WITH GRAPH PAPER

केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)
परीक्षार्थी प्रवेश–पत्र के अनुसार भरें

विषय Subject : Mathematics (041)

परीक्षा का दिन एवं तिथि
Day & Date of the Examination : 20 March '13, wednesday

उत्तर देने का माध्यम
Medium of answering the paper : English

प्रश्न पत्र के ऊपर लिखे कोड को दर्शाएं
Write Code No. as written on the top of Question Paper : 65/1

अतिरिक्त उत्तर-पुस्तिका (ओं) की संख्या
No. of Supplementary answer-book(s) used : Nil

किसी शारीरिक अक्षमता से प्रभावित हों तो संबधित वर्ग में ✓ का निशान लगाएं।
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B D H S C

B = दृष्टिहीन, D = मूक एवं बधिर, H = शारीरिक रूप से विकलांग, S = स्पास्टिक, C = डिस्लेक्सिक
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क्या लेखन - लिपिक उपलब्ध करवाया गया : हाँ / नहीं
Whether writer provided : Yes / No : NO

*एक खाने मे एक अक्षर लिखें। नाम के प्रत्येक भाग के बीच एक खाना रिक्त छोड़ दें। यदि परीक्षार्थी का नाम 24 अक्षरों से अधिक है, तो केवल नाम के प्रथम 24 अक्षर ही लिखें।

Each letter be written in one box and one box be left blank between each part of the name. In case Candidate's Name exceeds 24 letters, write first 24 letters.

कार्यालय उपयोग के लिए
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केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
Central Board of Secondary Education, Delhi

सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)
SENIOR SCHOOL CERTIFICATE EXAMINATION (CLASS XII)

प्रमाणित किया जाता है मैंने/हमने इस उत्तर पुस्तिका का मूल्यांकन प्रश्न पत्र के समुचित सेट के अनुसार और पूर्ण रूप से मूल्यांकन पद्धति के अनुसार किया है।
Certified that I/We have evaluated this answer-book according to the correct set of question paper and strictly as per the marking scheme.

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1. $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3}) = \frac{\pi}{3} - \left(\pi - \cot^{-1}(\sqrt{3})\right) = \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = \frac{\pi}{3} - \frac{5\pi}{6} = -\frac{3\pi}{6} = -\frac{\pi}{2}$

2. $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \tan^{-1}\left[2\sin\left(2\cdot\frac{\pi}{6}\right)\right] = \tan^{-1}\left[2\sin \pi/3\right] = \tan^{-1}\left(2\cdot\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

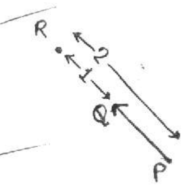
3. 'A' is skew symmytric when $a_{ij} + a_{ji} = 0$ so $a_{13} + a_{31} = 0 \Rightarrow x - 2 = 0 \Rightarrow x = 2$

4. $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, $A^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 2A$ $\because A^2 = 2A$ $\therefore k = 2$

5. $y = mx$ Differentiating wrt x, $\frac{dy}{dx} = m$ $\therefore y = x\frac{dy}{dx}$

6. $a_{32} = 5$, $A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = -(8-30) = 22$ $\therefore a_{32}A_{32} = 5\times 22 = 110$

7. $\vec{OR}\,(\vec{r}) = \frac{m\,\vec{OP} - n\,\vec{OQ}}{m-n} = \frac{\cancel{2}^{1}(3\vec{a} - 2\vec{b}) - 2(\vec{a} + \vec{b})}{\cancel{2}\,1 - 2} = \cancel{5\vec{a} - 3\vec{b}}\ \frac{\vec{a} - 4\vec{b}}{-1} = -\vec{a} + 4\vec{b}$



8. Given $\vec{a}$ is a unit vector, $|\vec{a}| = 1$
Now $(\vec{x} - \vec{a})\cdot(\vec{x} + \vec{a}) = \vec{x}\cdot\vec{x} - \vec{a}\cdot\vec{x} + \vec{x}\cdot\vec{a} - \vec{a}\cdot\vec{a} = 15$
Using $\vec{a}\cdot\vec{a} = |\vec{a}|^2$ and $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$, we get $|\vec{x}|^2 - |\vec{a}|^2 = 15 \Rightarrow |\vec{x}|^2 = 15 + 1 = 16$

$\therefore \quad |\vec{x}| = \pm 4 \quad$ but $|\vec{x}| > 0 \quad \therefore \quad |\vec{x}| = 4$ Ans

9. Using distance formula : $d = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}} = \dfrac{|(0)(2) + (0)(-3) + (0)(6) + 21|}{\sqrt{2^2 + (-3)^2 + 6^2}} = \dfrac{21}{7} = 3$ units

10. $R(x) = 3x^2 + 36x + 5$

$MR(x) = R'(x) = 6x + 36 \qquad R'(5) = 30 + 36 = 66$

Money for welfare of employees is a nice ~~thing~~ step.

