केन्द्रीय माध्यमिक शिक्षा बोर्ड,

Central Board of Secondary Edu

(परीक्षार्थी भरें To be filled in by the candidate)

परीक्षार्थी प्रश्न–पत्र के ऊपर लिखें कोड को दर्शाये गये बॉक्स में लिखें
Candidate should write code no. as written on the top of the question paper in this box → 30/1

अतिरिक्त उत्तर–पुस्तिका (ओं) की संख्या
No. of supplementary answer-book (s) used →

परीक्षा का नाम Name of the examination AISSE 2013

कक्षा Class X

विषय Subject (041) MATHEMATICS

परीक्षा का दिन एवं तिथि
Day & Date of the Examination SATURDAY, 02-03-2013

उत्तर देने का माध्यम Medium of answering the paper ENGLISH

किसी शारीरिक अक्षमता से प्रभावित हो तो सम्बन्धित वर्ग में ✓ का निशान लगायें

B	D	H	S	C

B = दृष्टिहीन, D =मूक व बधिर, H =शारीरिक रूप से विकलांग, S = स्पास्टिक C = डिस्लेक्सिक

If Physically challenged, tick the category

B = Blind, D= Deaf & Dumb, H = Physically Handicapped, S = Spastic, C = Dyslexic

क्या लेखन–लिपिक उपलब्ध करवाया गया : हां / नहीं
Whether writer provided : Yes / No.

12024

SECTION - D

(25) $\frac{1}{x-3} + \frac{2}{x-2} = \frac{8}{x}$

$\Rightarrow \frac{x-2+2(x-3)}{(x-3)(x-2)} = \frac{8}{x}$

$\Rightarrow \frac{x-2+2x-6}{x^2-2x-3x+6} = \frac{8}{x}$

$\Rightarrow \frac{3x-8}{x^2-5x+6} = \frac{8}{x}$

$\Rightarrow (3x-8)x = 8(x^2-5x+6)$

$\Rightarrow 3x^2-8x = 8x^2-40x+48$

$\Rightarrow 8x^2-3x^2+40x+8x+48=0$

$\Rightarrow 5x^2-32x+48=0$

$\Rightarrow 5x^2-20x-12x+48=0$

$\Rightarrow 5x(x-4)-12(x-4)=0$

$\Rightarrow (x-4)(5x-12)=0$

either, $x-4=0$ | or, $5x-12=0$

$\Rightarrow x=4$ | $5x=12$

$x = \frac{12}{5}$

so, $x = 4$ & $\frac{12}{5}$

(26) cards are 3, 5, 7, 9, ..., 35, 37.

Let total no. of cards = n.

$$37 = 3 + (n-1)2$$

$$\Rightarrow 37 = 3 + (n-1)2$$

$$\Rightarrow 37 - 3 = (n-1)2$$

$$\Rightarrow 34 = (n-1)2$$

$$\Rightarrow \frac{34}{2} = (n-1)$$

$$\Rightarrow 17 = n-1$$

$$\Rightarrow n = 17 + 1 = 18$$

Let A : no. on card is prime no.

$$A = \{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37\}.$$

$$P(A) = \frac{11}{18}$$

5

(27) $S_1 = 5(1)^2 + 3(1)$

$= 5 + 3 = 8$

$S_2 = 5(2)^2 + 3(2)$

$= 5 \times 4 + 6 = 20 + 6 = 26$

$a_1 = S_1 = 8$

$a_2 = S_2 - S_1 = 26 - 8 = 18$

$d = a_2 - a_1 = 18 - 8 = 10$

$a_m = 168$

$\Rightarrow a_1 + (m-1) d = 168$

$\Rightarrow 8 + (m-1) 10 = 168$

$\Rightarrow (m-1) 10 = 168 - 8$

$\Rightarrow (m-1) 10 = 160$

$\Rightarrow (m-1) = 16$

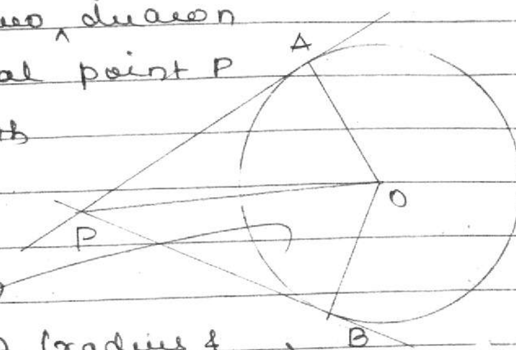
$\Rightarrow m = 17$

$a_{20} = a_1 + (20-1) d$

$= 8 + 19 d$

$= 8 + 19 \times 10 = 190 + 8 = 198.$

28. Given :- PA & PB are two tangents drawn from an external point P to a circle with centre O.



To Prove :- PA = PB

Construction :- Join PO

Proof :- OA ⊥ PA (radius & tangent)

