WITH GRAPH PAPER

केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली

सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)

परीक्षार्थी प्रवेश–पत्र के अनुसार भरें

विषय Subject : MATHEMATICS

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उत्तर देने का माध्यम
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प्रश्न पत्र के ऊपर लिखे कोड को दर्शाए
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अतिरिक्त उत्तर-पुस्तिका (ओ) की संख्या
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किसी शारीरिक अक्षमता से प्रभावित हो तो संबंधित वर्ग में ✓ का निशान लगाएं।
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B	D	H	S	C

B = दृष्टिहीन, D = मूक एवं बधिर, H = शारीरिक रुप से विकलांग, S = स्पस्टिक, C = डिस्लेक्सिक
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क्या लेखन – लिपिक उपलब्ध करवाया गया : हाँ / नहीं
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*एक खाने में एक अक्षर लिखें। नाम के प्रत्येक भाग के बीच एक खाना रिक्त छोड दें। यदि परीक्षार्थी का नाम 24 अक्षरों से अधिक है, तो केवल नाम के प्रथम 24 अक्षर ही लिखें।
Each letter be written in one box and one box be left blank between each part of the name. In case Candidate's Name exceeds 24 letters, write first 24 letters.

कार्यालय उपयोग के लिए
Space for office use

1. Since matrix is 3x3

$|2A| = 2^3|A| = 8(4)$

$= \underline{\underline{32}}$

2. $B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$ $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$

$\therefore A^T - B^T = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

3. $\int e^x(\tan x+1)\sec x\,dx$

$= \int (e^x \sec x \tan x + e^x \sec x)\,dx = e^x \sec x + c$ $\quad (\because \int e^x(f(x)+f'(x))\,dx = e^x f(x)+c$

$\therefore \quad f(x) = \underline{\underline{\sec x}}$

4. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \dfrac{\pi}{3} - \cos^{-1}\left(\dfrac{-1}{2}\right) = \dfrac{\pi}{3} - \left(\pi - \dfrac{\pi}{3}\right) = \dfrac{\pi}{3} - \pi + \dfrac{\pi}{3} = \underline{\underline{\dfrac{-\pi}{3}}}$

5. $(2*3)*4 = (2(2)+3)*4 = 7*4 = 2(7)+4 = 18$

$\therefore$ Ans: $\underline{\underline{18}}$

6. Given plane: $3x-4y+12z=3$

~~In normal form: 3x+~~

$\therefore$ Distance $= \dfrac{3}{\sqrt{3^2+4^2+12^2}} = \dfrac{3}{\sqrt{9+16+144}} = \underline{\underline{\dfrac{3}{13}}}$ units from origin

7. Given points: A(2,1), B(-5,7)

Vector AB → $-7\hat{i}+6\hat{j}$

Scalar components → $(-7,6)$

8. $\int_0^2 \sqrt{4-x^2}$

$= \int_0^2 \sqrt{2^2-x^2}$ $\quad \left(\because \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)$

$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$

$= 2\sin^{-1}1 = \frac{2\pi}{2} = \underline{\underline{\pi}}$

9. $\begin{pmatrix} 2x & 10 \\ 14 & 2y-6 \end{pmatrix} + \begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$

$\therefore 2x+3=7 \Rightarrow 2x=4 \Rightarrow x=2$

$2y-6+2=14$

$\Rightarrow 2y=18 \Rightarrow y=9$

$\therefore y-x = 9-2 = \underline{\underline{7}}$

10. $(\hat{k}\times\hat{j})\cdot\hat{i} + \hat{j}\cdot\hat{k}$

$= -\hat{i}.\hat{i} + 0 \quad (\because \hat{j}\cdot\hat{k}=0,\ \hat{i}\cdot\hat{i}=1)$

$= \underline{\underline{-1}}$

11. $y = \tan^{-1}\left[\dfrac{\sqrt{1+x^2}-1}{x}\right]$

Put $x=\tan\theta \quad (\Rightarrow \theta = \tan^{-1}x)$

$\therefore \dfrac{\sqrt{1+x^2}-1}{x} = \dfrac{\sqrt{1+\tan^2\theta}-1}{\tan\theta} = \dfrac{\sqrt{\sec^2\theta}-1}{\tan\theta} \quad (\because 1+\tan^2\theta=\sec^2\theta) \quad = \dfrac{\sec\theta-1}{\tan\theta} = \dfrac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}$

$$= \frac{1-\cos\theta}{\sin\theta} = \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$\because$ $1-\cos\theta = 2\sin^2\frac{\theta}{2}$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$= \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

$$\therefore y = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\tan^{-1}x = \frac{1}{2(1+x^2)}$$

$\because$ $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

$$\therefore \frac{dy}{dx} = \frac{1}{2(1+x^2)}$$

12. When cards are drawn, let $P(X)$ denote probability of event X.

$$P(\text{drawing 2 red cards}) = \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{26!}{2!\,24!} \times \frac{2!\,50!}{52!}$$

$$= \frac{26\times25}{2} \times \frac{2}{52\times51} = \frac{26\times25}{\underset{2}{52}\times51} = \frac{25}{102}$$

P(drawing 1 black, 1 red card) = $\frac{^{26}C_1 \times ^{26}C_1}{^{52}C_2} = \frac{26 \times 26 \times 2! \times 50!}{52!} = \frac{26 \times \cancel{26} \times 2}{\cancel{52} \times 51} = \frac{26}{51}$

P(drawing 2 black) = $\frac{^{26}C_2}{^{52}C_2} = \frac{25}{102}$

∴ Probability distribution of event E(X) of no. of red cards drawn:

X = no. of red cards	P(X)	X·P(X)	$X^2 P(X)$
0	$\frac{25}{102}$	0	0
1	$\frac{26}{51}$	$\frac{26}{51}$	$\frac{26}{51}$
2	$\frac{25}{102}$	$\frac{50}{102}$	$\frac{50}{51}$

μ = Mean = $\Sigma x P(x) = 0 + \frac{26}{51} + \frac{50}{102} = \frac{26+25}{51} = 1$

Variance = $\Sigma x^2 P(x) - \mu^2$

$= \frac{26+50}{51} - 1 = \frac{76}{51} - 1 = \frac{25}{51} = 0.49$

$$\begin{array}{r} 0.49 \\ 51 \overline{)250} \\ -204 \\ 460 \\ -459 \\ \hline 1 \end{array}$$

∴ Mean = 1

Variance = $\frac{25}{51} = 0.49$

13. Vector $\vec{p} \parallel \vec{a}\times\vec{b} = \cancel{\lambda(\vec{a}\times b} \; \lambda(\vec{a}\times\vec{b})$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$

$$= \hat{i}(28+4) - \hat{j}(7-6) + \hat{k}(-2-12)$$
$$= 32\hat{i} - \hat{j} - 14\hat{k}$$

$$\therefore \vec{p} = 32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}$$
$$\vec{p}\cdot\vec{c} = (32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k})(2\hat{i} - \hat{j} + 4\hat{k})$$
$$= 64\lambda + \lambda - 56\lambda$$
$$= \cancel{8} \; 9\lambda = 18 \text{ (given)}$$
$$\therefore \lambda = 2$$

$$\therefore \vec{p} = 64\hat{i} - 2\hat{j} - 28\hat{k}$$

$$\therefore \text{Vector } \vec{p} = 64\hat{i} - 2\hat{j} - 28\hat{k}$$

14. $x(x^2-1)\frac{dy}{dx}=1$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{x(x^2-1)} \Rightarrow dy=\frac{dx}{x(x^2-1)}$$

$$=\frac{x\,dx}{x^2(x^2-1)}$$ (Multiplying numerator and denominator by x)

Put $x^2=t$

$\Rightarrow 2x\,dx=dt \Rightarrow x\,dx=\frac{dt}{2}$

$$\therefore dy=\frac{dt}{2t(t-1)}$$

$$\frac{1}{t(t-1)}=\frac{A}{t}+\frac{B}{t-1}$$

$\Rightarrow A(t-1)+Bt=1$

$\Rightarrow A+B=0$

$-A=1 \Rightarrow A=-1, B=1$

$$\therefore \frac{1}{t(t-1)}=\frac{-1}{t}+\frac{1}{t-1}$$

$$\Rightarrow dy = \frac{1}{2}\left[-\frac{1}{t} + \frac{1}{t-1}\right] dt$$

$$= \frac{-dt}{2t} + \frac{dt}{2(t-1)}$$

Integrating both sides:

$$\int dy = \frac{-1}{2}\int \frac{dt}{t} + \frac{1}{2}\int \frac{dt}{t-1}$$

$$\Rightarrow y = \frac{-1}{2}\log|t| + \frac{1}{2}\log|t-1| + \log c' \quad \because \int \frac{dt}{t} = \log|t|$$

where c' is a constant

$$\Rightarrow y = \frac{1}{2}\log\left|\frac{t-1}{t}\right| + \log c'$$

$$= \frac{1}{2}\log\left(\left|\frac{x^2-1}{x^2}\right| \cdot c'^2\right)$$

$y = 0$ when $x = 2$

$$\Rightarrow 0 = \frac{1}{2}\log\left(\frac{4-1}{4}c'^2\right)$$

$$\Rightarrow \log \frac{3}{4}c'^2 = 0 \Rightarrow \frac{3}{4}c'^2 = 1 \Rightarrow c' = \frac{2}{\sqrt{3}}$$

$$\therefore y = \frac{1}{2}\log\left|\frac{x^2-1}{x^2}\right| + \log\frac{2}{\sqrt{3}}$$

$$= \log\sqrt{\frac{x^2-1}{x^2}}\cdot\left(\frac{2}{\sqrt{3}}\right)$$

$$e^y = \frac{2\sqrt{x^2-1}}{\sqrt{3}\,x}$$

$$\Rightarrow \sqrt{3}\,x\,e^y = 2\sqrt{x^2-1}$$

$$\Rightarrow 3x^2e^{2y} = 4(x^2-1)$$

$$\Rightarrow 3x^2e^{2y} = 4x^2-4$$

$$\Rightarrow x^2(4-3e^{2y}) = 4$$

is particular solution

15. given $a*b = |a-b|$

$$\therefore b*a = |b-a|$$

$$= |-(a-b)|$$

$$= |a-b| \quad (\because |-x| = |x|)$$

$$= a*b$$

$\therefore$ $*$ is commutative.

Checking if associative :

$a*(b*c) = a*|b-c| = |a-|b-c||$

$(a*b)*c = |a-b| * c = ||a-b|-c|$

~~For $a=-2, b=1, c=-3$~~

~~$a*(b*c) = |-2-|1+3||$~~

~~$= |-2-4| = 6$~~

~~$(a*b)*c = ||-2-1|+3|$~~

For $a=2, b=1, c=-3$

$a*(b*c) = |2-|1+3||$

$= |2-4| = 2$

$(a*b)*c = ||2-1|+3| = 4$

$\therefore a*(b*c) \neq (a*b)*c$

$\therefore$ <u>$*$ is not associative</u>

Consider $a \circ b = a$ for $a, b \in R$

$a \circ b = a$

$b \circ a = b$

For $a=2, b=3$

$a \circ b =$ ~~$2 \circ 3 = 2$~~ $2 \circ 3 = 2$

$b \circ a =$ ~~$3 \circ 2 = 3$~~ $3 \circ 2 = 3$

$\therefore a \circ b \neq b \circ a \Rightarrow$ $\circ$ is not commutative

For any $a, b, c \in R$,

$(a \circ b) \circ c = a \circ c = a$

$a \circ (b \circ c) = a \circ b = a$

$\therefore (a \circ b) \circ c = a \circ (b \circ c)$

$\therefore \circ$ is associative

Hence, $\circ$ is not commutative, but is ~~not~~ associative.