There should be a growth in raising funds for the welfare of the employees.

SECTION-B

11. Let $f(x) = x^2 + 4$

$f: R_+ \to [4, \infty)$

Let $f(x_1) = f(x_2) \quad \forall \; x_1, x_2 \in R_+$

$\Rightarrow \quad x_1^2 + 4 = x_2^2 + 4 \;\Rightarrow\; (x_1 - x_2)(x_1 + x_2) = 0$

$\because \; x_1 > 0, x_2 > 0 \;\therefore\; x_1 + x_2 > 0 \quad$ Hence $\; x_1 = x_2$

Thus f is one-one (injective)

Let $y = x^2+4 \Rightarrow x = \sqrt{y-4}$ $\{x = -\sqrt{y-4}$ rejected as $x>0\}$
Clearly, all values of y in the co-domain satisfy this relation to ~~give x~~ such that $f(x) = x^2+4$, hence f is an onto (surjective) function.

f is one-one as well as onto, thus f is invertible

Now let $\cancel{y=} f(y) = y^2+4$. Once again, $y = \sqrt{\cancel{4} f(y)-4}$ $\{$as $y>0$ ~~and $f(y) \in [4,\infty)$~~$\}$
Replacing y with $f^{-1}(y)$ and $f(y)$ with y, we get

$f^{-1}(y) = \sqrt{y-4}$ Hence shown.

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12. Let $\frac{3}{4} = \sin 2\theta \Rightarrow 2\theta = \sin^{-1}\left(\frac{3}{4}\right)$

so $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \tan\left(\frac{1}{2}\cdot 2\theta\right) = \tan\theta$

Let $\tan\theta = t$ then,

$\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$ $\Leftrightarrow \Rightarrow$ $\dfrac{3}{4} = \dfrac{2t}{1+t^2}$ $\Rightarrow$ $3+3t^2 = 8t$

$\Rightarrow$ $3t^2-8t+3=0$

$\cancel{\dfrac{\pi}{2}}$ $\quad \cancel{\theta < \pi}$ $\quad\Rightarrow$ $t = \dfrac{8 \pm \sqrt{64-4(9)}}{6} = \dfrac{4 \pm \sqrt{7}}{3}$

But $\sin 2\theta = \dfrac{3}{4}$ which implies that $0 < 2\theta < \dfrac{\pi}{2} \Rightarrow 0 < \theta < \dfrac{\pi}{4}$

Accordingly, $0 < \tan\theta < \tan(\pi/4)$ $\therefore$ $0 < \tan\theta < 1 \Rightarrow 0 < t < 1$

Thus $t = \dfrac{4+\sqrt{7}}{3}$ is to be rejected, giving $t = \tan^{-1}\left(\dfrac{1}{2}\sin^{-1}\dfrac{3}{4}\right) = \dfrac{4-\sqrt{7}}{3}$ H.P.

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13. Let $\Delta = \begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

Applying $R_1 \to R_1+R_2+R_3$, $\Delta = \begin{vmatrix} 3x+3y & 3x+3y & 3x+3y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$

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$$\Delta = 3(x+y)\begin{vmatrix} 1 & 1 & 1 \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$

$$\Delta = 3(x+y)\begin{vmatrix} 0 & 0 & 1 \\ y & -y & x+y \\ y & 2y & x \end{vmatrix}$$

Taking out common y from C_1 and y from C_2,

$$\Delta = 3(x+y)\,y^2\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & x+y \\ 1 & 2 & x \end{vmatrix}.$$

Expanding about R_1,

$$\Delta = 3y^2(x+y)\ \left[1\left(1\times 2 - (-1)(1)\right)\right] = 3\times 3y^2(x+y)$$

$\therefore\ \Delta = 9y^2(x+y)$ Hence Proved.

