केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं
परीक्षार्थी प्रवेश–पत्र के अनुसार भरें

विषय Subject : MATHEMATICS

विषय कोड Subject Code : 041

परीक्षा का दिन एवं तिथि
Day & Date of the Examination : MONDAY, 20.03.2017

उत्तर देने का माध्यम
Medium of answering the paper : ENGLISH

प्रश्न पत्र के ऊपर लिखे कोड को दर्शाएं :
Write code No. as written on the top of the question paper :

Code Number	Set Number
65/1	● ② ③ ④

अतिरिक्त उत्तर–पुस्तिका (ओं) की संख्या
No. of supplementary answer-book(s) used

विकलांग व्यक्ति : हाँ / नहीं
Person with Disabilities : Yes / No NO

किसी शारीरिक अक्षमता से प्रभावित हो तो संबंधित वर्ग में ✓ का निशान लगाएं।
If physically challenged, tick the category

B D H S C A

B = दृष्टिहीन, D = मूक व बधिर, H = शारीरिक रूप से विकलांग, S = स्पास्टिक
C = डिस्लेक्सिक, A = ऑटिस्टिक
B = Visually Impaired, D = Hearing Impaired, H = Physically Challenged
S = Spastic, C = Dyslexic, A = Autistic

क्या लेखन – लिपिक उपलब्ध करवाया गया : हाँ / नहीं
Whether writer provided : Yes / No NO

यदि दृष्टिहीन हैं तो उपयोग में लाए गये सॉफ्टवेयर का नाम :
If Visually challenged, name of software used : –

* एक खाने में एक अक्षर लिखें। नाम के प्रत्येक भाग के बीच एक खाना रिक्त छोड़ दें। यदि परीक्षार्थी का नाम 24 अक्षरों से अधिक है, तो केवल नाम के प्रथम 24 अक्षर ही लिखें।
Each letter be written in one box and one box be left blank between each part of the name. In case Candidate's Name exceeds 24 letters, write first 24 letters.

कार्यालय उपयोग के लिए
Space for office use

0390753
041/00309

SECTION-A

$A(\text{adj}A) = |A| I_n$

$A(\text{adj}A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$|A| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|A| = 8$

2) $f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3} & ; x \neq 3 \\ k & ; x = 3 \end{cases}$

$f(x)$ is continuous at $x = 3$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$

$\lim_{x \to 3} \dfrac{(x+3)^2 - (6)^2}{x-3} = k$

$\lim_{x \to 3} \dfrac{(x+3-6)(x+3+6)}{(x-3)} = k$

$\lim_{x \to 3} \dfrac{\cancel{(x-3)}(x+9)}{\cancel{(x-3)}} = k$

$$\lim_{x \to 3} (x+9) = k$$

$$12 = k$$

$$k = 12$$

3) $\displaystyle\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x}\,dx$

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$= \displaystyle -2\int \frac{\cos^2 x - \sin^2 x}{2\sin x \cos x}\,dx$

$= \displaystyle (-2)\int \frac{\cos 2x}{\sin 2x}\,dx$

$= \displaystyle (-2)\int \cot 2x\,dx$

$= (-2) \log|\sin 2x| \times \frac{1}{2} + C$ $\qquad \left[\int \cot x\,dx = \log|\sin x|\right]$

$= -\log|\sin 2x| + c$

$= \log|\operatorname{cosec} 2x| + c$

4) $P_1 \Rightarrow \quad 2x - y + 2z = 5$

$P_2 \Rightarrow \quad 5x - 2.5y + 5z = 20$

$\Rightarrow 2x - y + 2z = \frac{20}{2.5} = \frac{\cancel{20}^{4}}{\cancel{5}} \times 2 = 8$

$\text{distance} = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right| = \left| \frac{8 - 5}{\sqrt{2^2 + (-1)^2 + 2^2}} \right| = \left| \frac{3}{\sqrt{4+1+4}} \right| = \left| \frac{3}{\sqrt{9}} \right| = \frac{3}{3} = 1 \text{ units}$

SECTION B :

5) A is skew symmetric

$A = -A^T$

$|A| = (-1)|A^T|$

$|A| = -|A| \quad [\because \; |A| = |A^T|]$

$2|A| = 0$

$|A| = 0$

$\det A = 0$

2017

6) $f(x) = x^3 - 3x$

$f'(x) = 3x^2 - 3$

$\because$ $f(x)$ is a polynomial, it is continuous in the interval $[-\sqrt{3}, 0]$

$\because$ $f'(x)$ is a polynomial it is differentiable in the interval $(-\sqrt{3}, 0)$

$f(0) = 0 - 3(0) = 0$

$f(-\sqrt{3}) = (-\sqrt{3})^3 - 3(-\sqrt{3}) = -3\sqrt{3} + 3\sqrt{3} = 0$

$f(0) = f(-\sqrt{3})$

Hence there exists $c \in (-\sqrt{3}, 0)$ such that $f'(c) = 0$.

$f'(c) = 3c^2 - 3 = 0$

$3c^2 - 3 = 0$

$\cancel{3c^2}\ 3(c^2 - 1) = 0$

$c^2 - 1 = 0$

$c = \pm 1$

$+1$ doesn't exist between $(-\sqrt{3}, 0)$. Hence $c = -1$

7) $V = x^3$ V: Volume of the cube of side x

S: Surface area of the cube of side x.

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$3\cancel{9} = \cancel{3}(x^2)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{3}{x^2}$$

$S = 6x^2$

$$\frac{dS}{dt} = 12x\frac{dx}{dt} = 12\cancel{x}\left(\frac{3}{x^2}\right) = \frac{36}{x}$$

$$\left.\frac{dS}{dt}\right|_{x=10cm} = \frac{36}{10}\ cm^2/sec = 3.6\ cm^2/sec.$$

8) $f(x) = x^3 - 3x^2 + 6x - 100$

$f'(x) = 3x^2 - 6x + 6$

$= 3(x^2 - 2x + 2)$

discriminant of the formed quadratic $= b^2 - 4ac = (-2)^2 - 4(1)(2)$

$= 4 - 8 = -4$

$b^2 - 4ac < 0$

but $a > 0$ $[a = 1]$

Hence $x^2 - 2x + 2 > 0$ for all $x \in R$

$\therefore$ $3(x^2 - 2x + 2) > 0$; $x \in R$

$f'(x) > 0$; $x \in R$

Hence $f(x)$ is increasing on R.