16. $\int_{-1}^{2} |x^3 - x| \, dx$

Let $y = |x^3 - x|$

For $x^3 - x = 0$,

$x(x^2 - 1) = 0 \Rightarrow x = 0, x = \pm 1$

$-\quad +\quad -\quad +$

$-1 \quad 0 \quad 1$

$\therefore$ In $[-1, 2]$, for $x \in [-1, 0] \cup [1, 2]$, $|x^3 - x| = x^3 - x$

For $x \in [0, 1]$, $|x^3 - x| = x - x^3$

$\therefore \int_{-1}^{2} |x^3 - x| dx = \cancel{\int_{-1}^{0} (x - x^3) dx + \int_{0}} \int_{-1}^{0} (x^3 - x) dx + \int_{0}^{1} (x - x^3) dx + \int_{1}^{2} (x^3 - x) dx$

$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$ $\quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$

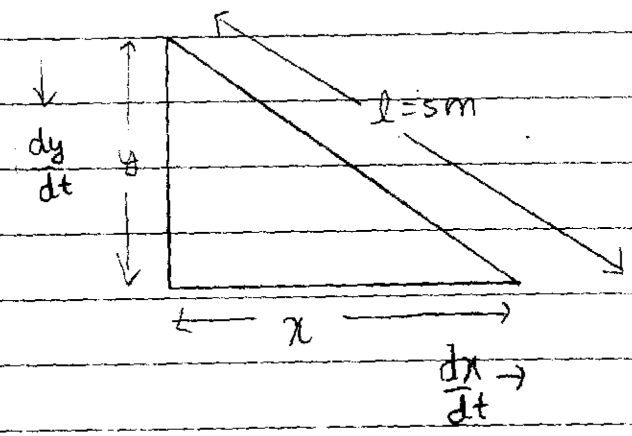
$$= -\left[\frac{(-1)^4}{4} - \frac{(-1)^2}{2}\right] + \left[\frac{1}{2} - \frac{1}{4}\right] + \left[\frac{2^4}{4} - \frac{2^2}{2} - \frac{1}{4} + \frac{1}{2}\right]$$

$$= -\left[\frac{1}{4} - \frac{1}{2}\right] + \left[\frac{1}{4}\right] + \left[4 - 2 + \frac{1}{4}\right]$$

$$= \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = 2 + \frac{3}{4} = \frac{11}{4}$$

$$\therefore \int_{-1}^{2} |x^3 - x|\, dx = \frac{11}{4}$$

17.



Let height of ladder on wall at any instant be y.
Let distance of foot of ladder from wall at any instant be x.

Length of ladder = constant = $l = 5m$

$l^2 = x^2 + y^2$ (By Pythagoras theorem)

Differentiating with respect to time t:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \qquad \left(\because \frac{d}{dt}(\text{constant}) = 0\right) \cancel{\frac{d}{dt}}$$

$$\Rightarrow x\frac{dx}{dt} = -y\frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

Given, $x = 4m$, $\frac{dx}{dt} = 2\,cm/s$

$$y = \sqrt{l^2 - x^2} = \sqrt{5^2 - 4^2} = \sqrt{9} = 3m$$

$$\therefore \frac{dy}{dt} = \cancel{\frac{-x}{3}} \; -\frac{4}{3} \times 2 \; cm/s = \frac{-8}{3}\,cm/s = -2.67\,cm/s$$

Hence, height of ladder is decreasing at rate of $2.67\,cm/s = \frac{8}{3}\,cm/s$

For given equation,

18. LHS: $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

~~$R_2 \to R_2 + R_1$~~

$R_1 \to R_1 - R_2 - R_3$

$$\begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Taking out 2 from R_1

$$= 2\begin{vmatrix} 0 & -c & -b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$R_3 \to R_3 + R_1$

$$= 2\begin{vmatrix} 0 & -c & -b \\ b & c+a & b \\ c & 0 & a \end{vmatrix}$$

$R_2 \to R_2 + R_1$

$$= 2\begin{vmatrix} 0 & -c & -b \\ b & a & 0 \\ c & 0 & a \end{vmatrix}$$

Taking determinant along R_1:

$$= 2\left[c(ab) - b(-ac)\right]$$
$$= 2\left[2abc\right]$$
$$= \underline{\underline{4abc}} = RHS$$

$\therefore$ Proved that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

To prove:

19. $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$

Let $\cos^{-1}\left(\frac{4}{5}\right) = x \Rightarrow \cos x = \frac{4}{5}$ $\quad \therefore \sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$

Let $\cos^{-1}\left(\frac{12}{13}\right) = y \Rightarrow \cos y = \frac{12}{13}$ $\quad \sin y = \sqrt{1-\cos^2 y} = \sqrt{1-\frac{12^2}{13^2}} = \frac{5}{13}$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$= \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13} = \frac{48-15}{65} = \frac{33}{65}$

$\therefore \cos\left(\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right)\right) = \frac{33}{65}$

$\Rightarrow \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$

Thus proved.