14. Given $y^x = e^{y-x}$

Taking natural logarithm both sides, $x\log y = y - x$ — ①

Differentiating with respect to x, $\log y + \dfrac{x}{y}\dfrac{dy}{dx} = \dfrac{dy}{dx} - 1$

$$\Rightarrow \frac{dy}{dx}\left(1 - \frac{x}{y}\right) = 1 + \log y$$

$$\Rightarrow \frac{dy}{dx} = \frac{1+\log y}{1-x/y} = \frac{y}{y-x}(1+\log y)$$

From eqn ①, $y-x = x\log y$ $\Rightarrow$ $\frac{dy}{dx} = \frac{y}{x}\frac{(1+\log y)}{\log y}$

Also from eq. ①, $y = x(1+\log y)$

$$\Rightarrow \frac{dy}{dx} = \frac{\cancel{x}(1+\log y)(1+\log y)}{\cancel{x}\log y}$$

$$\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$$ Proved.

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15. Given $f(x) = \sin^{-1}\left(\frac{2^{x+1}3^x}{1+(36)^x}\right) = \sin^{-1}\left(\frac{2(6^x)}{1+(6^x)^2}\right)$ — ①

Let $6^x = \tan\theta$

Differentiating wrt x, $6^x \log 6 = \sec^2\theta \frac{d\theta}{dx}$ — ②

Putting $6^x = \tan\theta$ in eqn ①, $f(\theta) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$

Thus $\frac{d}{d\theta}(f(\theta)) = 2$ ~~Also~~ Multiplying and dividing LHS by dx,

$$\frac{df(\theta)}{d\theta} \cdot \frac{dx}{dx} = \frac{df(\theta)}{dx} \cdot \frac{dx}{d\theta} = 2 \Rightarrow \frac{d(f(\theta))}{dx} = 2\frac{d\theta}{\cancel{dx}}$$

$$\Rightarrow \frac{d}{dx} f(x) = \frac{d}{dx}\left(\sin^{-1}\left(\frac{2^{x+1} \cdot 3^x}{1+36^x}\right)\right) = 2 \cdot \frac{6^x \log 6}{\sec^2\theta}$$

$$= \frac{(2\log 6)\, 6^x}{1+\tan^2\theta} = \frac{2 \cdot 6^x}{1+36^x} \log 6$$

$$= \left(\frac{2^{x+1} \cdot 3^x}{1+(36)^x}\right) \log 6 \quad \text{Ans}$$

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16. $$f(x) = \begin{cases} \dfrac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & -1 \le x < 0 \\[2ex] \dfrac{2x+1}{x-1}, & 0 \le x < 1 \end{cases}.$$

$$f(0^+) = \lim_{h \to 0} f(0+h) = \frac{2(0+h)+1}{0+h-1} = \frac{2h+1}{h-1} = \frac{0+1}{0-1} = -1$$

$$f(0) = \frac{0+1}{0-1} = -1$$

$$f(0^-) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sqrt{1+kh}-\sqrt{1-kh}}{h}$$

Multiplying and dividing by $\sqrt{1+kh}+\sqrt{1-kh}$ on both sides, we get

$$f(0^-) = \frac{(\sqrt{1+kh}-\sqrt{1-kh})(\sqrt{1+kh}+\sqrt{1-kh})}{h(\sqrt{1+kh}+\sqrt{1-kh})} = \lim_{h\to 0} \frac{1+kh-(1-kh)}{h(\sqrt{1+kh}+\sqrt{1-kh})}$$

$$= \lim_{h\to 0} \frac{2k\cancel{h}}{\cancel{h}(\sqrt{1+kh}+\sqrt{1-kh})} = \frac{2k}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2k}{2} = k$$

For continuity at $x=0$, $f(0^+) = f(0) = f(0^-)$

$$\Rightarrow -1 = k \quad \text{Ans}$$

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17. Let $I = \int \frac{x+2}{\sqrt{x^2+2x+3}}\, dx$

$$I = \frac{1}{2}\int \frac{(2x+2)+2}{\sqrt{x^2+2x+3}}\, dx = \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}\, dx + \int \frac{1}{\sqrt{x^2+2x+3}}\, dx$$

Let $x^2+2x+3=t$

$\Rightarrow 2x+2=\frac{dt}{dx}$ $\Rightarrow I=\frac{1}{2}\int\frac{dt}{\sqrt{t}}+\int\frac{dx}{\sqrt{(x+1)^2+2}}$

$=\frac{1}{2}\cdot 2\sqrt{t}+\log\left(x+1+\sqrt{(x+1)^2+2}\right)+c$ c= arbitrary constant

$I=\sqrt{x^2+2x+3}+\log\left(x+1+\sqrt{x^2+2x+3}\right)+c$ Ans

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18. Let $I=\int\frac{dx}{x^6(x^5+3)}$