$\angle PAO = 90^\circ$ — (I)

OB ⊥ PB (radius & tangent)

$\angle PBO = 90^\circ$ — (II)

In △PAO & △PBO

$\angle PAO = \angle PBO = 90^\circ$. (from (I) & (II))

PO = PO (common)

AO = BO (radii of same circle)

∴ △PAO ≅ △PBO (by RHS)

So, PA = PB (by CPCT)

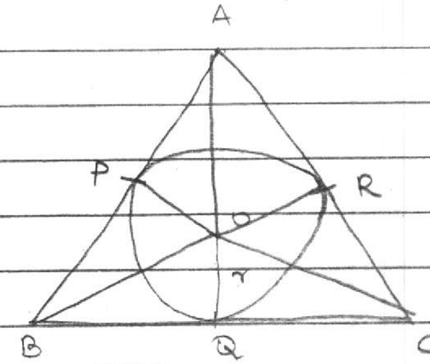
Hence proved

29.

(i) $AP = AR$ (tangents from A) — (I)

$BP = BQ$ (tangents from B) — (II)

$CQ = CR$ (tangents from C) — (III)



Adding (I), (II) & (III)

$AP + BP + CQ = AR + BQ + CR$

$(AP + BP) + CQ = (AR + CR) + BQ$

$AB + CQ = AC + BQ$

Proved

(ii) Area (ΔABC) = ar(ΔBOC) + ar(ΔBOA) + ar(ΔCOA)

$= \frac{1}{2} \times OQ \times BC + \frac{1}{2} \times OP \times AB + \frac{1}{2} \times OR \times AC$

$= \frac{1}{2} \times OQ \times BC + \frac{1}{2} \times OQ + AB + \frac{1}{2} \times OQ \times AC$

($\because OQ = OP = OR = r$ as radii of same circle)

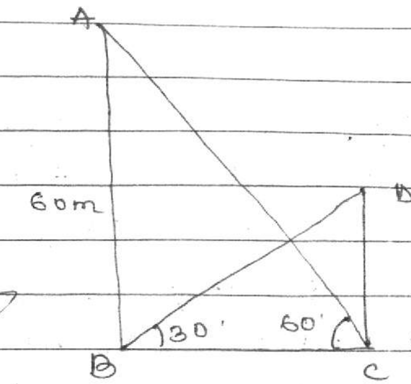
$= \frac{1}{2} OQ \left(BC + AB + AC \right)$

$= \frac{1}{2} r \ (AB + BC + AC)$

$= \frac{1}{2}$ (Perimeter of ΔABC) $\times r$

Proved

30. Let AB be the tower &
CD be the building
AB = 60 m.



In $\triangle ABC$,

$$\frac{AB}{BC} = \tan 60$$

$$\frac{60}{BC} = \sqrt{3}$$

$$BC = \frac{60}{\sqrt{3}} \text{ m.}$$

In $\triangle BDC$,

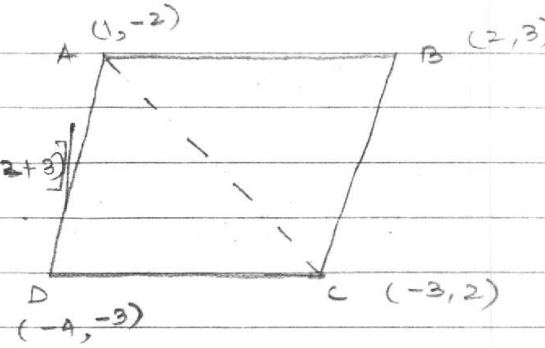
$$\frac{DC}{BC} = \tan 30$$

$$\frac{DC}{BC} = \frac{1}{\sqrt{3}}$$

$$DC = \frac{BC}{\sqrt{3}} \Rightarrow DC = \frac{60}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{60}{3}$$

$$= 20 \text{ m}$$

So, height of building is 20 m



(31) Ar (ΔADC)

