WITH GRAPH PAPER

केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली

सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)

परीक्षार्थी प्रवेश-पत्र के अनुसार भरें

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Write Code No. as written on the top of the Question paper : 65/1

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प्रमाणित किया जाता है कि ~~मैंने~~ इस उत्तर-पुस्तिका का मूल्यांकन [illegible] अनुसार किया है।

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2 3

[SECTION - C]

Ans-23>

$$4x + 3y + 2z = 60$$
$$x + 2y + 3z = 45$$
$$6x + 2y + 3z = 70$$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad D = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

Here,

$$AX = D$$

$A^{-1}(AX) = A^{-1}D$ multiplying both side by A^{-1} (A inverse)

$$\therefore (A^{-1}A)X = A^{-1}D$$

$\therefore IX = A^{-1}D$ $(\because A^{-1}A = I)$ (I - Identity Matrix)

$$\therefore X = A^{-1}D$$

Now, $A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$

$|A| = \operatorname{Det}(A)$

$\therefore |A| = 4\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 6 & 3 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 6 & 2 \end{vmatrix}$

$= 4(6-6) - 3(3-18) + 2(2-12)$

$= 0 - 3(-15) + 2(-10)$

$= 45 - 20 = 25$

$\therefore |A| = 25$

Now, $\operatorname{adj}(A) = \cancel{\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$

where A_{ij} = Co-factor of respective i^{th} row & j^{th} column

$\therefore$ $A_{11} = 0$ $\quad A_{12} = -(3-18) = 15$ $\quad A_{13} = 2-12 = (-10)$

$A_{21} = -(9-4) = (-5)$ $\quad A_{22} = 12-12 = 0$ $\quad A_{23} = -(8-18) = 10$

$A_{31} = 9-4 = 5$ $\quad A_{32} = -(12-2) = (-10)$ $\quad A_{33} = 8-3 = 5$

$$\therefore \ adj(A) = \begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$$

$$\therefore \ A^{-1} = \frac{1}{|A|} adj(A) = \frac{1}{25}\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}$$

$$\therefore \ X = A^{-1} D$$

$$= \frac{1}{25}\begin{bmatrix} 0 & -5 & 5 \\ 15 & 0 & -10 \\ -10 & 10 & 5 \end{bmatrix}\begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$$= \frac{1}{25}\begin{bmatrix} -5\times45 + 5\times70 \\ 15\times60 + 0\times45 - 70\times10 \\ -10\times60 + 10\times45 + 5\times70 \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 125 \\ 200 \\ 200 \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 125 \\ 200 \\ 200 \end{bmatrix}$$

$$\therefore \ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\therefore x = 5, \ y = 8, \ z = 8$

Ans-24)

A window,
rectangular surmounted
by an equilateral triangle.

Now, perimeter = 12

$\therefore 3x+2y = 12$

$y = \left(\frac{12-3x}{2}\right)$

Now, Area of window = Area of equilateral triangle + Area of rectangle

Area of equilateral (side x) triangle $= \frac{\sqrt{3}}{4}x^2$

Area of rectangle $= xy$

$\therefore$ Area (total) $\Rightarrow A = xy + \frac{\sqrt{3}}{4}x^2$

$= x\left(\frac{12-3x}{2}\right) + \frac{\sqrt{3}}{4}x^2$

$= \frac{12x - 3x^2}{2} + \frac{\sqrt{3}}{4}x^2 = \left(\frac{24x - 6x^2 + \sqrt{3}x^2}{4}\right)$

Now, for largest Area $\frac{dA}{dx} = 0$

and $\frac{dA^2}{dx^2}$ = (negative)

$\frac{dA^2}{dx^2} < 0$

$\therefore \frac{dA}{dx} = \frac{1}{4}\left(24 - 12x + 2\sqrt{3}x\right) = 0$

$\therefore 24 = 12x - 2\sqrt{3}x \Rightarrow x(12 - 2\sqrt{3}) = 24$

$x = \left(\frac{12}{6-\sqrt{3}}\right)$

Now, Second derivative test $\frac{dA^2}{dx^2} = \frac{1}{4}\left(-12 + 2\sqrt{3}\right) = \frac{2}{4}\left(\sqrt{3} - 6\right)$

$\therefore \frac{dA^2}{dx^2} < 0$

$\therefore$ At $x = \frac{12}{6-\sqrt{3}}$, the Area is Largest

$\therefore$ Area $= xy + \frac{\sqrt{3}}{4}x^2$

$= \frac{1}{4}\left(24x - 6x^2 + \sqrt{3}x^2\right)$

and $3x + 2y = 12$

$\therefore$ $y = \frac{12-3x}{2} = \frac{1}{2}\left(12 - 3\left(\frac{12}{6-\sqrt{3}}\right)\times\frac{(6+\sqrt{3})}{(6+\sqrt{3})}\right]$

$= \frac{1}{2}\left(12 - \frac{3\times12}{(36-3)}(6+\sqrt{3})\right]$

$= \frac{1}{2}\left(12 - \frac{3\times12}{33}(6+\sqrt{3})\right)$

$= \frac{1}{2}\left(12 - \frac{12}{11}(6+\sqrt{3})\right)$

$$= \frac{1}{2} \times 12 \left(\frac{11 - 6 - \sqrt{3}}{11} \right) = \frac{6}{11}\left(5-\sqrt{3}\right)$$