$=\int\frac{dx}{x(x^5(1+3x^{-5}))}=\int\frac{dx}{x^6(1+3x^{-5})}=\int\frac{x^{-6}}{3x^{-5}+1}dx$

Let $3x^{-5}+1=t$

$\Rightarrow -15x^{-6}dx=dt$ $\therefore I=\int\frac{-1}{15}\frac{dt}{t}=\frac{-1}{15}\int\frac{dt}{t}=\frac{-1}{15}\log t+c$

$=\frac{-1}{15}\log(3x^{-5}+1)+c$

$= \frac{-1}{15}\left[\log(3+x^5) - \log x^5\right] + c$

$= \frac{-1}{15}\log(x^5+3) + \frac{x}{3} + c$ ← Ans

19. Let $I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}}\,dx$

using property of definite integrals that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$,

$I = \int_0^{2\pi} \frac{1}{1+e^{\sin(2\pi-x)}}\,dx = \int_0^{2\pi} \frac{1}{1+e^{-\sin x}}\,dx \quad \{\sin(2\pi-x) = -\sin x\}$.

$= \int_0^{2\pi} \frac{1}{1+\frac{1}{e^{\sin x}}}\,dx = \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}}\,dx$

$I+I = 2I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}}\,dx + \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}}\,dx = \int_0^{2\pi}\left(\frac{1+e^{\sin x}}{1+e^{\sin x}}\right)dx = \int_0^{2\pi} 1\cdot dx = [x]_0^{2\pi}$

$\{\text{Using } \int f(x)\,dx + \int g(x)\,dx = \int (f(x)+g(x))\,dx\}$ $= 2\pi - 0 = 2\pi$

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$\because 2I = 2\pi \Rightarrow I = \pi$ Ans

20. given $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$

$\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

using additive properties of vectors, $\vec{a}+\vec{b} = \cancel{6i}\ (5+1)\hat{i} + (-1-1)\hat{j} + (\lambda+7)\hat{k}$

$= 6\hat{i} - 2\hat{j} + (\lambda+7)\hat{k}$

Also $\vec{a}-\vec{b} = (1-5)\hat{i} + (-1+1)\hat{j} + (7-\lambda)\hat{k}$

$= -4\hat{i} + (7-\lambda)\hat{k}$

For $\vec{a}+\vec{b}$ to be perpendicular to $\vec{a}-\vec{b}$, $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 0$

$\Rightarrow (6\hat{i} + -2\hat{j} + (7+\lambda)\hat{k})\cdot(-4\hat{i} + (7-\lambda)\hat{k}) = 0$

$\Rightarrow -24 + (49-\lambda^2) + 0 = 0$

$\Rightarrow \lambda^2 = 25$

$\Rightarrow \lambda = \pm 5$

21. Let the given lines be L_1 and L_2

$$L_1 \equiv \frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} \quad \text{and} \quad L_2 \equiv \frac{x-5}{3} = \frac{y+2}{2} = \frac{z-0}{6}$$

Any point $P(K)$ on line L_1 can be expressed as

$$P \equiv (K+3,\ 2K+2,\ 2K-4) \text{ — ① } K \in R$$

If L_1 and L_2 are intersecting (and are of course non-parallel), then there should be exactly one point of intersection, and in that case, only one real value of K should permit the exitence of that point on L_2, so

substituting x, y, z of L_2 by $P(K)$, we get

$$\frac{K+3-5}{3} = \frac{2K+2+2}{2}, \frac{2K-4-0}{6} \Rightarrow \frac{K-2}{3} = \overset{K+2}{\cancel{K}} = \frac{K-2}{3}.$$

which is true for $K=-4$ Hence the lines are intersecting

Putting this value in expression ①, $P \equiv \cancel{(2,0,-6)}\ (-1,-6,-12)$ Ans

22. Let E_1, E_2 ~~and~~ be events defined as

$E_1 \rightarrow$ A comes to school in time

$E_2 \rightarrow$ B comes in times

Given $p(E_1) = \frac{3}{7}$, $p(E_2) = \frac{5}{7}$. Also since E_1 and E_2 are independent events,

we get $p(E_1) \cdot p(E_2) = p(E_1 \cap E_2)$

$\Rightarrow p(E_1 \cap E_2) = \frac{3}{7} \cdot \frac{5}{7} = \frac{15}{49}$

$p(\text{only A coming in time}) = p(E_1) - p(E_1 \cap E_2)$

$= \cancel{pf} \qquad = \frac{3}{7} - \frac{15}{49} = \frac{6}{49} = p(\alpha)$

similarly, $p(\text{only B coming in time}) = p(E_2) - p(E_1 \cap E_2) = \frac{5}{7} - \frac{15}{49} = \frac{20}{49} = p(\beta)$