$= \frac{1}{2}\left|\left[1(-3-2) + (-4)(2+2) + (-3)(-2+3)\right]\right|$

$= \frac{1}{2}\left|\left[-5 + (-4)(4) + (-3)(1)\right]\right|$

$= \frac{1}{2}\left|\left[-5 - 16 - 3\right]\right|$

$= \frac{1}{2} \times |-24| = \frac{1}{2} \times 24 = 12$ sq. units

Ar (ΔABC)

$= \frac{1}{2}\left|\left[1(3-2) + 2(2+2) + (-3)(-2-3)\right]\right|$

$= \frac{1}{2}\left|\left[1 + 8 + 15\right]\right|$

$= \frac{1}{2} \times |24| = \frac{1}{2} \times 24 = 12$ sq. units

Ar (||gm ABCD) $= 12 + 12 = 24$ sq. units

$AB \times h = 24$

$\Rightarrow AB \times h = \cancel{48}\ 24$

$\Rightarrow \sqrt{(1-2)^2 + (-2-3)^2} \times h = \cancel{48}\ 24$

$\Rightarrow \left(\sqrt{(-1)^2 + (-5)^2}\right) \times h = \cancel{48}\ 24$

$= \left(\sqrt{1+25}\right) \times h = \cancel{48}\ 24$

$\Rightarrow \sqrt{26} \times h = \cancel{48}\ 24$

$\Rightarrow \boxed{h = \frac{48}{\sqrt{26}} = \frac{\overset{24}{\cancel{48}}\sqrt{26}}{\underset{13}{\cancel{26}}} = \frac{24}{13}\sqrt{26} \text{ units}}$

$\Rightarrow h = \frac{24}{\sqrt{26}} = \frac{24\sqrt{26}}{26} = \frac{12}{13}\sqrt{26}$ units. So, height of llgm is $\frac{12}{13}\sqrt{26}$

(32) Let water flows in 1 min $= h$.

$\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h = 192.5\ l.$

$\frac{\cancel{22}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{2} \times h = 192.5 \times 1000\ cm^3$

$\frac{11 \times 7}{2} \times h = 192500$

$h = \frac{\overset{\cancel{27500}\ 2500}{\cancel{192500}} \times 2}{\cancel{11 \times 7}}$

$= 5000\ cm$

water flows in 1 min = 5000 cm

water flows in $\frac{1}{60}$ hr = $\frac{5\cancel{000}}{100 \times 1\cancel{000}}$

water flows in 1 hr = $\frac{\cancel{5}}{\cancel{100}_{2}} \times \cancel{60}^{3}$

= 3 km

Rate of flow of water = 3 km/hr.

---

(33) Let Original speed of plane = x km/hr

Time taken to reach destination = $\frac{1500}{x}$ hr.

New speed = $(x+100)$ km/hr

New time = $\frac{1500}{x+100}$ hr

$$\frac{1500}{x} - \frac{1500}{x+100} = \frac{\cancel{30}}{2\cancel{0}\cancel{0}}$$

$$\Rightarrow 1500\left(\frac{1}{x} - \frac{1}{x+100}\right) = \frac{1}{2}$$

$\Rightarrow 1500 \left[\frac{\cancel{x}+100-\cancel{x}}{x(x+100)}\right] = \frac{1}{2}$

$\Rightarrow \frac{1500 \times 100}{x^2+100x} = \frac{1}{2}$

$\Rightarrow x^2+100x = 1500 \times 100 \times 2$

$\Rightarrow x^2+100x = 300000$

$\Rightarrow x^2+100x-300000 = 0$

$\Rightarrow x^2+600x-500x-300000 = 0$

$\Rightarrow x(x+600)-500(x+600) = 0$

$\Rightarrow (x+600)(x-500) = 0$

$x+600 = 0 \quad | \quad x-500 = 0$

$x = -600 \quad | \quad x = 500$

Since speed can't be negative

so, $x = 500$ km/hr.

So, original speed = 500 km/hr.

Yes, pilot is a responsible, honest, helpful, kind & generous person as he shown promptness towards the injured person. So, his values must be appreciated.

(34) Metal sheet required

= CSA of frustum + Area of bottom.

Here $h = 16$ cm

$r_1 = \frac{16}{2} = 8$ cm

$R = \frac{40}{2} = 20$ cm

$l = \sqrt{h^2 + (R-r)^2}$

$= \sqrt{(16)^2 + (20-8)^2}$

$= \sqrt{(16)^2 + (12)^2}$

$= \sqrt{256 + 144}$

$= \sqrt{400}$

$= 20$ cm.