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9) $P(2, 2, 1)$ $\quad Q(5, 1, -2)$

Direction ratios of the line PQ $= (5-2), (1-2), (-2-1)$

$= 3, -1, -3$

Equation of PQ

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$$

Any point on PQ is given by $(3\lambda+2, -\lambda+2, -3\lambda+1)$

$3\lambda + 2 = 4$

$3\lambda = 2$

$\lambda = 2/3$

z coordinate $= -3\lambda + 1 = -\cancel{3}\left(\frac{2}{\cancel{3}}\right) + 1 = -2 + 1 = -1$

y coordinate $= -\lambda + 2 = -\frac{2}{3} + 1 = \frac{1}{3}$

Hence point is $\left(4, \frac{1}{3}, -1\right)$

z coordinate $= -1$

10) A : number obtained is even

$= \{2, 4, 6\}$

B : number obtained is red

$= \{1, 2, 3\}$

$P(A) = \frac{3}{6} = \frac{1}{2}$

$P(B) = \frac{3}{6} = \frac{1}{2}$

$A \cap B$ = number obtained is red and even

$= \{2\}$

$P(A \cap B) = \frac{1}{6}$

$P(A)P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(A \cap B) \neq P(A)P(B)$ Hence events A and B are not independent events

11)

	Shirts	Trousers	Payment per day
(x) A	6	4	₹300
(y) B	10	4	₹ 400
	60	32	

Let A ~~for~~ work for x and B for y days.

To Minimize : $Z = 300x + 400y$

Constraints : $6x + 10y \geq 60$

$4x + 4y \geq 32$

$x \geq 0$

$y \geq 0$

PQR is the solution.

P (5,3) $\quad Z = 300(5) + 400(3) = ₹2700$

Q (0,8) $\quad Z = 400(8) = ₹3200$

R (10,0) $\quad Z = 300(10) = ₹3000$

Hence Z is minimum when A works for 5 days and B for 3 days.

1500
1200
1500
12000
2700

12) $\int \frac{dx}{5-8x-x^2}$

$= \int \frac{dx}{21-16-2\cdot x\cdot 4-x^2}$

$= \int \frac{dx}{21-(x^2+2\cdot x\cdot 4+(4)^2)}$

$= \int \frac{dx}{(\sqrt{21})^2-(x+4)^2}$

$\left[\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c \right]$

$= \frac{1}{2\sqrt{21}} \log \left| \frac{\sqrt{21}+x+4}{\sqrt{21}-x-4} \right| + c.$

$= \frac{1}{2\sqrt{21}} \log \left| \frac{x+\sqrt{21}+4}{\sqrt{21}-4-x} \right| + c.$

SECTION C:

13)

$$\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}.$$

$$\tan^{-1}\left(\frac{\frac{x-3}{x-4}+\frac{x+3}{x+4}}{1-\left(\frac{x-3}{x-4}\right)\left(\frac{x+3}{x+4}\right)}\right) = \tan^{-1}(1)$$

$$\left[\because \tan^{-1}\left(\frac{x-3}{x-4}\right) \leq \frac{\pi}{4}\right.$$

$$\tan^{-1}\left(\frac{x+3}{x+4}\right) \leq \frac{\pi}{4}$$

$$\therefore \tan^{-1}(\alpha) + \tan^{-1}(\beta) = \tan^{-1}\left.\left(\frac{\alpha+\beta}{1-\alpha\beta}\right)\right]$$

$$\tan^{-1}\left(\frac{\frac{(x-3)(x+4)+(x+3)(x-4)}{\cancel{x^2-16}}}{\frac{x^2-16-(x^2-9)}{\cancel{x^2-16}}}\right) = \tan^{-1}(1)$$

$$\tan^{-1}\left(\frac{x^2-\cancel{3x}+\cancel{4x}-12+x^2+\cancel{3x}-\cancel{4x}-12}{x^2-16-x^2+9}\right) = \tan^{-1}(1)$$

$$\tan^{-1}\left(\frac{2x^2-24}{-7}\right) = \tan^{-1}(1)$$

$$\frac{2x^2-24}{-7} = 1.$$

$$\cancel{2x^2 = 7} \quad 2x^2 - 24 = -7$$

$$2x^2 = -7+24 = 17$$

$$x^2 = \frac{17}{2} \Rightarrow x = \pm\sqrt{\frac{17}{2}}$$

$$\sqrt{\frac{17}{2}} < \sqrt{\frac{18}{2}}$$

$$\sqrt{\frac{17}{2}} < 3$$

If $x = +\sqrt{\frac{17}{2}}$

$\frac{x-3}{x-4}$ = positive

$\frac{x+3}{x+4}$ = positive

If $x = -\sqrt{\frac{17}{2}}$

$\frac{x-3}{x-4}$ = positive

$\frac{x+3}{x+4}$ = positive.

Hence for $x = \pm\sqrt{\frac{17}{2}}$ $\tan^{-1}\frac{x-3}{x-4}$ and $\tan^{-1}\left(\frac{x+3}{x+4}\right)$ lie in first quadrant.