$\therefore$ Dimension of rectangle $x \times y$

$$= \left(\frac{12}{6-\sqrt{3}} \times \frac{6}{11}(5-\sqrt{3}) \right)$$

$$x = \frac{12}{6-\sqrt{3}} \qquad y = \frac{6}{11}(5-\sqrt{3})$$

25) $$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\, dx$$

$$\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx \quad -(1)$$

Now, $\tan x = \dfrac{\sin x}{\cos x}$

Now, using property of definite Integral i.e $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$\therefore\ I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/2 - x)}}{\sqrt{\cos(\pi/2 - x)} + \sqrt{\sin(\pi/2 - x)}}\,dx$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx \quad -(2)$

Now,

adding eqn (1) & eqn (2)

$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx = \int_{\pi/6}^{\pi/3} 1\,dx$

$= \left[x\right]_{\pi/6}^{\pi/3} = \pi/3 - \pi/6 = \pi/6$

$\therefore\ I = \frac{\pi}{6 \times 2} = (\pi/12)$

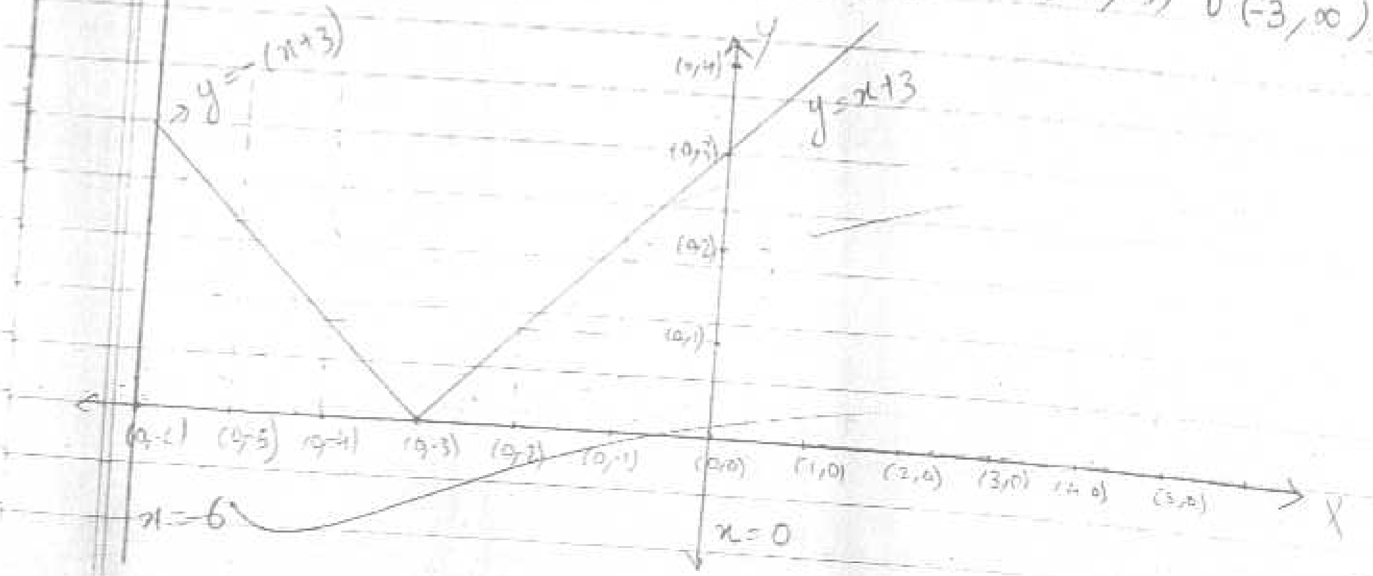
$$\therefore \int_{\pi/6}^{\pi/3} \frac{dx}{\sqrt{\tan x}+1} = \pi/12$$

Ans-26) $y = |x+3|$

$$|x+3| = 0$$

$$x = -3$$

$\therefore$ We have two intervals $(-\infty,-3) \cup (-3,\infty)$



Now,

for $x \in (-3, \infty)$, $y = (x+3)$

for $x \in (-\infty, -3)$ $y = -(x+3)$

$\therefore$ Area under the curve $y = |x+3|$ above x-axis and between $x = -6$ to $x = 0$

$$\text{Area} = \int_{-6}^{0} y \, dx = \int_{-6}^{-3} y \, dx + \int_{-3}^{0} y \, dx$$

for, $x \in (-6, -3)$, $y = -(x+3)$

$x \in (-3, 0)$ $y = (x+3)$

$$\therefore A = \int_{-6}^{-3} -(x+3) \, dx + \int_{-3}^{0} (x+3) \, dx$$

$$= (-1) \left[\frac{x^2}{2} + 3x \right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x \right]_{-3}^{0}$$

$$= (-1) \left[\left(\frac{9}{2} - 3(3) \right) - \left(\frac{6^2}{2} - 3 \times 6 \right) \right] + \left[(0+0) - \left(\frac{9}{2} - 3 \times 3 \right) \right]$$

$$= (-1) \times \left[\frac{9}{2} - 9 - (18-18) \right] + \left[9 - 9/2 \right]$$

$$= \frac{9}{2} + \frac{9}{2} = 9$$

Area $= 9$ unit^2

Ans-27) Eq^n of line : $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \to$ Vectorial form

$$\therefore \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda \to \text{Cartesian form}$$