Metal required

$= 3.14 \times 20 \, (R+r) + 3.14 \times r^2$

$= 3.14 \times 20 \, (20+8) + 3.14 \times 8^2$

$= 3.14 \times 20 \times 28 + 3.14 \times 64$

$= 3.14 \, (560 + 64)$

$= 3.14 \, (624) = 3.14 \times 624$

cost of metal required

$= \frac{1\cancel{0}}{10\cancel{0}} \times 3.14 \times 624$

$= \frac{196\cancel{0}\cancel{3}6}{\cancel{1}\cancel{0}} \quad \frac{195936}{10}$

$= \cancel{Rs\ 196.036} = Rs\ 195.936$

$\boxed{\text{Section - 'C'}}$

15. $(k+4)x^2 + (k+1)x + 1 = 0$

$a = k+4, \quad b = k+1, \quad c = 1$

For equal roots,

$b^2 - 4ac = 0$

$b^2 = 4ac$

$(k+1)^2 = 4(k+4)1$

$k^2 + 1 + 2k = 4k + 16$

$k^2 + 2k - 4k + 1 - 16 = 0$

$k^2 - 2k - 15 = 0$

$k^2 - 5k + 3k - 15 = 0$

$k(k-5) + 3(k-5) = 0$

$(k-5)(k+3) = 0$

$K-5=0$ | $K+3=0$
$K=5$ | $K=-3$

So, values of K are 5 & -3.

~~Let a be the 1st term & d be common difference~~

16 $a_{19} = 3a_6$

$a+18d = 3(a+5d)$

$a+18d = 3a+15d$

$2a = 3d$ — 1

$a_9 = 19$

$a+8d = 19$

$2(a+8d) = 19 \times 2$

$2a+16d = 38$

$3d+16d = 38$

$19d = 38$

$d = \frac{\cancel{38}\,2}{\cancel{19}}$

$d = 2$

Putting value of d in eq^n (1).

$2a = 3(2)$

$2a = 6$

$a = 3$

So, AP is 3, 5, 7, 9 . . . .