14) $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$= \begin{vmatrix} a^2+\cancel{2a}-\cancel{2a}-1 & 2a+1 & 1 \\ 2a+1-a-2 & a+2 & 1 \\ 0 & 3 & 1 \end{vmatrix}$ $[C_1 \to C_1 - C_2]$

$= \begin{vmatrix} a^2-1 & 2a+1 & 1 \\ a-1 & a+2 & 1 \\ 0 & 3 & 1 \end{vmatrix}$

$= (a-1) \begin{vmatrix} a+1 & 2a+1 & 1 \\ 1 & a+2 & 1 \\ 0 & 3 & 1 \end{vmatrix}$ [Taking $(a-1)$ common out of C_1]

$= (a-1) \begin{vmatrix} a+1 & 2a-2 & 0 \\ 1 & a-1 & 0 \\ 0 & 3 & 1 \end{vmatrix}$ $\begin{bmatrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{bmatrix}$

$\neq$

$$= (a-1)\begin{vmatrix} a+1 & 2(a-1) & 0 \\ 1 & (a-1) & 0 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= (a-1)^2 \begin{vmatrix} a+1 & 2 & 0 \\ 1 & 1 & 0 \\ 0 & 3 & 1 \end{vmatrix}$$ [Taking $(a-1)$ common out of C_2]

$$= (a-1)^2 \begin{vmatrix} a+1 & 2 \\ 1 & 1 \end{vmatrix}$$

$$= (a-1)^2 \,(a+1-2)$$ [On expanding the determinant]

$$= (a-1)^2 (a-1)$$

$$= (a-1)^3$$

Proved.

s) $e^y(x+1) = 1$.

$e^y = \frac{1}{x+1}$ —— (I)

On differentiating both sides w.r.t x.

$e^y \frac{dy}{dx} = \frac{d}{dx}(x+1)^{-1}$.

$e^y \frac{dy}{dx} = -1(x+1)^{-1-1}$

$e^y \frac{dy}{dx} = \frac{-1}{(x+1)^2}$

$\frac{dy}{dx} = \frac{-1}{(x+1)^2 e^y} = \frac{-1}{(x+1)^2}\frac{(x+1)}{1} = \frac{-1}{(x+1)}$ —— (II).

On differentiating (II) wrt x on both sides.

$\frac{d^2y}{dx^2} = -\frac{d}{dx}(x+1)^{-1} = (-1)(-1)(x+1)^{-2} = \frac{1}{(x+1)^2}$ —— (III)

$\frac{d^2y}{dx^2} = \left(\frac{-1}{x+1}\right)^2 = \left(\frac{dy}{dx}\right)^2$ [Using equation (II)]

Hence $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ Proved.

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16) $I = \int \dfrac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}\, d\theta.$

$= \int \dfrac{\cos\theta}{(4+\sin^2\theta)(5-4(1-\sin^2\theta))}\, d\theta.$

$= \int \dfrac{\cos\theta}{(4+\sin^2\theta)(1+4\sin^2\theta)}\, d\theta$

Let $\sin\theta = t$

$\cos\theta\, d\theta = dt$

$= \int \dfrac{dt}{(4+t^2)(1+4t^2)}$

$\dfrac{1}{(4+t^2)(1+4t^2)} = \left[\dfrac{(1+4t^2) - 4(4+t^2)}{(4+t^2)(1+4t^2)}\right] \times \dfrac{1}{-15}$

$= \dfrac{-1}{15}\left(\dfrac{1}{4+t^2} - \dfrac{4}{1+4t^2}\right)$

$= \dfrac{4}{15}\left(\dfrac{1}{1+4t^2}\right) - \dfrac{1}{15}\left(\dfrac{1}{4+t^2}\right)$

$$I = \int \left[\frac{4}{15}\left(\frac{1}{4t^2+1}\right) - \frac{1}{15}\left(\frac{1}{4+t^2}\right)\right] dt$$

$$= \frac{4}{15}\int \frac{1}{1+4t^2}\,dt - \frac{1}{15}\int \frac{1}{2^2+t^2}\,dt$$

$$= \frac{1}{15}\int \frac{1}{\left(\frac{1}{2}\right)^2+t^2}\,dt - \frac{1}{15}\int \frac{1}{2^2+t^2}\,dt$$

$$\left[\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$= \frac{1}{15}\times\frac{1}{1/2}\tan^{-1}\left(\frac{t}{1/2}\right) - \frac{1}{15}\times\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right) + c$$

$$= \frac{2}{15}\tan^{-1}(2t) - \frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right) + c$$

$$= \frac{2}{15}\tan^{-1}(2\sin\theta) - \frac{1}{30}\tan^{-1}\left(\frac{\sin\theta}{2}\right) + c$$

$$\cancel{= \frac{1}{30}\left(\tan^{-1}(2\sin\theta)\right.} \quad = \frac{1}{30}\left(4\tan^{-1}(2\sin\theta) - \tan^{-1}\left(\frac{\sin\theta}{2}\right)\right) + c$$

17). $f(x) = |x-1| + |x-2| + |x-4|$

$$= \begin{cases} (x-1) - (x-2) - (x-4) & ; 1 \leq x < 2 \\ (x-1) + (x-2) - (x-4) & ; 2 \leq x \leq 4 \end{cases}$$

1.5

+ve −ve −ve

3

+ve +ve −ve.

$$= \begin{cases} x - 1 - x + 2 - x + 4 & ; 1 \leq x < 2 \\ x - 1 + x - 2 - x + 4 & ; 2 \leq x \leq 4 \end{cases}$$

$$= \begin{cases} 5 - x & ; 1 \leq x < 2 \\ x + 1 & ; 2 \leq x \leq 4 \end{cases}$$

$$I = \int_1^4 \{|x-1| + |x-2| + |x-4|\} dx$$

$$= \int_1^2 (5-x)\, dx + \int_2^4 (x+1)\, dx$$

$$\left[\because \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)\, dx \text{ when } a \leq c \leq b\right]$$

$$= \left[5x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} + x\right]_2^4$$

$$= \left[10 - 2\right] - \left[5 - \frac{1}{2}\right] + \left[8 + 4\right] - \left[2 + 2\right]$$