Let, there be pt on line. $[P((3\lambda+2)\hat{i} + (4\lambda-1)\hat{j} + (2\lambda+2)\hat{k})]$

Equation of plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ (vectorial form)

$x - y + z = 5$ (cartesian eq^n)

Now, the point $P((3\lambda+2)\hat{i} + (4\lambda-1)\hat{j} + (2\lambda+2)\hat{k})$ will lie on the plane and intersect the plane

$$\therefore (3\lambda+2) - (4\lambda-1) + (2\lambda+2) = 5$$

$$\lambda + 2 + 1 + 2 = 5 \qquad (\lambda = 0)$$

$\therefore$ The point is $(3(0)+2)\hat{i} + (4\times0-1)\hat{j} + (2\times0+2)\hat{k} \Rightarrow P$

$\therefore$ Point P is $2\hat{i} - \hat{j} + 2\hat{k}$

Now, we have to find the distance of this point with other point $Q(-\hat{i} - 5\hat{j} - 10\hat{k})$

then we have to apply Distance formula

$$D = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$

for pts (x_1, y_1, z_1) & (x_2, y_2, z_2) in a plane

Similarly,

Here, distance between Point P and Q

is $D_{PQ} = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$

$= \sqrt{9 + 16 + 144} = \sqrt{169} = 13$

$\therefore$ The distance of point $(-1, -5, -10)$ from the point

of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$
and the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$ is 13.

QS-28

Boxes	I	II	III
coins	G, G	S, S	G, S

Boxes contains coins

G = Gold , S = Silver

A person chooses a box at random & takes out a coin. If the coin is of gold, ~~~~ the probability that the other coin in the box is also of gold.

For this we will use Baye's theorem,
According to which,

$$P(E_i/A) = \frac{P(E_i \cap A)}{P(A)}$$

$$P(A) = P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + \ldots + P(E_n).A(E/E_n)$$

$$\therefore P(E_i/A) = \frac{P(E_i).\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i).\,P(A/E_i)}$$

Here, $P(A) \rightarrow$ the probality of choosing gold coin

$E_1 \rightarrow$ probability of other coin of gold in Box I
$E_2 \rightarrow$ Event of other coin of gold Box II
$E_3 \rightarrow$ Event of other coin of gold Box III

$$\therefore P(E_1/A) = \frac{P(E_1).\,P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + P(E_3).P(A/E_3)}$$

$$= \frac{1/3 \times 1}{1/3 \times 1 + 1/3 \times 0 + 1/3 \times 1/2} = \frac{1}{1 + 1/2} = \left(\frac{2}{3}\right)$$

Ans-29) let no. of desktop computers be x units and portable model be y units

Now, According to the conditions, $[x, y \geq 0]$

$$\left[x+y \leq 250\right] \quad \frac{x}{250}+\frac{y}{250} \leq 1$$

$$25{,}000x + 40{,}000y \leq 70{,}00{,}000$$

$$25x + 40y \leq 70{,}00 \qquad \cancel{25x + 40y \leq 7{,}000}$$

$$\left[5x+8y \leq 1400\right]$$

$$\frac{x}{280}+\frac{y}{175} \leq 1$$

Now, objective is $z = [4500x + 5000y]$ (Maximise)

$= 500(9x + 10y)$

Now, the corner points and the feasible region are shown in the graph

In the graph the points encircled ⊙ → corner points and the shaded region is the feasible region

For optimum solution.

$$z = 500(9x + 10y)$$

Maximise $z = 500(9x+10y)$ (in RS)

Corner points	Maximize $z = 500(9x+10y)$
(0,0)	Rs 0
(250,0)	Rs 11,25,000
(0,175)	Rs 8,75,000
(200,50)	$\boxed{\text{Rs } 11,50,00}$

Intersection of

$5x+8y=1400$

$x+y=250$

$5x+5y=1250$

$3y=150$

$y=50$

$x=200$

∴ Thus to get maximum profit, the merchant should cost 200 desktop models and 50 portable model. And the maximum profit is RS 11,50,000

Reference : Graph

[SECTION B]

11) $f: R \to R \quad f(x) = 10x + 7$

$g: R \to R$

To find: $g: R \to R$, such that $fog = fog = I_R$

Therefore the function $g(x)$ is must be the inverse of $f(x)$

Now, $f(x)$ is one & onto (bijective)

as $f(x_1) = f(x_2)$ $\quad f(x) = 10x_1 + 7$

$10x_1 + 7 = 10x_2 + 7$

$x_1 = x_2$

and Now, $f(x) = y = 10x + 7$

$10x = y - 7 \qquad x = \frac{y-7}{10}$

$\therefore g(y) = x = \frac{y-7}{10} \qquad \therefore g(x) = \frac{x-7}{10}$

Now, checking fog

$f(g(x)) = fog(x)$

LHS: $f(g(x)) = f\left(\frac{x-7}{10}\right)$ $\left[\because g(x) = \frac{x-7}{10}\right]$