(17) Steps of construction :-

1) Draw a line segment OA of length 4 cm

2) Taking OA radius draw a circle with centre O

3) Draw 120° on OA taking O as centre & let the ray intersect circle at B

4) Taking B & A as draw 90° on each

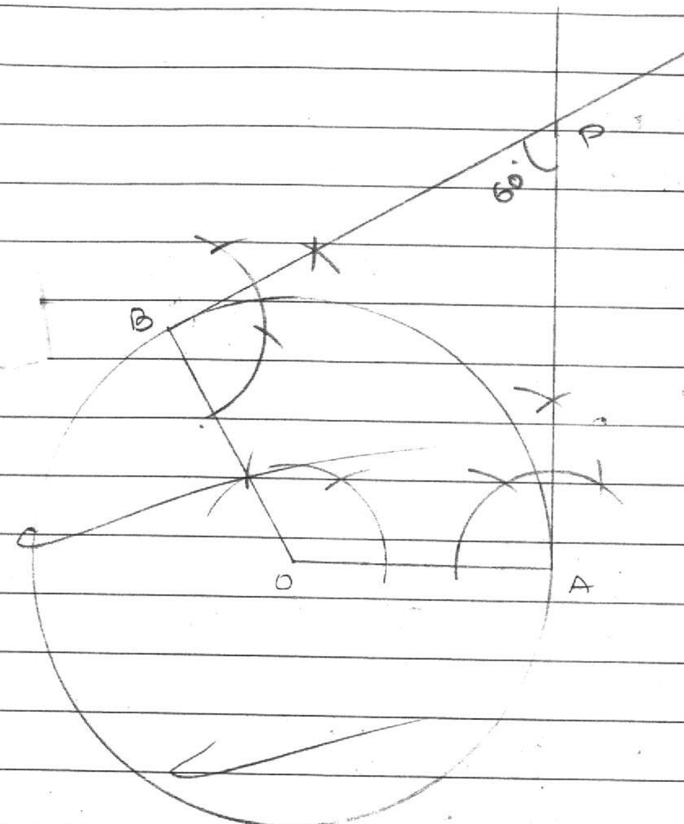
5) ~~&~~ Let these rays intersect at P

6) PA & PB are required tangents

Justification :- $\angle BOA = 120°$, $\angle PBO = 90° = \angle OAP$

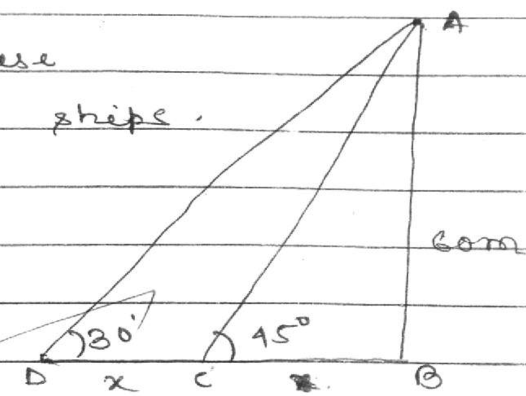
So, $\angle BPA = \cancel{180}\ 360 - (120 + 90 + 90)$

$= 360 - 300$



PA & PB are required tangents

(18) Let AB be the lighthouse
and C & D be the two ships.
Let $BC = x \cdot m$.



In ΔABC,

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{60}{BC} = 1$$

$$60 = BC$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30$$

$$\frac{60}{BC+CD} = \frac{1}{\sqrt{3}}$$

$$\frac{60}{60+x} = \frac{1}{\sqrt{3}}$$

$$60\sqrt{3} = 60+x$$

$$x = 60(\sqrt{3}-1)$$

$$x = 60(1.732-1)$$

$$= 60(0.732)$$

$= 43.92\ m.$

so, Distance between two ships $= 43.92\ m.$

(19.) $AB = \sqrt{(2+2)^2 + (3-2)^2}$

$= \sqrt{4^2 + 1^2} = \sqrt{16+1} = \sqrt{17}$

$BC = \sqrt{(-2+1)^2 + (2+2)^2}$

$= \sqrt{(-1)^2 + (4)^2} = \sqrt{16+1} = \sqrt{17}$

$CD = \sqrt{(-1-3)^2 + (-2+1)^2}$

$= \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$

$AD = \sqrt{(2-3)^2 + (3+1)^2}$

$= \sqrt{(-1)^2 + (4)^2} = \sqrt{16+1} = \sqrt{17}$

$AC = \sqrt{(2+1)^2 + (3+2)^2}$

$= \sqrt{3^2 + 5^2} = \sqrt{9+25} = \sqrt{34}$

$BD = \sqrt{(-2-3)^2 + (2+1)^2}$

$= \sqrt{(-5)^2 + (3)^2} = \sqrt{25+9} = \sqrt{34}$

since $AB = BC = CD = AD$ & $AC = BD$.

so, all sides are equal & diagonals are equal

so, ABCD is a square

20. Let P (−1, y) divides AB in k : 1.

$$\underset{(-3,10)}{A}\text{ ------ }\overset{(-1,y)}{P}\text{ ------ }\underset{(6,-8)}{B}$$

$$(-1, y) = \left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$$

So, $\frac{6k-3}{k+1} = -1$

$\Rightarrow 6k - 3 = -k - 1$

$\Rightarrow 7k = 2$

$\Rightarrow k = \frac{2}{7}$

So, P divides AB in ratio 2 : 7.

Now, $\frac{-8k+10}{k+1} = y$

$\Rightarrow \frac{-8\left(\frac{2}{7}\right)+10}{\frac{2}{7}+1} = y$

$$\Rightarrow \frac{\frac{-16}{7}+10}{\frac{2+7}{7}} = y$$

$$\Rightarrow \frac{\frac{-16+70}{\cancel{7}}}{\frac{9}{\cancel{7}}} = y$$

$$\Rightarrow \frac{\cancel{54}\,6}{\cancel{9}} = y$$

$\Rightarrow y = 6$

So, $y = 6$.

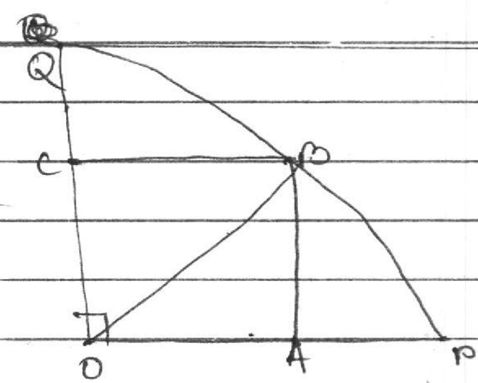
(21) OABC is a square

So, $\angle OAB = 90^\circ$

$OA^2 + AB^2 = OB^2$

$(21)^2 + (21)^2 = OB^2$

$OB^2 = 2 \times (21)^2$



Ar (shaded region) = ar (quadrant OPBQ) − ar (sq. OABC)