$$= 8 - \frac{9}{2} + 12 - 4$$

$$= 16 - \frac{9}{2}$$

$$= \frac{23}{2} = 11.5$$

$32 - 9$

$= 23$

$5 \frac{1}{2} = \frac{9}{2}$

$$\begin{array}{r} 32 \\ -\ 9 \\ \hline 23 \end{array}$$

18) $(\tan^{-1}x - y)dx = (1+x^2)dy$

$$\left(\frac{\tan^{-1}x - y}{1+x^2}\right) = \frac{dy}{dx}.$$

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{\tan^{-1}x}{1+x^2}$$

$$\frac{dy}{dx} + Py = Q$$

Hence its a linear differential equation solution

$$P = \frac{1}{1+x^2}$$

$$\text{Integration factor} = e^{\int P dx} = e^{\int \frac{1}{1+x^2}dx}$$

$$= e^{\tan^{-1}x}$$

$$e^{\tan^{-1}x}\left(\frac{dy}{dx} + \frac{y}{1+x^2}\right) = e^{\tan^{-1}x}\frac{\tan^{-1}x}{1+x^2}$$

[On multiplying both sides by integration factor].

$$\frac{d}{dx}\left(y e^{\tan^{-1}x}\right) = e^{\tan^{-1}x}\frac{\tan^{-1}x}{1+x^2}$$

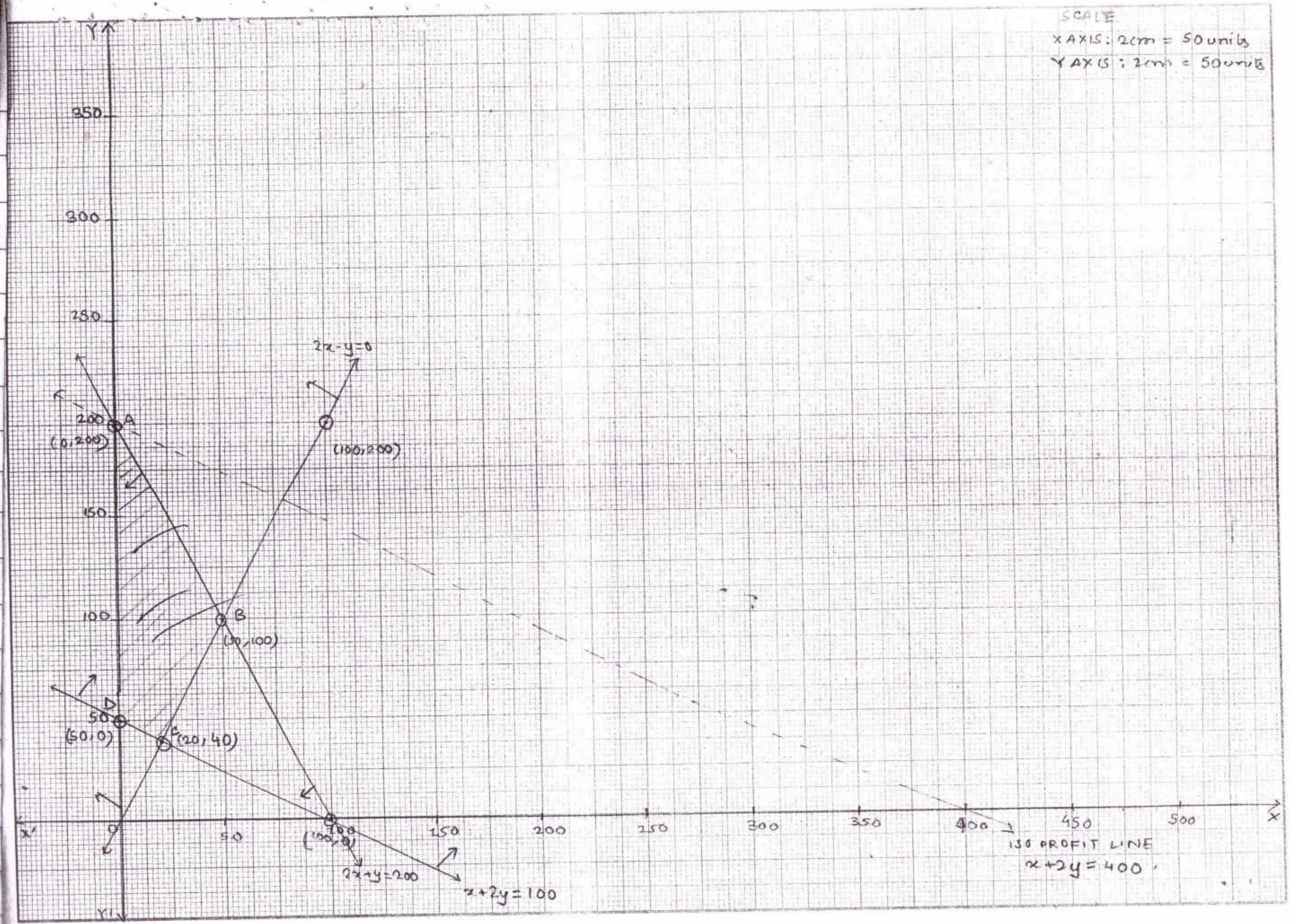
$$y e^{\tan^{-1}x} = \int e^{\tan^{-1}x}\frac{\tan^{-1}x}{1+x^2}dx + c$$