$\therefore f(g(x)) = f\left(\frac{x-7}{10}\right)$

$= 10\left(\frac{x-7}{10}\right) + 7 = x - 7 + 7 = x$ = RHS

$\therefore fog = x$ (Proved) $fog = I_R$ ($R \in$ Real nos)

Now,

LHS = $g(f(x)) = g(10x+7)$ $f(x) = 10x+7$

$= \frac{(10x+7) - \cancel{7}}{10} = \frac{10x}{10} = x$ = RHS

Thus fog and gof are identity function

This means that $g(x)$ is the inverse of $f(x)$

$\therefore g(x) = \frac{x-7}{10}$ ($g : R \longrightarrow R$)

Domain → R, Range → R

~~And, thus LHS = RHS~~

Ans-12) $\tan^{-1}\left[\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}x$

$-\frac{1}{\sqrt{2}} \le x \le 1$

Now, LHS $= \tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \times \dfrac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

$= \tan^{-1}\left(\dfrac{(\sqrt{1+x})^2-(\sqrt{1-x})^2}{(\sqrt{1+x}+\sqrt{1-x})^2}\right)$

$= \tan^{-1}\left(\dfrac{(1+x)-(1-x)}{1+x+1-x+2\sqrt{1-x^2}}\right) = \tan^{-1}\left(\dfrac{2x}{2(1+\sqrt{1-x^2})}\right)$

$= \tan^{-1}\left(\dfrac{x}{1+\sqrt{1-x^2}}\right)$

Now, putting $x = \sin\theta$

$= \tan^{-1}\left(\dfrac{\sin\theta}{1+\sqrt{1-\sin^2\theta}}\right)$

Now, $1-\sin^2\theta = \cos^2\theta$

$= \tan^{-1}\left(\dfrac{\sin\theta}{1+\cos\theta}\right)$

Now, $\sin\theta = 2\sin\theta/2\,\cos\theta/2$

$1+\cos\theta = 2\cos^2\theta/2$

$$= \tan^{-1}\left(\frac{2\sin\theta/2\,\cos\theta/2}{2\cos\theta/2\,\cos\theta/2}\right) = \tan^{-1}(\tan\theta/2)$$

$$= \theta/2 \qquad (\theta = \sin^{-1}x)$$

$$= \frac{1}{2}(\sin^{-1}x)$$

Now $\sin^{-1}x + \cos^{-1}x = \pi/2$

$\sin^{-1}x = \pi/2 - \cos^{-1}x$

$$= \frac{1}{2}(\pi/2 - \cos^{-1}x)$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x = RHS$$ proved

Thus as LHS = RHS

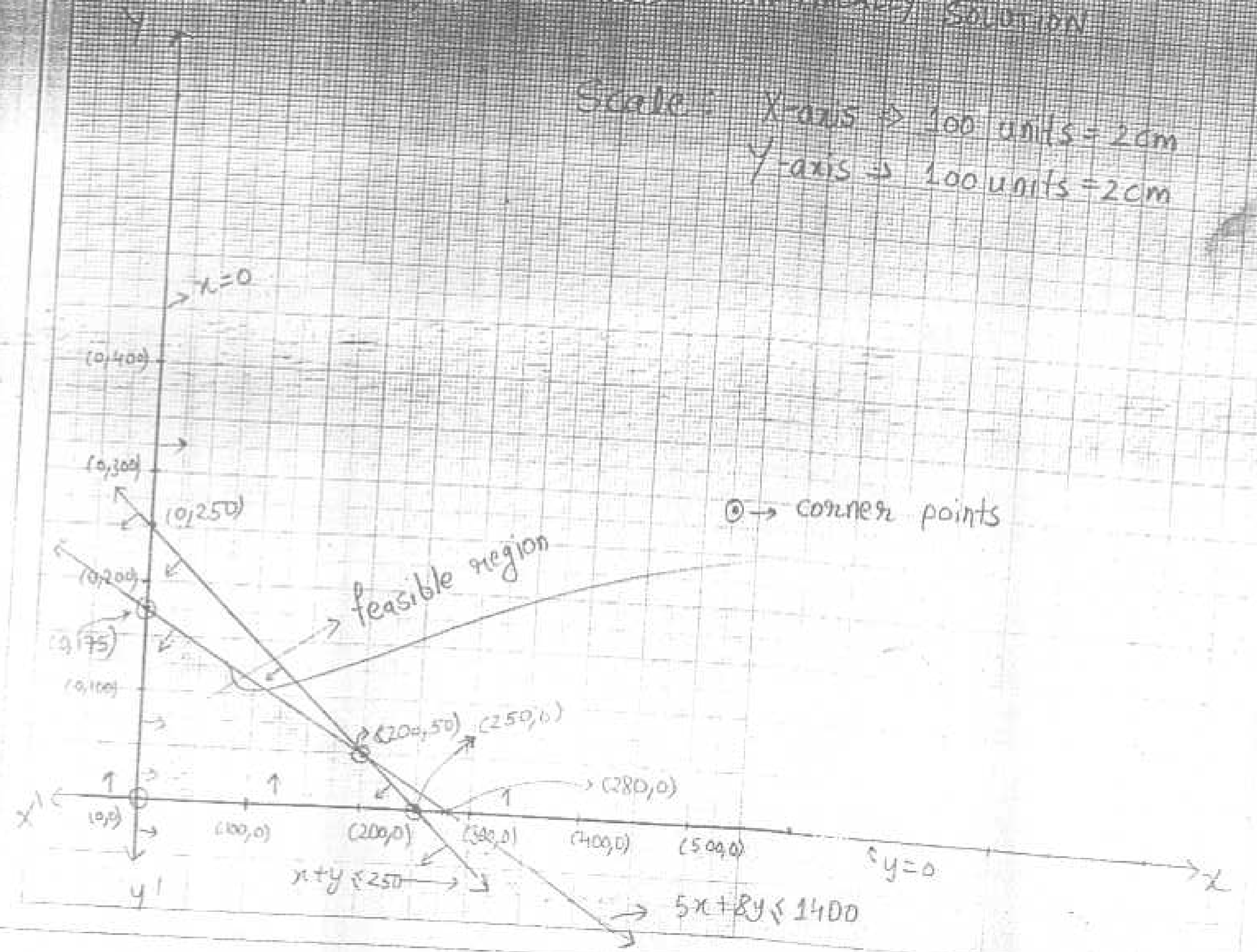
and $\tan^{-1}\left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right] = \pi/4 - \frac{1}{2}\cos^{-1}x$