$= \frac{1}{4} \times \pi \times r^2 - (OA)^2$

$= \frac{1}{4} \times \frac{22}{7} \times (OB)^2 - (OA)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 2 \times 21 \times 21 - 21 \times 21$

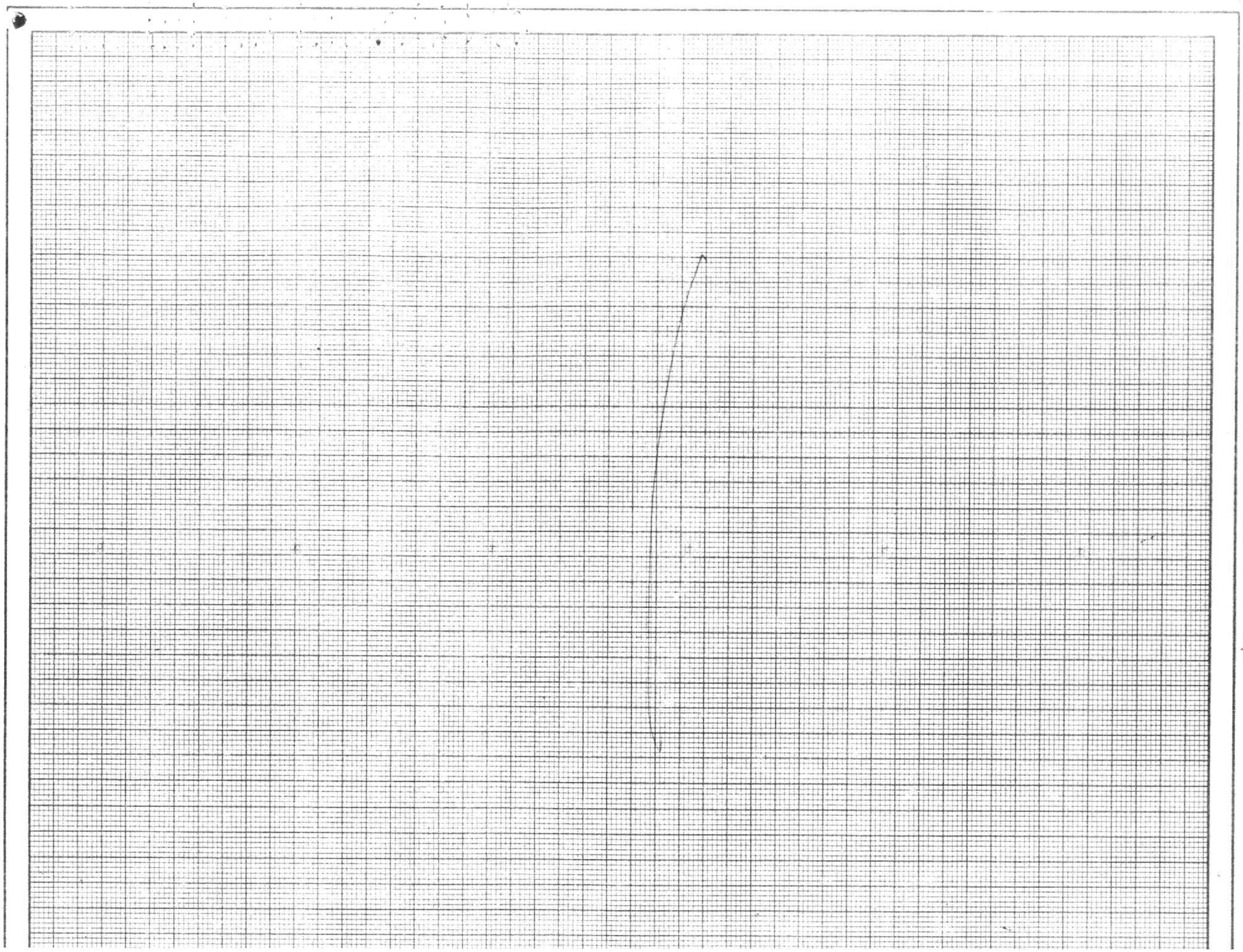
$= 21 \times 21 \left(\frac{1}{4} \times \frac{22}{7} \times 2 - 1 \right)$