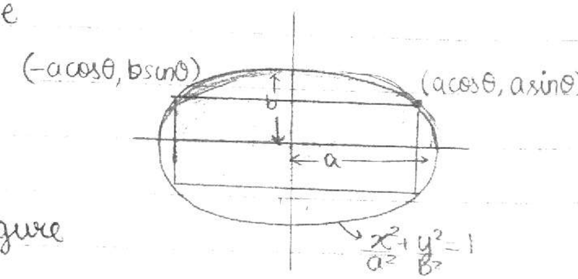
$p(\text{only either A or B coming in time}) = p(\alpha) + p(\beta) = \frac{6}{49} + \frac{20}{49} = \frac{26}{49}$ Ans

✓ One's sense of discipline improves if he/she comes to school in time.

SECTION-C

23. Let any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be parametrically represented as

$$P(\theta) = (a\cos\theta, b\sin\theta)$$



Then, area of rectangle shaded in the figure

$$A = l \cdot b$$

$$= (2a\cos\theta)(2b\sin\theta)$$

$$A = f(\theta) = 2ab\sin 2\theta$$

Clearly, $f(\theta)$ is maximum when $\sin 2\theta$ is max. i.e. $2\theta = \pi/2$ or $\theta = \pi/4$.

<u>Using calculus</u>: $A = f(\theta) = 2ab\sin 2\theta$ — (1)

$$f'(\theta) = 4ab\cos 2\theta$$

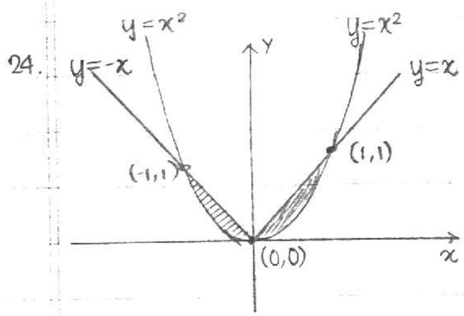
$$f'(\theta) = 0 \quad \text{at} \quad 2\theta = \frac{\pi}{2} \text{ or } \theta = \pi/4$$

$$f''(\theta) = -8ab\sin 2\theta \qquad f''(\pi/4) = -8ab < 0$$

Thus $f(\theta)$ is maximum at $\theta = \pi/4$.

Putting $\theta = \pi/4$ in eqn ①, $\therefore A_{max} = 2ab$ ~~Ans~~ sq. units   Ans

24.



The shaded area is the required area since both $|x|$ and x^2 are symmetrical about the y axis, the total area is twice of any one shaded part. Hence

$$A = \text{required area} = 2\left[\int_0^1 x\,dx - \int_0^1 x^2\,dx\right]$$

$$= 2\int_0^1 (x - x^2)\,dx = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left[\frac{1}{2} - \frac{1}{3} - (0 - 0)\right]$$

$$= 2\left(\frac{3-2}{6}\right) = \frac{1}{3} \text{ sq. units} \quad \text{Ans}$$

25. Given $(\tan^{-1}y - x)\,dy = (1+y^2)\,dx$

Let $y = \tan\theta$ $\therefore dy = \sec^2\theta\, d\theta$ $\Rightarrow (\theta - x)\cancel{\sec^2\theta}\, d\theta = (1+\tan^2\theta)\,dx$

$= \cancel{\sec^2\theta}\, dx$

$$\Rightarrow \frac{dx}{d\theta} + x = \theta$$

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$

$$I.F. = e^{\int 1 \cdot d\theta} = e^{\theta}$$

Multiplying by I.F. on both sides, $dx\, e^{\theta} + e^{\theta} d\theta\, x = \theta e^{\theta} d\theta$

$$\Rightarrow \int d(xe^{\theta}) = \int \theta e^{\theta} d\theta$$

$$\Rightarrow xe^{\theta} = e^{\theta}\cdot\theta - \int 1\cdot e^{\theta} d\theta$$

$$= e^{\theta}\theta - e^{\theta} + c$$

$$\Rightarrow x = \theta - 1 + ce^{-\theta}$$

Putting back $\theta = \tan^{-1}y$

$$x = \tan^{-1}y - 1 + ce^{-\tan^{-1}y}$$

At $x=0, y=0$, so $0 = 0 - 1 + ce^{0} \Rightarrow c = 1$

$$\Rightarrow x = \tan^{-1}y - 1 + e^{-\tan^{-1}y}$$ Ans

26. Let the required line be L

L passes through (1,2,3) $L \equiv \frac{x-1}{a} = \frac{y-2}{b} = \frac{z-3}{c}$ —① a, b, c being the D.R.s of the line L