SECTION C - Q29 continued

Q29) GRAPHICALLY SOLUTION

Scale: X-axis ⇒ 100 units = 2cm

Y-axis ⇒ 100 units = 2cm



ns-13)
$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0 \quad \text{To Prove}$$

$$LHS = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix}$$

$\cancel{C_1 \to C_1 + (C_2 + C_3)}$

$$= \begin{vmatrix} \cancel{x-2+(2x-3)+3x-4} & \cancel{2x-3} & \cancel{3x-4} \\ \cancel{x-4+2x-9+3x-16} & \cancel{2x-9} & \cancel{3x-16} \\ \cancel{x-8+2x-27+3x-64} & \cancel{2x-27} & \cancel{3x-64} \end{vmatrix}$$

$$\cancel{= \begin{vmatrix} 6x-9 \\ 6x- \end{vmatrix}}$$

$R_1 \to R_1 - R_2$

$$= \begin{vmatrix} 2 & 6 & 12 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 6 & 12 \\ 4 & 18 & 48 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} \quad R_2 \to R_2 - R_3$$

$$= 2 \times 2 \begin{vmatrix} 1 & 3 & 6 \\ 2 & 9 & 24 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix}$$

$C_2 \to C_2 - 3C_1$ $\qquad$ $C_3 \to C_3 - C_1$

$$= 4 \begin{vmatrix} 1 & 0 & 6 \\ 2 & 3 & 24 \\ x-8 & -x-3 & 3x-64 \end{vmatrix} = 4 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 12 \\ x-8 & -x-3 & -3x-16 \end{vmatrix} = 0$$

$$\therefore \frac{1}{3}\left(3x(-1)(3x+16) + (x+3)(12) \right) = 0$$

$$4(x+3) - (3x+16) = 0$$

$$4x + 12 - 3x - 16 = 0$$

$$\underline{x = 4}$$

Ans $\Rightarrow$ $\underline{x=4}$

-16 $=0$

Ans-14) $x^y = e^{x-y}$

let $x^y = u$ $\qquad e^{x-y} = v$

Now $\log u = y \log x$ (using product rule)

$\frac{1}{u}\frac{du}{dx} = \log x . \frac{dy}{dx} + \frac{y}{x}$ $\qquad \left(\because \frac{d \log x}{dx} = 1/x\right)$

$\frac{du}{dx} = x^y \left(\log x . \frac{dy}{dx} + y/x\right)$

$\log v = x - y$

$\frac{1}{v}\frac{dv}{dx} = 1 - \frac{dy}{dx}$ $\qquad \therefore \frac{dv}{dx} = e^{x-y}\left(1 - \frac{dy}{dx}\right)$

$\because u = v$

$\therefore$ differentiating both side with respect to x

$\therefore \frac{du}{dx} = \frac{dv}{dx}$

$\therefore x^y \left(\log x . \frac{dy}{dx} + y/x\right) = e^{x-y}\left(1 - \frac{dy}{dx}\right)$

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$\therefore x^y \cdot \log x \cdot \frac{dy}{dx} + y \cdot x^{y-1} = e^{x-y} - e^{x-y}\frac{dy}{dx}$

$\frac{dy}{dx}\left(x^y \cdot \log x + e^{x-y}\right) = e^{x-y} - y\frac{x^y}{x}$

LHS $= \frac{dy}{dx} = \frac{e^{x-y} - y/x \times e^{x-y}}{e^{x-y}\log x + e^{x-y}}$ $\quad (\because x^y = e^{x-y})$

$\therefore \frac{dy}{dx} = \frac{e^{x-y}(x-y) \times 1/x}{e^{x-y}(\log x + \log e)}$ $\quad (\because \log e = 1$

$= \frac{(x-y)}{x\{\log(ex)\}}$ $\quad$ Now, $e^{x-y} = x^y$

$= \frac{y \log x}{x\{\log(ex)\}} = \frac{\log x}{\frac{x}{y}(\log(ex))}$ $\quad \therefore$ $x - y = y\log x$