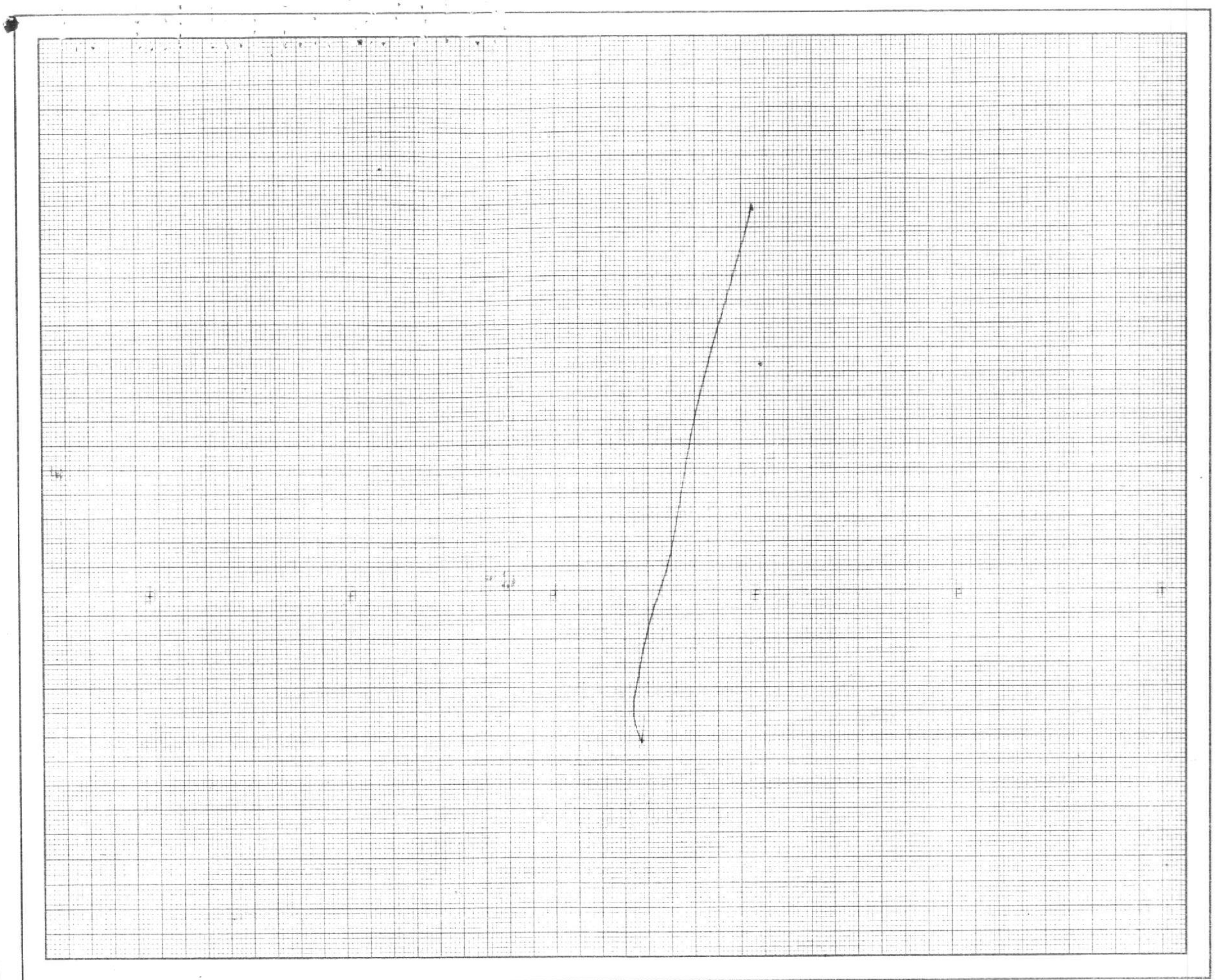
$= 21 \times 21 \left(\frac{11}{7} - 1 \right)$

$= 441 \left(\frac{11-7}{7} \right)$

$= 441 \left(\frac{4}{7} \right)$ 63

$= 252 \text{ cm}^2$



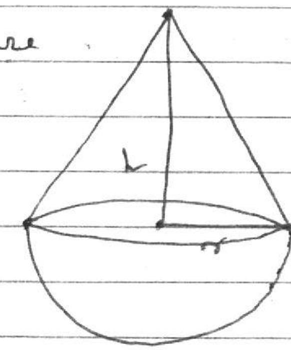


22. Radius of cone = Radius of hemisphere

= 7 cm.

height of cone = 31 − 7

= 24 cm



TSA of toy = CSA of cone + CSA of hemisphere

$l = \sqrt{h^2 + r^2} = \sqrt{(24)^2 + (7)^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625}$

= 25 cm

TSA of toy = $\frac{22}{7} \times r \times l + 2\pi r^2$

$= \frac{22}{\cancel{7}} \times \cancel{7} \times 25 + 2 \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7$

= 550 + 308

= 858 cm².