Let $\int e^{\tan^{-1}x}\frac{\tan^{-1}x}{1+x^2}dx$ be I_1

$\tan^{-1}x = t$

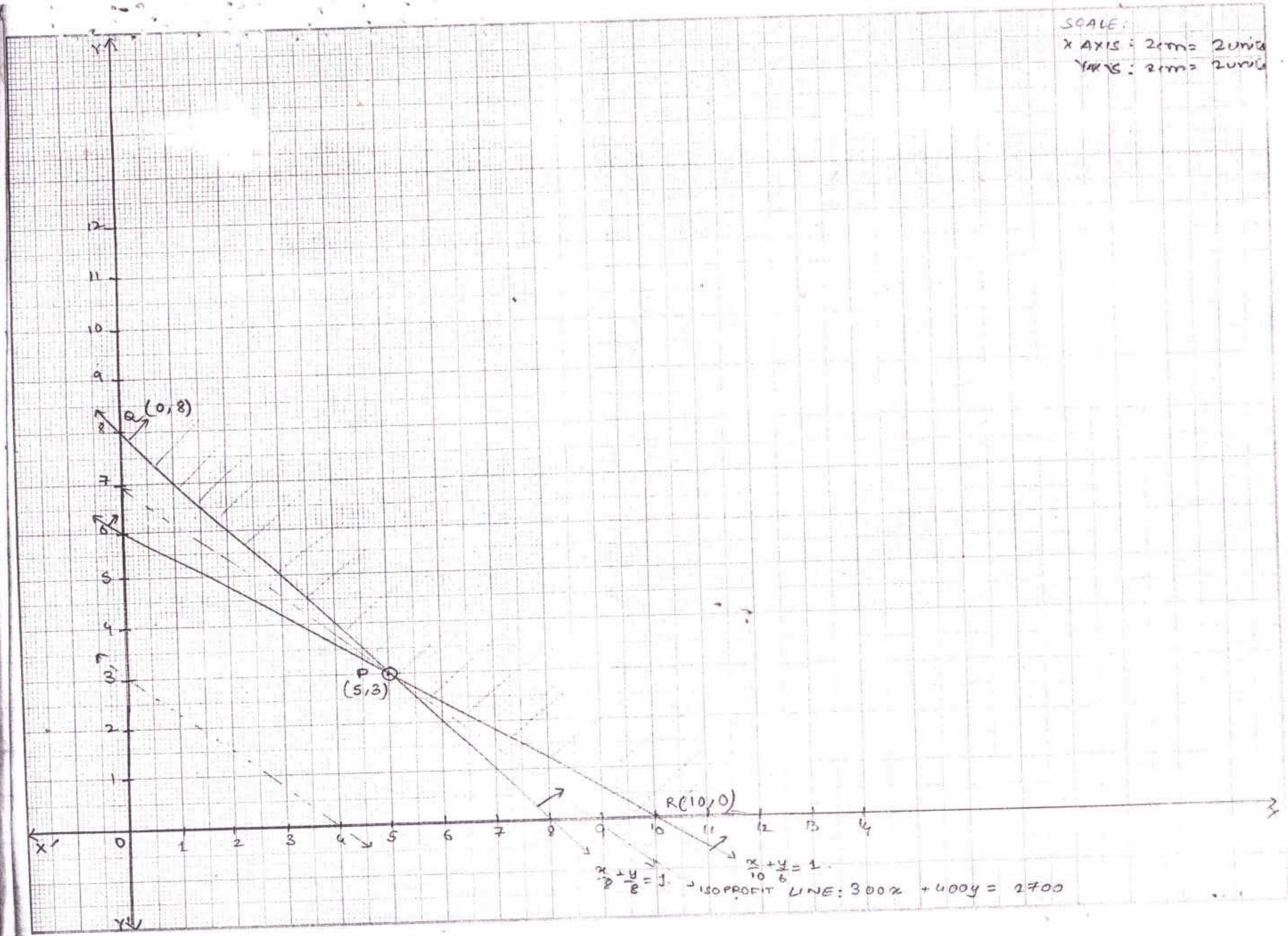
$\frac{1}{1+x^2}dx = dt$

$$I_1 = \int e^t t\, dt$$



SCALE
X AXIS : 2cm = 50 units
Y AXIS : 2cm = 50 units
Y
X'
X
Y'
350
300
250
200
150
100
50
0
50
100
150
200
250
300
350
400
450
500
2x-y=0
A
(0,200)
(100,200)
B
(50,100)
D
(50,0)
C(20,40)
(100,0)
2x+y=200
x+2y=100
ISO PROFIT LINE
x+2y=400

20



SCALE:
X AXIS : 2cm = 2units
Y AXIS : 2cm = 2units
Y
X'
Y'
Q (0,8)
P (5,3)
R(10,0)
x/10 + y/6 = 1.
x/8 + y/8 = 1.
ISOPROFIT LINE: 300x + 400y = 2700

20

$$I_1 = \int \underset{II}{e^t}\, \underset{I}{t}\, dt$$

$$= t \int e^t dt - \int \left(\frac{d(t)}{dt} \int e^t dt \right) dt \;\cancel{+c}$$

$$= te^t - \int 1 \cdot e^t \, dt$$

$$= te^t - e^t$$

$$= e^t (t-1)$$

$$= e^{\tan^{-1}x} (\tan^{-1}x - 1)$$

$$\therefore\; y e^{\tan^{-1}x} = e^{\tan^{-1}x}(\tan^{-1}x - 1) + c.$$

$$y = (\tan^{-1}x - 1) + \frac{c}{e^{\tan^{-1}x}}$$

$$y = \tan^{-1}x - 1 + c e^{-\tan^{-1}x}$$

20

19a) $\vec{OA} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\vec{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$= -\hat{i} - 2\hat{j} - 6\hat{k}$ $\qquad |\vec{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{41}$ units

36, 25 + 9 + 1, 4 + 1 + 1

$\vec{AC} = \vec{OC} - \vec{OA} = \hat{i} - 3\hat{j} - 5\hat{k}$ $\qquad |\vec{AC}| = \sqrt{1^2 + (-3)^2 + (-5)^2} = \sqrt{35}$ units

$\vec{BC} = \vec{OC} - \vec{OB} = 2\hat{i} - \hat{j} + \hat{k}$ $\qquad |\vec{BC}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{6}$ units

$\vec{AC} \cdot \vec{BC} = 2 + 3 - 5 = 0$. Hence $\vec{AC} \perp \vec{BC}$ Hence $\angle C = 90°$

$|\vec{BC}|^2 + |\vec{AC}|^2 = |\vec{AB}|^2$

Hence ~~tria~~ ΔABC is right angled at C.