$\frac{x}{y} = \log(ex)$

$= \frac{\log x}{\{\log(ex)\}^2} = $ RHS $\quad$ $x/y = 1 + \log x$

$\left(\because \frac{x-y}{xy} = \log x\right)$

Ans-15) Now, radius of sphere = 9 cm

$\Delta r = 0.03$ cm

Now, Area $= 4\pi r^2$ (Sphere)

$A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

Now, $\frac{dA}{dr} = \frac{\Delta A}{\Delta r}$ (when $\Delta r \to 0$)

$\therefore \Delta A = \Delta r \times 8\pi r$ (at $r = 9$ cm)

$\Delta A = 0.03 \times 8 \times \pi (9)$ cm^2

$= 2.16\pi$ cm^2

≈ 6.7824 cm^2

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Ans-16) $x = \tan\left(\frac{1}{a}\log y\right)$ (To prove ;
$(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$

Now, differentiating with respect to x on both side

$1 = \sec^2\left(\frac{1}{a}\times\log y\right)\frac{d}{dx}\left(\frac{1}{a}\log y\right)$

$1 = \sec^2\left(\frac{1}{a}\log y\right)\cdot\frac{1}{a}\frac{d(\log y)}{dx}$ (using chain rule)

$1 = \sec^2\left(\frac{1}{a}\log y\right)\times\frac{1}{a}\times\frac{1}{y}\frac{dy}{dx}$

$\therefore ay = \sec^2\left(\frac{1}{a}\log y\right)\frac{dy}{dx}$

$\therefore ay = (1+x^2)\frac{dy}{dx}$

Now, $\sec^2\left(\frac{1}{a}\log y\right) = 1+\tan^2\left(\frac{1}{a}\log y\right)$

$\sec^2\left(\frac{1}{a}\log y\right) = 1+x^2$

$\left(\because x = \tan\left(\frac{1}{a}\log y\right)\right)$

$$ay = (1+x^2)\frac{dy}{dx}$$

Now, differentiating with respect to x on both side

$$a\frac{dy}{dx} = (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} \quad \left(\because \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right) \text{ (Product rule)}$$

$$\therefore (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - a\frac{dy}{dx} = 0$$

$$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(2x-a) = 0$$

Thus proved

Ans-17)
$$\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x}\,dx$$

let
$$\int \frac{x+\sin x}{1+\cos x}\,dx = I$$

$$\therefore I = \int \frac{x+\sin x}{1+\cos x} \times \frac{1-\cos x}{1-\cos x}\, dx$$

$$= \int \frac{(x+\sin x)(1-\cos x)}{1-\cos^2 x}\, dx$$

$$= \int \frac{x - x\cos x + \sin x - \sin x \cos x}{\sin^2 x}\, dx$$

$$= \int x\,\mathrm{cosec}^2 x - x\cot x\,\mathrm{cosec}\, x + \mathrm{cosec}\, x - \cot x)\, dx$$

Now, $\int x f'(x) + f(x) = x f(x) + c$

$$= \int x \frac{d(-\cot x)}{dx} + (-\cot x) + x\frac{d(\mathrm{cosec}\, x)}{dx} + \mathrm{cosec}\, x \; dx$$

$$= -x\cot x + x\,\mathrm{cosec}\, x + c \qquad \left(\because \frac{d\,\mathrm{cosec}\, x}{dx} = -\cot x\,\mathrm{cosec}\, x\right)$$

$$= x\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right) + c$$

$$= x\left(\frac{1-\cos x}{\sin x}\right) + c = \frac{x \times 2\sin^2 x/2}{2\sin x/2 \cos x/2} + c$$

$$\left(\because 1-\cos x = 2\sin^2 x/2 \;\&\; \sin x = 2\sin x/2 \cos x/2\right.$$

$I = x\tan x/2 + C \quad = \left(x\tan\frac{x}{2}\right) + c$ $c \Rightarrow$ constant

$$\therefore \int_0^{\pi/2} I\,dx = \left[x\tan x/2 + c\right]_0^{\pi/2}$$

$$= \left[\frac{\pi}{2}\tan\left(\frac{\pi}{4}\right) + c - 0\tan 0 - c\right]$$

$$= \left(\frac{\pi}{2}\right)(1) + 0$$

$$\therefore \int_0^{\pi/2} \frac{x+\sin x}{1+\cos x}\,dx = \frac{\pi}{2}$$

18) $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$$x\,dy = dx \cdot \left(y + \sqrt{x^2+y^2}\right)$$

$$\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$$

$c \to$ constant

The equation ~~is~~ in the form of homogeneous equation

$\therefore \dfrac{dy}{dx} = \dfrac{y + \sqrt{x^2+y^2}}{x}$

Now, $\dfrac{dy}{dx} = y/x + \sqrt{1+(y/x)^2}$

let $y/x = t$ $\quad y = xt$

$\dfrac{dy}{dx} = t + x\dfrac{dt}{dx}$

$\therefore t + x\dfrac{dt}{dx} = t + \sqrt{1+t^2}$

$\therefore x\dfrac{dt}{dx} = \sqrt{1+t^2}$ $\quad \therefore \dfrac{1}{\sqrt{1+t^2}}\,dt = \dfrac{1}{x}\,dx$