L is parallel to planes P_1 and P_2, and hence perpendicular to their normals whose D.R.s are respectively a_1, b_1, c_1 and a_2, b_2, c_2 $a_1 = 1, b_1 = -1, c_1 = 2$
$a_2 = 3, b_2 = 1, c_2 = 1$

Thus, $aa_i + bb_i + cc_i = 0$ $i = 1, 2$

$\Rightarrow a - b + 2c = 0$

and $3a + b + c = 0$

Using cross multiplication to solve a, b, c, $\frac{a}{-3} = \frac{+b}{+5} = \frac{c}{4}$

substituting these ratios in the equation of line L,

$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

Its vector equation is $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$ Ans

27. Let N and S be two events defined as

S → the captain gets the number '6' on the die

$\bar{S} = N$ → the captain does not get the number '6' on the die

Obviously $p(S) = \frac{1}{6}$ and $p(N) = p(\bar{S}) = 1 - p(S) = \frac{5}{6}$.

Case 1: A wins

'A' can win if it gets a 6 before B. The following subcases are possible

S, NNS, NNNNS, NNNNNNS, ----∞

This suggests that either S occurs, or both A & B lose and the next time A wins, and so on.

Total probability for A winning

$$p(A) = p(S) + p(N)\,p(N)\,p(S) + (p(N))^4\,p(S) + \dots \infty$$

$$= \frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right)^2 + \frac{1}{6}\left(\frac{5}{6}\right)^4 + \dots \infty = \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \dots \infty\right] = \frac{1}{6}\,\frac{1}{1-\frac{25}{36}}$$

$= \frac{1}{6} \cdot \frac{36}{11} = \frac{6}{11}$

Case 2: B wins

Either A wins or B wins, so $p(A) + p(B) = 1 \Rightarrow p(B) = 1 - p(A) = 1 - 6/11 = \frac{5}{11}$

As it can be seen, the decision of referee was not correct because probability of winning of 'A' team ~~is~~ (6/11) is more than that of 'B' (5/11)

28. Let

x = number of units of A

y = number of units of B

workers required for making 1 unit of A = 2

" " " of x units of A = $2x$

workers required for making y units of B = $3y$

so total no. of workers required = $2x+3y$

This number should be less than or equal to no. of available workers i.e. 30

$\Rightarrow 2x+3y \leq 30 \quad \Leftrightarrow \quad \frac{x}{15} + \frac{y}{10} \leq 1$

similarly, no of required units of capital $= 3x+y$
This should be less than or equal to available capital

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$\Rightarrow 3x+y \leq \overset{32}{17} \quad \Leftrightarrow \quad \frac{x}{17/3} + \frac{y}{17} \leq 1$ Also, ~~profit~~ revenue $Z = 100x + \cancel{200y}\ 120y$