area $= \frac{1}{2} |\vec{AC} \times \vec{BC}|$

$$= \frac{1}{2} \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -5 \\ 2 & -1 & 1 \end{vmatrix} \right| = \frac{1}{2} \left| \hat{i}(-8) - \hat{j}(11) + \hat{k}(5) \right|$$

$$= \frac{1}{2}\sqrt{(-8)^2 + (-11)^2 + (5)^2} = \cancel{\frac{1}{2}\sqrt{209}\text{ sq. units}}$$

$$= \frac{1}{2}\sqrt{210}\text{ sq. units}$$

$$= \sqrt{\frac{210}{4}} = \sqrt{52.5}\text{ sq. units.}$$

121, 25, 46, 64, 10

64, 121, 25, 209

35 × 6 = 210

52.5 × 4 = 210.0

20) $\overrightarrow{OA} = 3\hat{i} + 6\hat{j} + 9\hat{k}$

$\overrightarrow{OB} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{OC} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\overrightarrow{OD} = 4\hat{i} + 6\hat{j} + \lambda\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -2\hat{i} - 4\hat{j} - 6\hat{k} = \vec{a}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -\hat{i} - 3\hat{j} - 8\hat{k} = \vec{b}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \hat{i} + 0\hat{j} + (\lambda - 9)\hat{k} = \vec{c}$

Scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}] = 0$ $\because$ $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

$$\begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & \lambda-9 \end{vmatrix} = 0$$

$$\begin{array}{r} 2_1 7 \\ \times 4 \\ \hline 68 \\ 50 \end{array}$$

$$(-2)(-3\lambda+27) + 4(-\lambda+9+8) - 6(3) = 0.$$

$$6\lambda - 54 - 4\lambda + 68 - 18 = 0.$$

$$2\lambda = 54 + 18 - 68$$

$$2\lambda = 72 - 68 = 4$$

$$\lambda = 2$$

21)

X	$P(X)$	$X\,P(X)$	$P(X)\,X^2$	
4	$P((1,3),(3,1))$ $= \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} = \frac{2}{12}$	$\frac{\cancel{2}}{\cancel{12}_{\cancel{6}\,3}}\times \cancel{4}^{2} = \frac{2}{3}$	$\frac{2}{3}\times 4 = \frac{8}{3}$	
6	$P((1,5),(5,1))$ $= \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} = \frac{2}{12}$	$\frac{2}{12}\times 6 = 1$	$1\times 6 = 6$	
8	$P((1,7),(7,1),(3,5),(5,3))$ $= \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} = \frac{4}{12}$	$\frac{\cancel{4}}{\cancel{12}_{3}}\times 8 = \frac{8}{3}$	$\frac{8}{3}\times 8 = \frac{64}{3}$	40.667 30 70.667 64.000 6.667
10	$P((3,7),(7,3))$ $= \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} = \frac{2}{12}$	$10\times\frac{\cancel{2}}{\cancel{12}_{6}} = \frac{5}{3}$	$\frac{5}{3}\times 10 = \frac{50}{3}$	
12	$P((5,7),(7,5))$ $= \frac{1}{4}\times\frac{1}{3} + \frac{1}{4}\times\frac{1}{3} = \frac{2}{12}$	$\cancel{12}\times\frac{2}{\cancel{12}} = 2$	$2\times 12 = 24$	

$$\bar{x} = \sum_{i=4}^{12} P(x_i)\, x_i$$

$$= \frac{2}{3} + 1 + \frac{8}{3} + \frac{5}{3} + 2$$

$$= 3 + \cancel{\frac{15}{3}}\ 5 = 8$$

Mean $= 8$

$$\text{Variance} = \cancel{\left(\sum_{i=4}^{12} P(x_i)\, x_i\right)^2} \quad \sum_{i=4}^{12} P(x_i)\, x_i^2 - \left(\sum_{i=4}^{12} P(x_i)\, x_i\right)^2$$

$$= \frac{122}{3} + 30 - 64$$

$$= 40.667 + 30 - 64$$

$$= 70.667 - 64$$

$$= 6.667.$$

2) E_1 : Event that students have 100% attendance

$P(E_1) = \frac{30}{100}$

E_2 : Event that students are irregular

$P(E_2) = \frac{70}{100}$

A : Student has grade A

$P(A|E_1) = \frac{70}{100}$

$P(A|E_2) = \frac{10}{100}$

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{30}{100} \times \frac{70}{100}}{\frac{30}{100} \times \frac{70}{100} + \frac{70}{100} \times \frac{10}{100}}$$

$$= \frac{21\cancel{0}\cancel{0}}{28\cancel{0}\cancel{0}} = \frac{3}{4}$$

2100
700
2800

$\frac{21}{21+7} = \frac{\cancel{21}\,3}{\cancel{28}\,4}$

Yes regularity is required in school for discipline as well as scoring well in academics.

23) $Z = x + 2y$

Constraints: $x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x \geq 0$

$y \geq 0$.

$L_1 \Rightarrow x + 2y = 100$

$\frac{x}{100} + \frac{y}{50} = 1.$

Pts: $(100, 0)$ $(\cancel{50}\ 0, 50)$

$L_2 \Rightarrow 2x - y = 0$

$y = 2x$

Pts: $(50, 100)$, $(100, 20$

$L_3 \Rightarrow 2x + y = 200$

$\frac{x}{100} + \frac{y}{200} = 0.$

Pts $(100, 0)$, $(0, 200)$

Solution is in the region ABCD.

