR

$\frac{4}{x} + 3y = 8$ .... (i), $\frac{6}{x} - 4y = -5$ ....(ii)

(i) $\times$ 3 $\Rightarrow \frac{12}{x} + 9y = 24$ and (ii) $\times$ 2 $\Rightarrow \frac{12}{x} - 8y = -10$ 1½ m

Solving to get y = 2 and x = 2 1½ m

21. Correct construction of triangle Δ PQR 1 m

Correct construction of a triangle similar to Δ PQR 2m

22. Total number of cards = 18 ½ m

(i) Prime numbers less than 15 are 3, 5, 7, 11, 13 – Five in number ½ m

$\therefore$ P (a prime no. less than 15) $= \frac{5}{18}$ 1m

(ii) numbers divisible by 3 and 5 is only 15 (one in number) ½ m

$\therefore$ P (a no. divisible by 3 and 5) $= \frac{1}{18}$ ½ m

23. If A $\left(-\frac{2}{5}, 6\right)$, P (m, 3) and B (2, 8) are in the same line

$\Rightarrow$ Area (Δ APB) = 0 1 m

$\therefore -\frac{2}{5}(3-8) + m(8-6) + 2(6-3) = 0$ 1 m

$\Rightarrow 2 + 2m + 6 = 0 \Rightarrow m = -4$ 1m

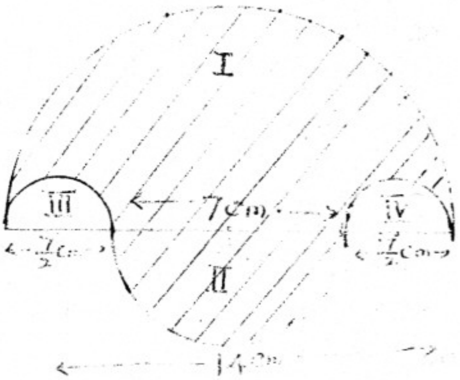
24. $\dfrac{AP}{AB} = \dfrac{1}{3} \Rightarrow \dfrac{AP}{PB} = \dfrac{1}{2}$ ½ m

$$\underset{A(2,1)}{} \overset{P\ (x,\ y)}{\underline{\hspace{2cm}}}_{1:2} \ B\ (5,-8)$$

$x = \dfrac{4+5}{3} = 3,\ \ y = \dfrac{-8+2}{3} = -2 \Rightarrow P(3,-2)$ 1+½ =1½ m

P lies on $2x - y + k = 0 \Rightarrow 6 + 2 + k = 0 \Rightarrow k = -8$ 1 m

25.



Area of semi-circle I $= \dfrac{1}{2} \times \dfrac{22}{7} \times 7 \times 7\text{cm}$

$= 77\ \text{cm}^2$ ½ m

Area of semi-circle II $= \dfrac{1}{2} \times \dfrac{22}{7} \times \dfrac{7}{2} \times \dfrac{7}{2}\ \text{cm}^2$

$= \dfrac{77}{4}\ \text{cm}^2$ ½ m

Combined area of semi-circles III and IV

$= \dfrac{22}{7} \times \dfrac{7}{4} \times \dfrac{7}{4}\ \text{cm}^2 = \dfrac{77}{8}\ \text{cm}^2$ 1m

$\therefore$ Area of shaded region $= 77\left[\dfrac{1}{4} - \dfrac{1}{8} + 1\right]\ \text{cm}^2$

$= \dfrac{693}{8}\ \text{cm}^2$ or $86.625\ \text{cm}^2$ 1m

OR

$AB^2 = AC^2 + BC^2$ (as $\angle ACB = 90^\circ$)

$= 24^2 + 10^2 = 26^2 \Rightarrow AB = 26\text{cm}$ ½ m

$\therefore$ Area of semi-circle ACBOA $= \left(\dfrac{1}{2} \times 3.14 \times 13 \times 13\right)\text{cm}^2$ 1m

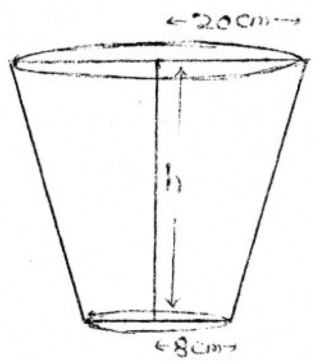
$\therefore$ Area of $\Delta ACB = \left(\frac{1}{2} \times 24 \times 10\right) \text{cm} = 120 \text{ cm}^2$ ½ m

$\therefore$ Area of shaded region $= \left[\left(\frac{1}{2} \times 3.14 \times 13 \times 13\right) - 120\right] \text{cm}^2$

$= (265.33 - 120)$ or 145.33 cm^2 1m

SECTION D

26.



Volume of bucket $= 10459\frac{3}{7} \text{ cm}^3$

$= \frac{73216}{7} \text{ cm}^3$

$\therefore \frac{73216}{7} = \frac{1}{3} \times \frac{22}{7} \times h \left[20^2 + 8^2 + 20 \times 8\right]$ 1m

$\therefore h = \frac{73216}{7} \times \frac{21}{22} \times \frac{1}{624} = 16 \text{ cm}$ 1m

$\ell^2 = h^2 + (r_2 - r_1)^2 = 16^2 + (20-8)^2$

$= 400$

$\Rightarrow \ell = 20 \text{ cm}$ 1m

Total surface area of metal sheet used

$= \frac{22}{7} \times 20 \times (20+8) + \frac{22}{7}(8)^2 \text{ cm}^2$ 2 m

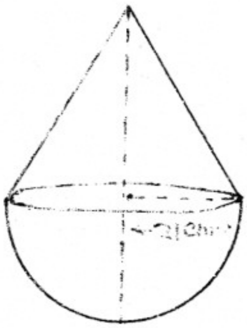
$= \left(1760 + \frac{1408}{7}\right) \text{cm}^2$