20 Given, $y = (\tan^{-1}x)^2$

$\Rightarrow \frac{dy}{dx} = \cancel{2\tan^{-1}x\left(\frac{1}{1+x^2}\right)}\ 2\tan^{-1}x \frac{d}{dx}(\tan^{-1}x)$

$= \frac{2\tan^{-1}x}{1+x^2}$ $\left(\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\right)$

$\frac{d^2y}{dx^2} = \frac{(1+x^2)\frac{d}{dx}(2\tan^{-1}x) + (2\tan^{-1}x)\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$ $\left(\because \left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}\right)$

$$\therefore \frac{d}{d} = \frac{\frac{(1+x^2)(2) - (2\tan^{-1}x)(2x)}{(1+x^2)}}{(1+x^2)^2}$$

$$= \frac{2 - 4x\tan^{-1}x}{(1+x^2)^2}$$

$$\therefore (x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx}$$

$$= (x^2+1)^2 \frac{(2-4x\tan^{-1}x)}{(1+x^2)^2} + 2x(x^2+1)\frac{(2\tan^{-1}x)}{(x^2+1)}$$

$$= 2-4x\tan^{-1}x + 2x(2\tan^{-1}x)$$

$$= 2 - 4x\tan^{-1}x + 4x\tan^{-1}x$$

$$= 2$$

$\therefore$ Thus proved that:

$$(x^2+1)^2\frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2.$$

21 $\frac{dy}{dx} + y\cot x = 4x\,\mathrm{cosec}\,x$

is ~~of~~ form $\frac{dy}{dx} + py = q$ where $p = \cancel{y}\cot x$, $q = 4x\,\mathrm{cosec}\,x$

~~Integr~~ This is a linear differential equation

Integrating factor, $IF = e^{\int p\,dx}$

$= e^{\int \cot x\,dx}$

$= e^{\log|\sin x|\,dx}$

$= \sin x$

$\therefore$ Solution is given by $y \times IF = \int q \times IF$

$\Rightarrow y\sin x = \int 4x\,\mathrm{cosec}\,x\,\sin x\,dx$

$= \int 4x\,dx \qquad (\because \sin x = \frac{1}{\mathrm{cosec}\,x})$

$= \frac{4x^2}{2} + c$

$= 2x^2 + c$

$\therefore y\sin x = 2x^2 + c$

$y = 0$ when $x = \frac{\pi}{2}$

$\therefore \cancel{y\sin\left(\frac{\pi}{2}\right)} \cancel{= 2}\ (0)\sin\left(\frac{\pi}{2}\right) = 2\left(\frac{\pi}{2}\right)^2 + c$

$\Rightarrow c + \frac{\pi^2}{2} = 0 \Rightarrow c = -\frac{\pi^2}{2}$

$\therefore y\sin x = 2x^2 - \frac{\pi^2}{2}$ is particular solution

22. Line passes through $(3,-4,-5)$ and $(2,-3,1)$

$\therefore$ direction ratios are $\rightarrow$ $2-3, -3+4, 1+5 = -1, 1, 6$ Should not be in bracket

Line passes through $(3,-4,-5)$ $\therefore$ Equation in Cartesian form is:

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$$

$\therefore$ Any point of line is of form $(-\lambda+3, \lambda-4, 6\lambda-5)$

~~Since this line point~~ At point where line meets plane, equation of plane: $2x+y+z=7$ is satisfied.

$\Rightarrow 2(-\lambda+3) + (\lambda-4) + (6\lambda-5) = 7$

$\Rightarrow -2\lambda+6 + \lambda-4 + 6\lambda-5 = 7$

$\Rightarrow 5\lambda-3 = 7 \Rightarrow 5\lambda = 10 \therefore \lambda = 2$

$\therefore$ given point is $(-\lambda+3, \lambda-4, 6\lambda-5)$

$= (-2+3, 2-4, 12-5) = (1,-2,7)$

$\therefore$ Required point is $\underline{\underline{(1,-2,7)}}$

27 According to data :

	Food I	Food II
Vitamin A (units/kg)	2	1
Vitamin C (units/kg)	1	2
Cost per kg (Rs)	5	7

Let x be no. of units of ~~purch~~ food I purchased

$y \rightarrow$ no. of units of food II purchased.

$\therefore$ Total cost $= 5x+7y$

Objective function, Z = Minimise $5x+7y$

subject to constraints:

$2x+y \geq 8 \Rightarrow \frac{x}{4} + \frac{y}{8} \geq 1$

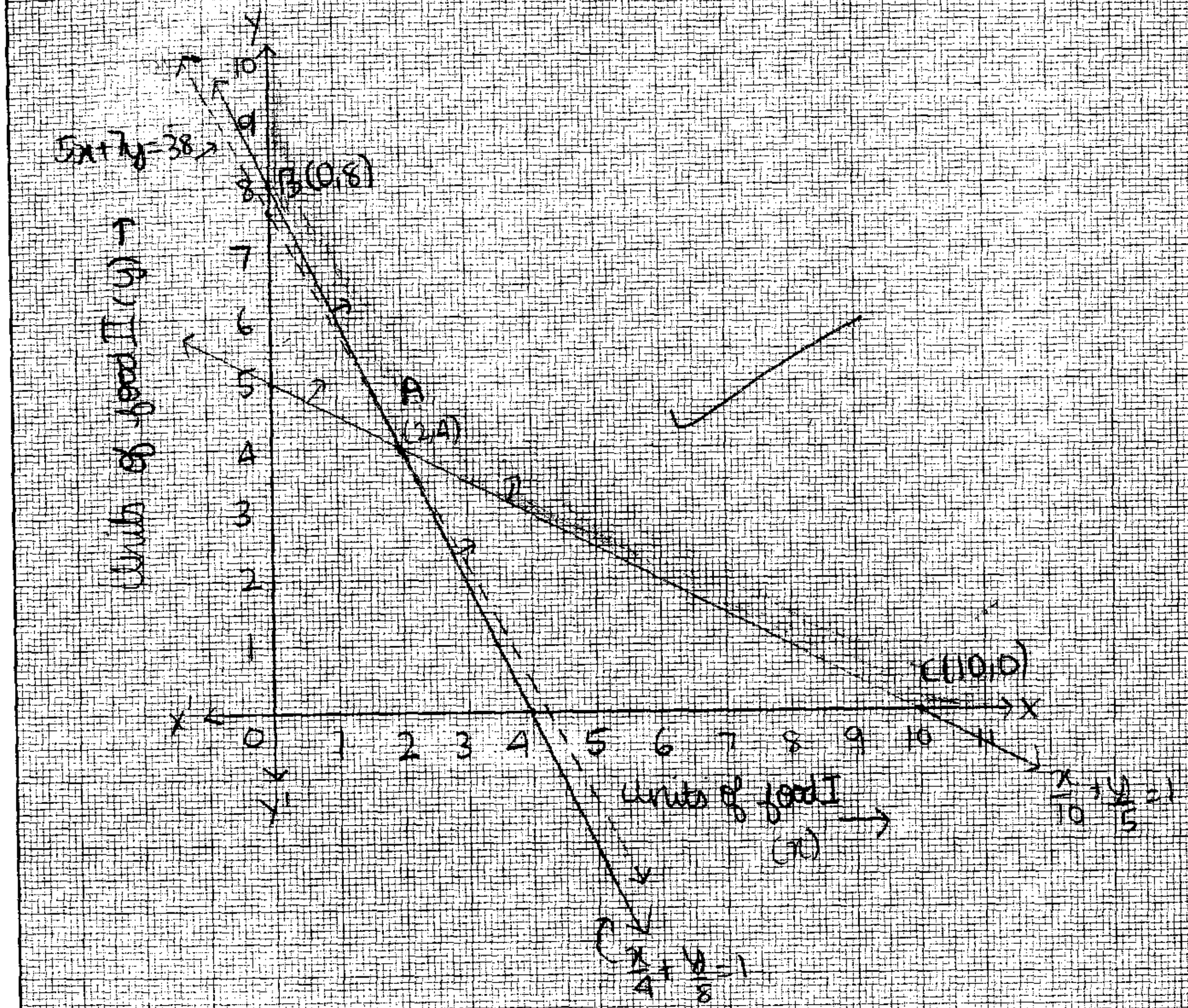
~~$x+2y$~~

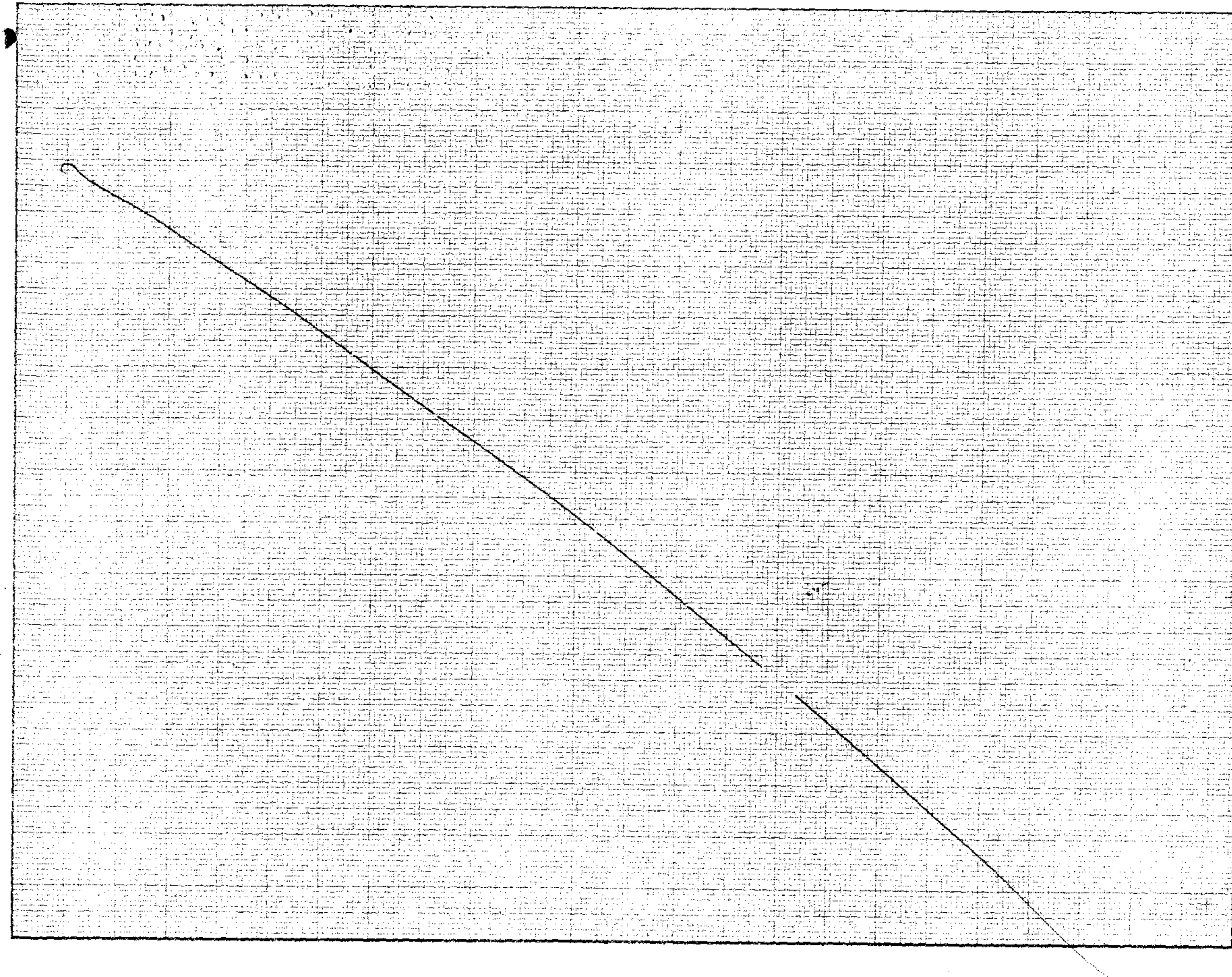
$x+2y \geq 10 \Rightarrow \frac{x}{10} + \frac{y}{5} \geq 1$