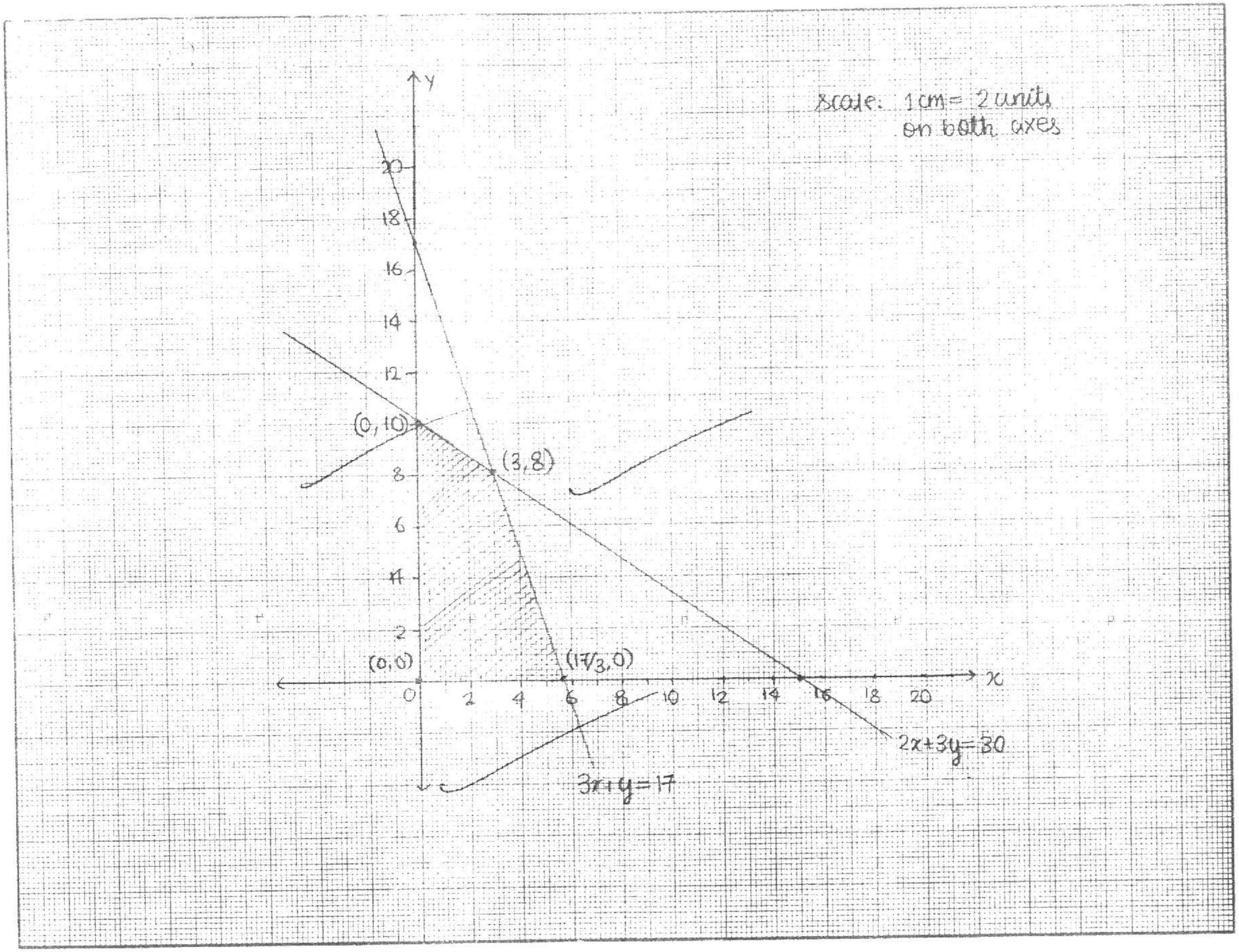
Also, $x \geq 0$ and $y \geq 0$ {no. of items cannot be negative}

Plotting the graphs of $\frac{x}{15} + \frac{y}{10} = 1$, $\frac{x}{17/3} + \frac{y}{17} = 1$, $x \geq 0$ and $y \geq 0$, and