$\therefore$ Cost of metal sheet $= \text{Rs}\left(1760 + \frac{1408}{7}\right)\frac{14}{10}$

$= \text{Rs } 2745.60$ 1 m

OR

$$\text{Volume of hemisphere} = \frac{2}{3}\times\frac{22}{7}\times(21)^3\ \text{cm}^3 \qquad 1\text{ m}$$



$$\therefore\ \text{Volume of cone} = \left(\frac{2}{3}\cdot\frac{2}{3}\ \frac{22}{7}\times 21\times 21\times 21\right)\ \text{cm}^3$$

$$= \frac{1}{3}\times\frac{22}{7}\times(21)^2\times h \qquad 1\tfrac{1}{2}\text{ m}$$

$$\Rightarrow h = \frac{2\times 2\times 22\times 7\times 21}{22\times 21} = 28\text{ cm} \qquad 1\text{ m}$$

$$\therefore\ \ell^2 = h^2 + r^2 = 28^2 + 21^2 = 1225 = (35)^2$$

$$\Rightarrow \ell = 35\text{ cm} \qquad 1\text{ m}$$

$$\text{Surface area of toy} = 2\pi r^2 + \pi r \ell$$

$$= \left(2\times\frac{22}{7}\times 21\times 21 + \frac{22}{7}\times 21\times 35\right)\ \text{cm}^2 \qquad 1\tfrac{1}{2}\text{ m}$$

$$= 5082\text{ cm}^2$$

27. Correctly stated Given, To Prove, Construction and correct Figure $\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)$ 2 m

Correct Proof 2 m

$$AD^2 = AC^2 + CD^2 \qquad 1\text{ m}$$

$$= AC^2 + \left(\frac{1}{2}\ BC\right)^2 \qquad \tfrac{1}{2}\text{ m}$$

$$4\,AD^2 = 4\,AC^2 + BC^2 \qquad \tfrac{1}{2}\text{ m}$$

28. Let the three consecutive numbers be $x, x+1, x+2$ 1 m

According to the question

$$x^2 + (x+1)(x+2) = 46$$

or $2x^2 + 3x - 44 = 0 \quad \Rightarrow \quad 2x^2 + 11x - 8x - 44 = 0$ 2m

$\Rightarrow \quad (x-4)(2x+11) = 0$ 1m

As x is positive $\Rightarrow \quad x = 4 \left(x = \dfrac{-11}{2} \text{ rejected} \right)$ 1 m

$\therefore$ The numbers are 4, 5, 6 1 m

OR

Let the two numbers be x, y where $x > y$

$\therefore \quad x^2 - y^2 = 88$ ....(i)

2 m

Also, $x = 2y - 5$ ....(ii)

From (i) and (ii), $(2y-5)^2 - y^2 = 88$

$\Rightarrow 3y^2 - 20y - 63 = 0$ 1 m

or $3y^2 - 27y + 7y - 63 = 0 \quad \Rightarrow \quad (3y + 7)(y - 9) = 0$

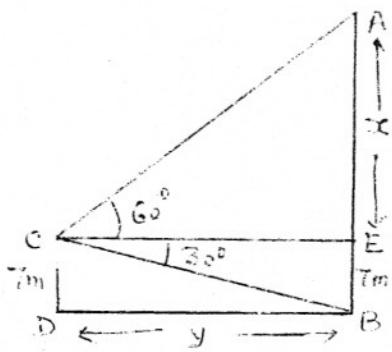
$\Rightarrow \quad y = 9, -7/3$ 1 m

$x = 2y - 5 = 13$ or $x = -\dfrac{29}{3}$ 1 m

$\therefore$ The numbers are 13, 9 (Rejecting $x = \dfrac{-29}{3}$

and $y = -7/3$) 1 m

29. Let CD be the building and AB, the tower



Correct Figure 1m

Writing trigonometric equations

(i) $\dfrac{7}{y} = \tan 30^\circ = \dfrac{1}{\sqrt{3}} \quad \Rightarrow \quad y = 7\sqrt{3}$

$= 12.124$ m 1½ m

$\therefore \frac{x}{y} = \tan 60^\circ = \sqrt{3}$ 1½ m

or $\frac{x}{7\sqrt{3}} = \sqrt{3} \Rightarrow x = 21$ 1 m

$\therefore$ Height of tower = (21 + 7) m or 28 m 1 m

30.

Classes	0–10	10–20	20–30	30–40	40–50	50–60	60–70
class marks (x_i)	5	15	25	35	45	55	65
$d_i = \frac{x_i - 35}{10}$	–3	–2	–1	0	1	2	3
f_i	8	7	15	20	12	8	10
$f_i d_i$	–24	–14	–15	0	12	16	30

$\sum f_i = 80, \ \sum f_i d_i = 5$ 1 m

(i) $\bar{x} = 35 + \frac{5}{80} \times 10 = 35.63$ 1 m

(ii) Modal Class = 30 – 40 ½ m

Mode $= 30 + \frac{20-15}{40-27} \times 10 = 33.85$ 1½ m

(iii) Median Class = 30 – 40 ½ m

Median $= 30 + \frac{40-30}{20} \times 10 = 35.0$ 1½ m

Note : If a candidate finds any two measures of central tendency correctly and finds the third correctly using Empirical formula, give full credit