$x \geq 0, \; y \geq 0$

22

SCALE:
X axis → 1 cm = 1 unit
Y axis → 1 cm = 1 unit





Shaded region is solution region.
Taking value of Z at corner points A, B, C:

$Z_A = 5(2) + 7(4) = 10 + 28 = 38$
$Z_B = 5(0) + 7(8) = 56$
$Z_C = 5(10) + 7(0) = 50$

$\therefore$ Minimum value of Z obtained is 38.
The shaded solution region is unbounded.
$\therefore$ To check if Z is minimum at A, graph of $5x + 7y < 38$ is drawn.
The region $5x + 7y < 38$ has no points in common with solution region.
$\therefore$ Z is minimum at (2, 4) and
minimum cost = Rs. 38

(PTO)

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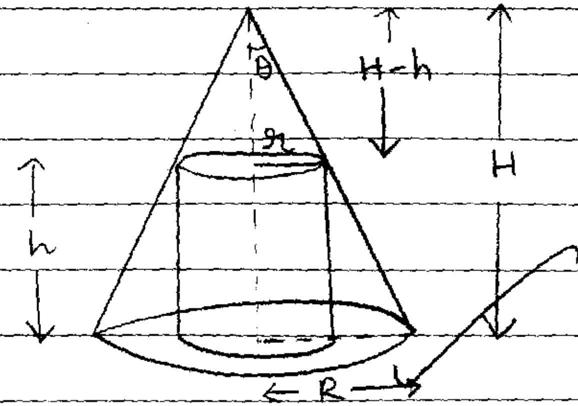


Consider cone of radius R, height H in which is ~~inscr~~ inscribed a right circular cylinder of height h, radius r.

Let semivertical angle of cone be ~~an~~ θ.

From figure, $\tan\theta = \dfrac{r}{H-h} = \dfrac{R}{H}$

$\therefore r = (H-h)\tan\theta$

Curved surface area of cylinder, S

$= 2\pi r h$

$= 2\pi h(H-h)\tan\theta$

$= 2\pi h H \tan\theta - 2\pi h^2 \tan\theta$

~~Differentiating with respect to h,~~

$$\frac{dS}{dh} = 2\pi H \tan\theta - 4\pi h \tan\theta$$

$$\frac{d^2S}{dh^2} = -4\pi \tan\theta$$

For maximum curved surface area, S,

$$\frac{dS}{dh} = 0$$

$$\Rightarrow 2\pi H \tan\theta = 4\pi h \tan\theta$$

$$\Rightarrow H = 2h \Rightarrow h = \frac{H}{2}$$

$$\frac{r}{H-h} = \frac{R}{H} \Rightarrow \frac{r}{R} = \frac{H-h}{H} = \frac{H}{2H} = \frac{1}{2}$$

$$\therefore r = \frac{R}{2}$$

$$\frac{d^2S}{dh^2} = -4\pi \tan\theta < 0$$ ~~for $\theta \in [0, \pi$~~ ~~($\because$ for right circular cone, $\tan\theta \in [0, \frac{\pi}{2}]$)~~

$\left(\because \text{for cone, } \theta \in \cancel{[0, \tfrac{\pi}{2}]} \Rightarrow \cancel{\tan}\ \theta \in \left(0, \tfrac{\pi}{2}\right) \Rightarrow \tan\theta > 0\right)$

$\therefore$ ~~r is max~~ S is maximum when $r = \frac{R}{2}$

∴ Proved that ~~eqbi~~ for cylinder of greatest curved surface area,
radius of cylinder ; $r = \frac{\text{Radius of cone}(R)}{2}$

24 $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

Put $x = \sin\theta \quad \Rightarrow \theta = \sin^{-1}x$

$\Rightarrow dx = \cos\theta\, d\theta$

$I = \int \frac{\sin\theta\,(\sin^{-1}(\sin\theta))}{\sqrt{1-\sin^2\theta}} \cos\theta\, d\theta$

$= \int \frac{\theta \sin\theta}{\cos\theta} \times \cos\theta\, d\theta \quad \because \sin^{-1}(\sin\theta) = \theta$, $\sqrt{1-\sin^2\theta} = \cos\theta$

$= \int \theta \sin\theta\, d\theta = I$

Integrating by parts with first function $=\theta$, second function $= \sin\theta$

$I = \theta \int \sin\theta\, d\theta - \left[\int \frac{d\theta}{d\theta} \cdot \left(\int \sin\theta\, d\theta\right)\right]$