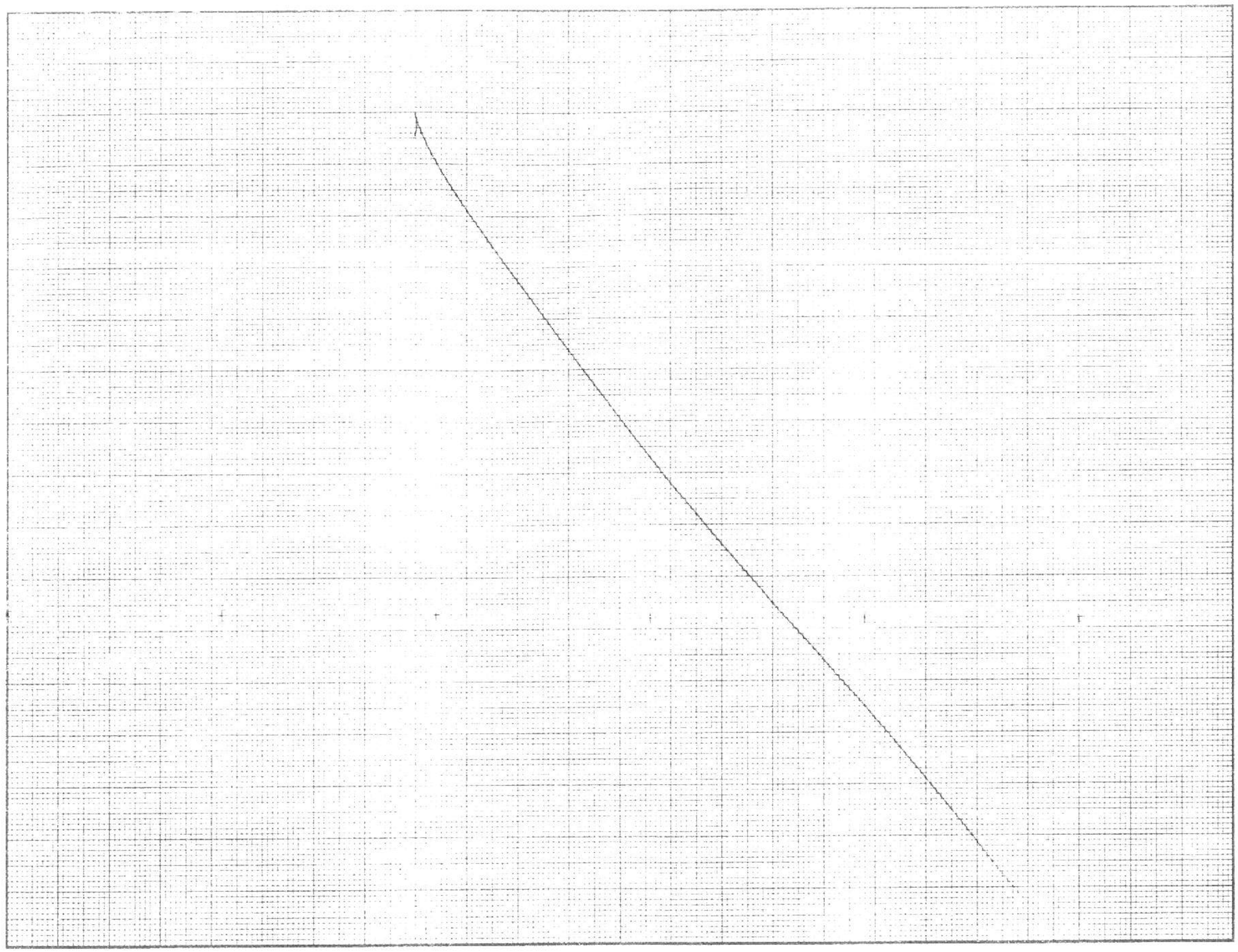
satisfying an arbitarary point (say (0,0)) in these equations, we get the shaded region. since an extremum lies on sharp points only, we plot a table for Z for 4 points only.

x	0	0	17/3	3
y	0	10	0	8
Z	0	$\boxed{\cancel{2000}}$ 1200	~~567~~ 567	~~1000~~ 1260

maximize(Z) : ~~x=0, y=10~~
~~at~~ $x=3$, $y=8$



Scale: 1 cm = 2 units on both axes
y
x
(0,10)
(3,8)
(0,0)
(17/3, 0)
2x+3y=30
3x+y=17



Thus, ~~profit~~ revenue will be maximized if ~~10~~ 3 units of B and ~~10~~ 8 units of A are manufactured. The corresponding revenue is ₹ ~~2000~~. 1260

✓ Yes. The manufacturer is a noble person in thinking that men and women are equally capable of doing work and as such ~~should~~ be paid equally.

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29. Let the number of awardees for honesty, helping others and supervisors be x, y and z respectively. We have

$$x+y+z=12 \quad \Rightarrow \quad x+y+z=12$$
$$3(y+z)+2x=33 \qquad 2x+3y+3z=33$$
$$x+z=2y \qquad x-2y+z=0$$

Let $A=\begin{bmatrix}1&1&1\\2&3&3\\1&-2&1\end{bmatrix}$, $B=\begin{bmatrix}12\\33\\0\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$

Then $AX=B$ yields the aforementioned equations

X can be found as $X=A^{-1}B$

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$|A|= 1(3+6)-(2-3)+(-4-3)$

$= 9+1-7=3\neq 0 \qquad \because |A|\neq 0 \therefore A^{-1}$ exists.

Now $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$

$A_{11} = +\begin{vmatrix} 3 & 3 \\ -2 & 1 \end{vmatrix} = 9$ $\quad A_{12} = -\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 1$ $\quad A_{13} = +\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -7$

$A_{21} = -\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -3$ $\quad A_{22} = +\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$ $\quad A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = 3$

$A_{31} = +\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} = 0$ $\quad A_{32} = -\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -1$ $\quad A_{33} = +\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1$

$$\text{adj} A = (\text{cof}\cdot A)^T = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A \qquad \therefore X = A^{-1}B = \frac{1}{|A|} \text{adj}A \cdot B = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 6 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\Rightarrow x=3, y=4, z=5$

$\therefore$ ~~ato~~ 3 awardees for honesty, 4 for cooperation and 5 for supervision.

✓ ~~Unity~~ Leadership is ~~any~~ an important quality that should be included in award categories, to ensure that there are some people that can take important decisions for the benefit of the colony.