$\int \dfrac{1}{\sqrt{1+t^2}}\,dt = \int \dfrac{1}{x}\,dx$

$\ln(t + \sqrt{t^2+1}) = \ln x + \ln c$ $\quad \ln c \to$ const

$(\because \int \dfrac{1}{\sqrt{\cdots}}\,dx = \ln(x+\sqrt{x^2+1}) + c$

$\therefore \ln\left(t+\sqrt{t^2+1}\right) = \ln(c \times x)$

$\therefore$ taking antilog

$t + \sqrt{t^2+1} = cx$ $\quad c \to$ constant

$\therefore \frac{y}{x} + \sqrt{y^2/x^2+1} = cx$

$y + \sqrt{x^2+y^2} = cx^2$

$\therefore \sqrt{x^2+y^2} + y - cx^2 = 0$

Ans-(9) $\quad (y+3x^2)\frac{dx}{dy} = x$

$\therefore \frac{dy}{dx} = \frac{y+3x^2}{x}$

$\therefore \frac{dy}{dx} = \frac{y}{x} + 3x$

$\therefore \frac{dy}{dx} + y\left(\frac{-1}{x}\right) = 3x$

Now, it is in the form Linear equation or Integrating Factor (I.f)

$\therefore I.f = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = e^{\ln(1/x)} = 1/x$

$\left(\because e^{\ln(a)} = a\right)$

$\left(\because \frac{dy}{dx} + y\,P(x) = Q(x)\right.$

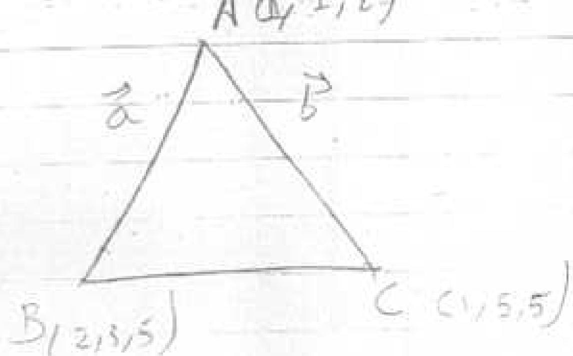
$I.f = e^{\int P(x)\,dx}$

and Solution is $y \times I.f = \int Q(x).(I.f)\,dx$

$\therefore$ Solution is $y \times \frac{1}{x} = \int 3x \times \frac{1}{x}\,dx = \int 3\,dx = 3x + c$

$\therefore y/x = 3x + c$ $\therefore \left(y = 3x^2 + cx\right)$

Ans-20) ΔABC with vertices $A(1,1,2)$, $B(2,3,5)$, $C(1,5,5)$



Now, If we know position vector of two sides of triangle & suppose they are $\vec{a}$ and $\vec{b}$

then Area $= \frac{1}{2} |\vec{a} \times \vec{b}|$

Now, Here, let us suppose side AB has position vector $\vec{a}$ and side AC has P.V $\vec{b}$.

$\therefore \vec{a} = \vec{OB} - \vec{OA}$

where, $\vec{OB} = 2\hat{i} + 3\hat{j} + 5\hat{k}$, $\vec{OA} = \hat{i} + \hat{j} + 2\hat{k}$

$\vec{OC} = \hat{i} + 5\hat{j} + 5\hat{k}$

$\therefore \vec{a} = 2\hat{i} + 3\hat{j} + 5\hat{k} - (\hat{i} + \hat{j} + 2\hat{k})$
$= \hat{i} + 2\hat{j} + 3\hat{k}$

Similarly $\vec{b} = \vec{OC} - \vec{OA} = \hat{i} + 5\hat{j} + 5\hat{k} - (\hat{i} + \hat{j} + 2\hat{k})$
$= 4\hat{j} + 3\hat{k}$

$\therefore$ Area $= \frac{1}{2}|\vec{a} \times \vec{b}|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = \hat{i}(6-12) - \hat{j}(-3) + \hat{k}(4)$$
$= (-6\hat{i} + 3\hat{j} + 4\hat{k})$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{36+9+16}$
$= \sqrt{61}$

Area $= \frac{1}{2}\sqrt{61}$

Ans-21) $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$

$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

Vector eq^{n}s :

$\therefore \vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + t(-\hat{i} + \hat{j} - 2\hat{k})$

$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$

Now, here, $\vec{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{a_2} = (\hat{i} - \hat{j} - \hat{k})$

$\therefore \vec{a_1} - \vec{a_2} = (\hat{i} - 2\hat{j} + 3\hat{k}) - (\hat{i} - \hat{j} - \hat{k})$

$= 0\hat{i} - \hat{j} + 4\hat{k} = (-\hat{j} + 4\hat{k})$

$\vec{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$

$\vec{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = \hat{i}(-2+4) - \hat{j}(2+2) + \hat{k}(-2-1)$

$= 2\hat{i} - 4\hat{j} - 3\hat{k}$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{4+16+9} = \sqrt{29}$

Now, distance between skew lines

$$d = \left| \frac{(\vec{a}_1 - \vec{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\therefore d = \left| \frac{(-\hat{j} + 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k})}{\sqrt{29}} \right|$$

$$= \left| \frac{(4 - 12)}{\sqrt{29}} \right| = \frac{8}{\sqrt{29}} = \left(\frac{8}{29} \sqrt{29} \right)$$