$= -\theta\cos\theta - \left[\int (-\cos\theta)\, d\theta\right]$

$= -\theta\cos\theta + \int \cos\theta\, d\theta$

$= -\theta\cos\theta + \sin\theta + C$ $\quad (\because \int \cos\theta \, d\theta = \sin\theta + c)$ $\quad$ ~~θ sinθ~~

$\theta = \sin^{-1}x$

$x = \sin\theta$

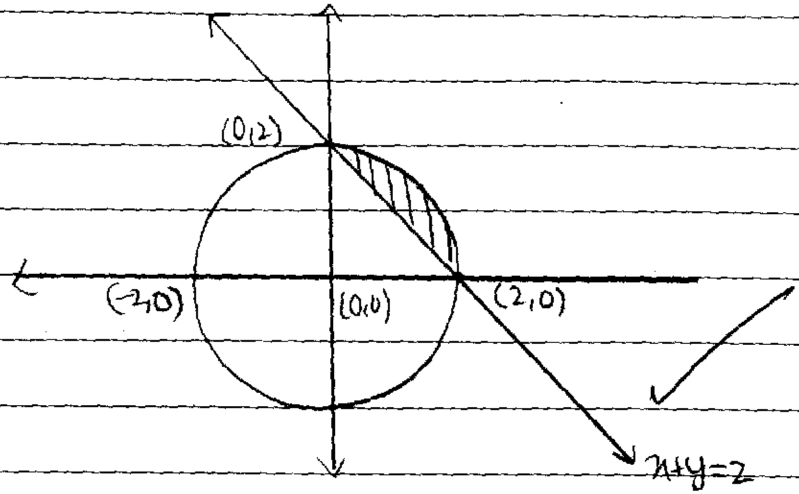
$\Rightarrow \cos\theta = \sqrt{1-x^2}$

$\therefore I = -(\sin^{-1}x)(\sqrt{1-x^2}) + x + C$

$= -\sqrt{1-x^2}\,\sin^{-1}x + x + C$

(PTO)

25.



X, Y should be written

Region I: $x^2+y^2 \leq 4$ represents a region enclosed by circle with centre $(0,0)$, radius 2 units

Region 2: $x+y \geq 2$ represents region under line $x+y=2$.

Finding ~~they~~ the points of intersection of $x^2+y^2=4$, $x+y=2$

$x = 2-y$

$\Rightarrow (2-y)^2 + y^2 = 4$

$\Rightarrow 4+y^2-4y+y^2=4 \Rightarrow 2y^2-4y=0 \Rightarrow 2y(y-2)=0$

$\therefore y=0$ or $y=2$

$x=2$ or $x=0$

$\therefore$ Intersection points are $(0,2)$ and $(2,0)$

For circle,

$x^2+y^2=4$

$\Rightarrow y=\sqrt{4-x^2}$

For line

$x+y=2$

$\Rightarrow y=2-x$

Area required $= \int_0^2 y\,dx$ for circle $- \int_0^2 y\,dx$ for line.

$$= \int_0^2 \sqrt{4-x^2}\,dx \quad \int_0^2 (2-x)\,dx$$

$$= \cancel{\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}}$$

$$= \int_0^2 \sqrt{2^2-x^2}\,dx - \int_0^2 (2-x)\,dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x-\frac{x^2}{2}\right]_0^2$$

$$= \frac{2\pi}{2} - \left[4-\frac{4}{2}\right]_0^2 = (\pi-2) \text{ sq. units}$$

$\therefore$ Area of region $=$ $(\pi-2)$ square units

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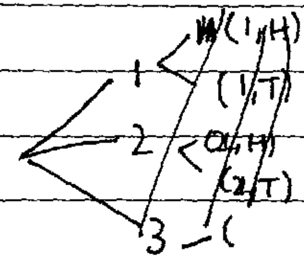
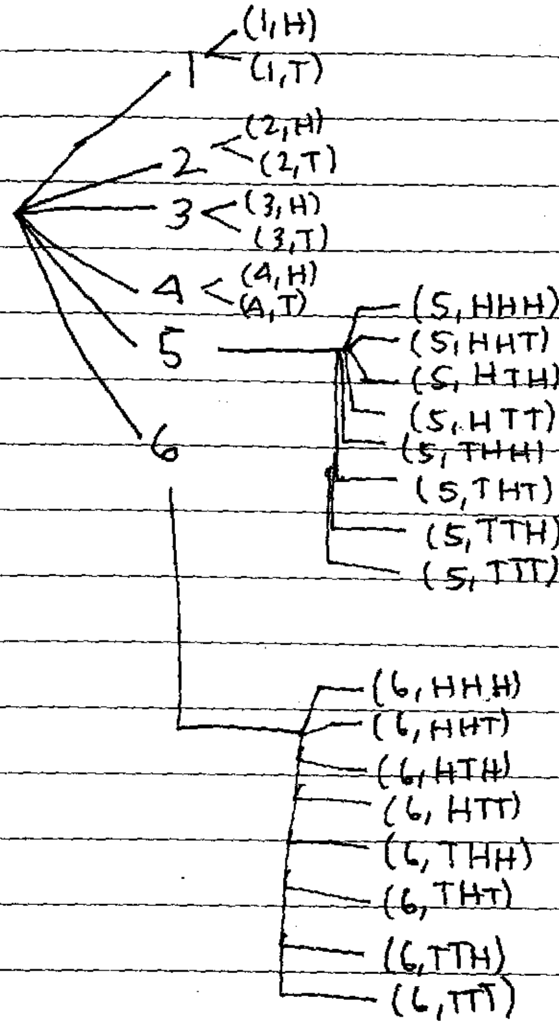


Sample space :