A	$(0, 200)$	$Z = x + 2y = 400$
B	$(50, 100)$	$Z = x + 2y = 50 + 200 = 250$
C	$(20, 40)$	$Z = x + 2y = 20 + 80 = 100$
D	$(50, 0)$	$Z = x + 2y = 50$

Hence Z is maximum when $x = 0$ $y = 200$ [at $(0, 200)$]

SECTION D :-

4)
$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-4)1 + 4(1) + 4(2) & -4(-1) + 4(-2) + 4(1) & -4(1) + 4(-2) + 4(3) \\ -7(1) + 1(1) + 3(2) & -7(-1) + 1(-2) + 3(1) & -7(1) + 1(-2) + 3(+3) \\ 5(1) - 3(1) - 1(2) & 5(-1) - 3(-2) - 1(1) & 5(1) - 3(-2) - 1(3) \end{bmatrix}$$

$7 + 2$
-6
$7 + 2 + 3,$
$\cancel{-1 + 6 - 5}$
$5 + 6 - 3 \; 3$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$x = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -4(4) + 4(9) + 4(1) \\ -7(4) + 1(9) + 3(1) \\ 5(4) - 3(9) - 1(1) \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -16 + 36 + 4 \\ -28 + 9 + 3 \\ 20 - 27 - 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

Hence $x = 3$

$y = -2$

$z = -1$

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} A = 8I$$

$\cancel{AA^{-1} = I}$

$\cancel{\frac{1}{8}A = A^{-1}}$

$$A^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

28
-12
16

-19
$+3$
-16

5) $f: R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ $\qquad f(x) = \frac{4x+3}{3x+4}$

Let $f(x_1) = f(x_2)$

$$\frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4}$$

$$(4x_1+3)(3x_2+4) = (4x_2+3)(3x_1+4)$$

$$\cancel{12x_1x_2} + 9x_2 + 16x_1 + \cancel{12} = \cancel{12x_1x_2} + 9x_1 + 16x_2 + \cancel{12}$$

$$7x_1 = 7x_2$$

$$x_1 = x_2$$

Hence $f(x)$ is one to one.

Let $y = \frac{4x+3}{3x+4}$

$$y(3x+4) = 4x+3$$

$$3xy + 4y - 4x = 3$$

$$x(3y-4) = 3-4y.$$

$$x = \frac{3-4y}{(3y-4)}$$

Hence for every $y \in R - \left\{\frac{4}{3}\right\}$ there exists x such that $x \in R - \left\{\frac{-4}{3}\right\}$.

$$\frac{-4}{3} = \frac{3-4y}{3y-4}$$

$$-12y + 16 = 9 - 12y$$

$$16 = 9$$

which is not possible ; Hence x cannot be $-\frac{4}{3}$.

Hence $f(x)$ is onto function.

$\therefore$ $f(x)$ is a bijective. Hence f^{-1} exists

~~$f(f^{-1})$~~ ~~$fof(x) = x$~~ By the definition of inverse

$$f(f^{-1}(x)) = x$$

$$\frac{4f^{-1}(x) + 3}{3f^{-1}(x) + 4} = x.$$

$$4f^{-1}(x) + 3 = 3xf^{-1}(x) + 4x$$

$$f^{-1}(x)(4 - 3x) = 4x - 3$$

$$f^{-1}(x) = \frac{4x-3}{4-3x} = \frac{3-4x}{3x-4}$$

$$f^{-1}(0) = \frac{3-4(0)}{3(0)-4} = \frac{3}{-4} = \frac{-3}{4}$$

$$f^{-1}(x) = 2$$

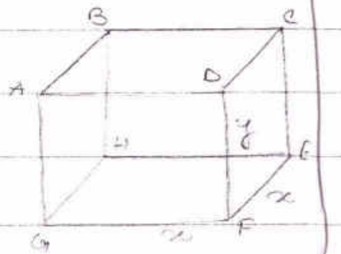
$$\frac{3-4x}{3x-4} = 2 \Rightarrow 3 - 4x = 6x - 8$$

$$10x = 11$$

$$x = \frac{11}{10}$$

Let a closed cuboid have a base of $x \times x$ and a height y

Let its volume be V and surface area be S



$$V = x^2 y \qquad S = 2(x^2 + 2xy)$$

$$y = \frac{V}{x^2} \qquad = 2x^2 + 4xy$$

$$= 2x^2 + 4x\frac{V}{x^2}$$

$$= 2x^2 + \frac{4V}{x}.$$

$$\frac{dS}{dx} = \frac{d}{dx}\left(2x^2 + \frac{4V}{x}\right) = 4x + \frac{4V(-1)}{x^2} = 4x - \frac{4V}{x^2}$$

$$\frac{d^2S}{dx^2} = 4 - \frac{4V(-2)}{x^3} = 4 + \frac{8V}{x^3}.$$

To maximize or minimize S, $\frac{dS}{dx} = 0$

$$4x - \frac{4V}{x^2} = 0.$$

$$\cancel{4}x = \frac{\cancel{4}V}{x^2}$$

$$x^3 = V$$

$$x = V^{1/3}$$

$$\left.\frac{d^2S}{dx^2}\right|_{x = V^{1/3}} = 4 + \frac{8V}{V} = 12 \text{ which is } > 0$$

Hence S is minimum at $x = V^{1/3}$

$$x = v^{1/3}.$$
$$x^3 = v$$
$$x^3 = x^2 y$$
$$x^3 - x^2 y = 0$$
$$x^2(x-y) = 0$$
$$x = y.$$

Hence the given cuboid is a cube of side x.