$\frac{8}{\sqrt{29}}$ is the Shortest distance between the lines

Ans-22) Now $\sum_{i=1}^{n} P(x_i) =$

Here $\sum_{i=0}^{7} P(x_i) =$

$\therefore K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$9K + 10K^2 = 1$

$10K^2 + 9K - 1 = 0$

$10K^2 + 10K - K - 1 = 0$

$10K(K+1) - 1(K+1) = 0$

$K = 1/10$ or $K = -1$

but $K \neq -1$

$\therefore K = 1/10$

$\therefore$ i) $K = 1/10$

ii) $P(X<3) = P(0) + P(1) + P(2)$

$= 0 + K + 2K = 3K = 3(1/10) = 3/10$

(iii) $P(X > 6) = P(7)$

$= 7K^2 + K = K(7K + 1)$

$= \frac{1}{10}\left(\frac{7}{10} + 1\right) = \frac{1}{10}\left(\frac{17}{10}\right) = \frac{17}{100}$

(iv) $P(0 < X < 3) = P(1) + P(2)$

$= K + 2K = 3K = \frac{3}{10}$

nutri
(2)

(SECTION A)

1) $f : A \to B$

$A = \{1,2,3\}$ $B = \{4,5,6\}$

Now, $f(1) = 4$, $f(2) = 5$ $f(3) = 6$

$\therefore f(1) \neq f(2) \neq f(3)$

$\therefore$ The function f is one-one

Ans-2) $\cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \pi$

$= \frac{2\pi}{3} + \sin^{-1}(\sin(\pi - \pi/3))$

$= \frac{2\pi}{3} + \sin^{-1}(\sin \pi/3) = \frac{2\pi}{3} + \frac{\pi}{3} = (\pi)$

Ans $= \pi$

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Fictitious Roll No.
(To be entered by Board)

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Please do not write your Roll Number on this Answer-Book

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Supplementary Answer-Book(s) No. ____

(Section - A) Continued

3) Evaluate $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix} = 0$

$= \cos 15^\circ \cdot \cos 75^\circ - \sin 15^\circ \cdot \sin 75^\circ$

$= \cos(15^\circ + 75^\circ) = \cos 90 = 0$

$(\because \cos(A+B) = \cos A \cos B - \sin A \sin B)$

4) $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

$A.A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 19 & 0 \\ 0 & 19 \end{bmatrix}$

$= 19 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 19 I_2$ (Identity matrix of order 2)

$\therefore A^2 = 19 I$

Now

multiplying by A^{-1} on both side

$$A^{-1} A^2 = 19 \times A^{-1} I$$
$$(A^{-1}A) \cdot A = 19 A^{-1}$$
$$\therefore I A = 19 A^{-1}$$
$$A^{-1} = \frac{1}{19} A$$

$A^{-1} = \frac{1}{19} A$ $\quad A^{-1} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$

Ans-5) Elements : 5

Then orders are : 1×5 , 5×1

There are two possible orders

6) $\int (ax+b)^3 \, dx = \frac{(ax+b)^4}{4 \times a} + c$

Ans $= \frac{(ax+b)^4}{4a} + c$

7) $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1}\left(\frac{x}{1}\right) + c$

Ans: $\sin^{-1}(x) + c$

8) Let pts be $A(1,0,0)$ & $B(0,1,1)$

$\therefore \vec{AB} = -\hat{i} + \hat{j} + \hat{k}$

Now $|\vec{AB}| = \sqrt{1+1+1} = \sqrt{3}$

$\therefore$ Direction cosines are $l = -1/\sqrt{3}, m = 1/\sqrt{3}, n = 1/\sqrt{3}$

$(-1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$

Ans-~~(8)~~ 9) let $\vec{a} = \hat{i} - \hat{j}$

(9) $\vec{b} = \hat{i} + \hat{j}$

Now, projection of $\vec{a}$ on $\vec{b}$

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$\vec{a} \cdot \hat{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{(\hat{i}-\hat{j}).(\hat{i}+\hat{j})}{|\vec{b}|}$

$= \frac{1-1}{\sqrt{2}} = 0$

$\therefore$ Projection of $\hat{i}-\hat{j}$ on vector $\hat{i}+\hat{j} = 0$

Ans 10) $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$

$\vec{r} = (5\hat{i}-4\hat{j}+6\hat{k}) + \lambda(3\hat{i}+7\hat{j}+2\hat{k})$

$\vec{r} = 5\hat{i}-4\hat{j}+6\hat{k} + \lambda(3\hat{i}+7\hat{j}+2\hat{k})$

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$x - 2$

$x - 4$

$x - 8$

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$x - 2$
$x - 4$
$x - 8$

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$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix}$$

$$\begin{vmatrix} x-2^1 & 2x-3 & 3x-4^1 \\ x-2^2 & 2x-3^2 & 3x-4^2 \\ x-2^3 & 2x-3^3 & 3x-4^3 \end{vmatrix}$$

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Rough

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix}$$

$$\begin{vmatrix} x-2^1 & 2x-3 & 3x-4^1 \\ x-2^2 & 2x-3^2 & 3x-4^2 \\ x-2^3 & 2x-3^3 & 3x-4^3 \end{vmatrix}$$

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