~~Let A be event~~ A → event that she gets exactly one head

$E_1 \rightarrow$ event she got 1, 2, 3 or 4 with the die

$E_2 \rightarrow$ event she got 5, 6 with die.

$$P(E_1) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}} = \cancel{\frac{4}{6} = \frac{2}{3}}$$

$$P\left(\frac{A}{E_1}\right) = \frac{P(A \cap E_1)}{P(E_1)} = \cancel{P(\text{exactly one head}) \text{ if she}} \quad \frac{1}{2}$$

(∵ there are equal chances of getting head or tail when the coin is tossed)

$$P\left(\frac{A}{E_2}\right) = \frac{P(A \cap E_2)}{P(E_2)}$$

$$\cancel{P\left(\frac{A}{E_1}\right) = \frac{P(A \cap E_1)}{P(E_1)}}$$

Each of probability of getting 1, 2, 3 or 4 $= \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$

Each of (1,H), (1,T), (2,H), (2,T), (3,H), (3,T), (4,H), (4,T), (5,H), (5,T), (6,H), (6,T) has probability $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

For each of 5, 6, probability $= \frac{1}{6}$

$\therefore$ Each outcome of coin toss after getting 5 or 6 has

probability $\frac{1}{6} \times \frac{1}{8} = \frac{1}{48}$.

$$P(E_1) = \frac{2}{3}, \quad P(E_2) = \frac{1}{3}$$

$$P\left(\frac{A}{E_1}\right) = \frac{P(A \cap E_1)}{P(E_1)} = \frac{\frac{1}{12} \times 4}{\frac{2}{3}} = \frac{1}{2} = P(\text{exactly 1 head if 1, 2, 3 or 4})$$

$$P\left(\frac{A}{E_2}\right) = \frac{P(A \cap E_2)}{P(E_2)} = \cancel{\frac{1}{48}} \times 3 \; \frac{\frac{6}{48}}{\frac{1}{6} \times 2} = \frac{\frac{1}{8}}{\frac{1}{3}} = \frac{3}{8} = P(\text{exactly 1 head if 5, 6})$$

$\therefore$ ~~P(exactly 0~~

$\therefore$ By Bayes theorem,

P(1, 2, 3, 4 if exactly 1 head)

$$= P\left(\frac{E_1}{A}\right) = \frac{P\left(\frac{A}{E_1}\right) P(E_1)}{P\left(\frac{A}{E_1}\right) P(E_1) + P\left(\frac{A}{E_2}\right) \cdot P(E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{2}{3}}{\frac{1}{2}\times\frac{2}{3}+\frac{3}{8}\times\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{8}} = \frac{\frac{8}{24}}{\frac{11}{24}} = \frac{8}{11} = 0.727$$

$\therefore$ Probability of (1,2,3,4) if exactly one head $= \frac{8}{11}$

28. Given system can be represented in matrix form as

$$AX = B \Rightarrow X = A^{-1}B$$

where $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$

$\therefore X = \cancel{A^{-1}B =}$

$$\text{adj}(A) = \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$$

$$|A| = 1(-21+1) - 1(-14-3) - 1(-2-9)$$

$$= -20 + 17 + 11 = 8$$

$$A^{-1} = \frac{1}{|A|}\,\mathrm{adj}(A) = \frac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\therefore x = 3,\ y = 1,\ z = 1$
is solution of given system.

29 Let perpendicular from P (7,14,5) intersect plane at ~~(x,y of p,q,r)~~ point O.

∴ ~~dr of direction ratio of OP = (p-7, q-14, r-5)~~

For given plane, $2x+4y-z=2$

direction ratios of normal → $2,4,-1$

= direction ratios of OP.

OP passes through $(7,14,5)$ and has direction ratios $(2,4,-1)$

∴ ~~Any point on OP is of form~~

equation of OP:

$$\frac{x-7}{2} = \frac{y-14}{4} = \frac{z-5}{-1} = \lambda$$

∴ Any point on OP is of form

$(2\lambda+7, 4\lambda+14, -\lambda+5)$

Since point O is of this form and lies on plane $2x+4y-z=2$,

$2(2\lambda+7) + 4(4\lambda+14) - (-\lambda+5) = 2$

$\Rightarrow 4\lambda+14 + 16\lambda + 56 + \lambda - 5 = 2$

$\Rightarrow 21\lambda = -63 \Rightarrow \lambda = -3$

∴ Foot of the perpendicular $= (2(-3)+7, 4(-3)+14, 3+5)$

$= (1, 2, 8)$

Let image of point be at $R(r,s,t)$

$\therefore O$ is midpoint of PR ie midpoint of $(7,14,5)$ and $(1,2,8)$

$$\Rightarrow (1,2,8) = \left(\frac{7+r}{2}, \frac{14+s}{2}, \frac{5+t}{2}\right)$$

$$\Rightarrow \frac{7+r}{2} = 1 \Rightarrow r = -5$$

$$\frac{14+s}{2} = 2 \Rightarrow s = -10$$

$$\frac{5+t}{2} = 8 \Rightarrow t = 11$$

$\therefore$ Image is at $R(-5,-10,11)$

Length of $\perp$ = distance PO from $P(7,14,5)$ to $O(1,2,8)$

$$= \sqrt{(7-1)^2 + (14-2)^2 + (5-8)^2}$$

$$= \sqrt{36+144+9} = \sqrt{189} = \sqrt{9 \times 21}$$

$$= 3\sqrt{21} \text{ units}$$

∴ Foot of perpendicular → (1, 2, 8)

Image of point in plane → (−5, −10, 11)

Length of ⊥ OP = $3\sqrt{21}$ units