7) $4y = 3x^2$

$\Rightarrow y = \dfrac{3x^2}{4}$

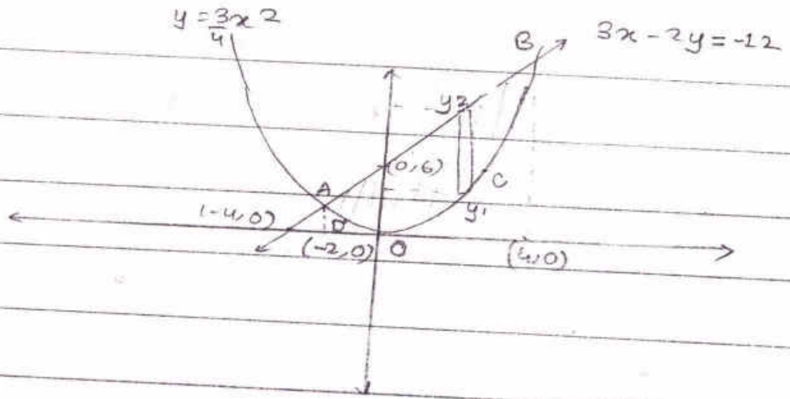
$3x - 2y + 12 = 0$

$\Rightarrow 2y - 3x = 12$

$\Rightarrow \dfrac{y}{6} - \dfrac{x}{4} = 0$

$\Rightarrow \dfrac{y}{6} + \dfrac{x}{-4} = 0$

$\Rightarrow 2y = 3x + 12$.



To find points of intersection :-

$4y = 3x^2$

$2(2y) = 3x^2$

$2(3x + 12) = 3x^2$

$6x + 24 = 3x^2$

$3x^2 - 6x - 24 = 0$

$x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0$

$x(x-4) + 2(x-4) = 0$

$(x+2)(x-4) = 0$.

$x = -2$ or 4

Hence of ODABCO

area = $\int_{-2}^{4} (y_2 - y_1)\,dx$

$= \int_{-2}^{4} \frac{3x+12}{2} - \frac{3x^2}{4}\,dx$

$= \left[\frac{3}{2}\frac{x^2}{2} + 6x - \frac{\cancel{3}}{4}\frac{x^3}{\cancel{3}}\right]_{-2}^{4}$

$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4}\right]_{-2}^{4}$

$= \left[\frac{3}{\cancel{4}}(\cancel{16}^{4}) + 6(4) - \frac{\cancel{64}^{16}}{\cancel{4}}\right] - \left[\frac{3}{\cancel{4}} \times \cancel{4} - 12 - \frac{(\cancel{-8})^{2}}{\cancel{4}}\right]$

$= [12 + 24 - 16] - [3 - 12 + 2]$

$= 20 + 7$

$= 27$ sq. units

$= 27$ sq. units

$$(x-y)\frac{dy}{dx} = x+2y.$$

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

$$\frac{dy}{dx} = \frac{1+2\left(\frac{y}{x}\right)}{1-\left(\frac{y}{x}\right)}$$

$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ Hence the solution of differential equation is that of a homogenous solution.

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}.$$

$$v + x\frac{dv}{dx} = \frac{1+2v}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+2v}{1-v} - v = \frac{1+2v-v+v^2}{1-v} = \frac{1+v+v^2}{1-v}.$$

$$\frac{1-v}{1+v+v^2}\,dv = \frac{dx}{x}.$$

On integrating both sides :-

$$\int \frac{1-v}{1+v+v^2}\,dv = \int \frac{dx}{x} + c.$$

$$\int \frac{1-v}{1+v+v^2}\,dv = \ln|x| + c.$$

$$\int \frac{1-v}{1+v+v^2}\,dv = \ln|x| + c$$

$$\frac{1}{2}\int \frac{2-2v}{1+v+v^2}\,dv = \ln|x| + c$$

$$\frac{-1}{2}\left[\int \frac{2v+1}{1+v+v^2}\,dv - 3\int \frac{dv}{1+v+v^2}\right] = \ln|x| + c$$

$$-\frac{1}{2}\left[\int \frac{dv}{1+v+v^2} - 3\int \frac{dv}{v^2+2\cdot v\cdot\frac{1}{2}+\frac{1}{4}+\frac{3}{4}}\right] = \ln|x| + c$$

$$\frac{-1}{2}\left[\int \frac{dt}{t} - 3\int \frac{dv}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right] = \ln|x| + c$$

Let $1+v+v^2 = t$

$(1+2v)\,dv = dt$

$$\frac{-1}{2}\left[\ln|t| - 3\,\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{v+1/2}{\sqrt{3}/2}\right)\right] = \ln|x| + c$$

$\left[\because \int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$$-\frac{1}{2}\left[\ln|1+v+v^2| - 2\sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right)\right] = \ln|x| + c.$$

$$-\frac{1}{2}\ln\left|1+\frac{y}{x}+\frac{y^2}{x^2}\right| + \sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) - \ln|x| = c$$

$$2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = \ln|x^2| + \ln\left|1+\frac{y}{x}+\frac{y^2}{x^2}\right| + 2c$$

$2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = \ln|x^2+xy+y^2| + c'$ $[c' = 2c]$

$y=0$, $x=1$

$30^\circ = \frac{\cancel{30}}{\cancel{180}}$
6

$2\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \ln|1| + c'$

$c' = \cancel{2}\sqrt{3} \times \frac{\pi}{\cancel{6}3} = \frac{\sqrt{3}\pi}{3} = \frac{\pi}{\sqrt{3}}$.

Hence ;

$2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = \ln|x^2+xy+y^2| + \frac{\pi}{\sqrt{3}}$.

29) ~~Let the plane~~

~~A~~ Let $A(a, 0, 0)$

$B(0, b, 0)$

$C(0, 0, c)$

Then centroid of ΔABC is $G\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$.

~~Let the~~ The equation of the plane is given by :-

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-a & y & z \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0$$

$\Rightarrow (x-a)(bc) - y(-ac) + z(ab) = 0$

$xbc - abc + acy + abz = 0$

$bcx + acy + abz - abc = 0 \quad \Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} - 1 = 0$

distance of the origin from the plane $= 3p$

$$\left| \frac{-1}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}} \right| = 3p$$

$$\frac{1}{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2} = 9p^2$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2}$$

Let the centroid G be at (x, y, z)

$x = \frac{a}{3}$ $y = \frac{b}{3}$ $c = \frac{z}{3}$

$$\left(\frac{3}{a}\right)^2 + \left(\frac{3}{b}\right)^2 + \left(\frac{3}{c}\right)^2 = \frac{1}{p^2}$$

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$$

Proved.

Hence locus of G is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$