23.

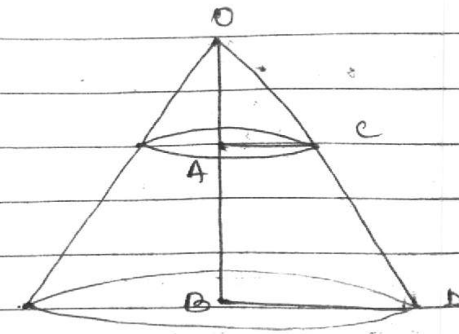
~~In AP~~

Let $AC = r$ & $BD = R$

$OA = AB = \frac{OB}{2}$

Let $OB = H$.

$OA = \frac{H}{2}$



In ΔOAC & ΔOBD

$\angle AOC = \angle BOD$ (common)

$\angle OAC = \angle OBD$ (90° each)

$\therefore \Delta OAC \sim \Delta OBD$ (by AA)

$\frac{OA}{OB} = \frac{AC}{BD}$

$\frac{\cancel{H}}{2} \times \frac{1}{\cancel{H}} = \frac{r}{R}$

$\frac{r}{R} = \frac{1}{2} \Rightarrow r = \frac{R}{2}$

$$\frac{\text{Vol. of small cone}}{\text{Vol of frustum}}$$

$$= \frac{\cancel{\frac{1}{3}} \cancel{\pi} r^2 \frac{H}{2}}{\cancel{\frac{1}{3}} \cancel{\pi} \left(H - \frac{H}{2}\right) (r^2 + R^2 + Rr)}$$

$$= \frac{r^2 \cancel{\frac{H}{2}}}{\cancel{\frac{H}{2}} (r^2 + R^2 + Rr)}$$

$$= \frac{\frac{R^2}{4}}{\frac{R^2}{4} + R^2 + R\left(\frac{R}{2}\right)}$$

$$= \frac{\frac{R^2}{4}}{\frac{R^2 + 4R^2 + 2R^2}{4}} = \frac{\frac{R^2}{\cancel{4}}}{\frac{7R^2}{\cancel{4}}}$$

$$= \frac{\cancel{R^2}}{7\cancel{R^2}} = \frac{1}{7}$$

so, ratio of vol. of two parts of cone is 1:7

24. radius of sphere $(r) = \frac{8}{2} = 4\,cm = \frac{4}{100}\,m$.

height of cylindrical wire $(H) = 12\,m$.

let radius of cylindrical wire ~~(R)~~ = R.

Vol of sphere = vol. of cylindrical wire.

$$\frac{4}{3}\cancel{\pi}r^3 = \cancel{\pi}R^2H$$

$$\frac{4}{3} \times \frac{4}{100} \times \frac{4}{100} \times \frac{4}{100} = R^2 \times 12$$

$$R^2 = \frac{4 \times 4 \times 4 \times 4}{100 \times 100 \times 100 \times 3 \times 12}$$

$$R^2 = \frac{4 \times 4 \times 4 \times 4}{100 \times 100 \times 10 \times 10 \times 2 \times 2 \times 3 \times 3}$$

$$R = \frac{\cancel{4}^{2} \times 4}{100 \times 10 \times \cancel{2} \times 3}$$

$$= \frac{8}{100 \times 10 \times 3}$$

$$= \frac{2.666}{1000}$$

$$= 0.002666 \text{ m}$$

$$= \frac{2666}{10000\cancel{00}} \times \cancel{100}$$

$$= 0.2666 \text{ cm.}$$

$$\text{width} = 2R = 2 \times 0.2666$$

$$= 0.5332 \text{ cm.}$$

$$\boxed{\text{Section - B}}$$

9.

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

$$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$$

$$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$$

$$(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$$

$\sqrt{2}x + 5 = 0$ $\sqrt{2}\,x = -5$ $x = \frac{-5}{\sqrt{2}}$ $= \frac{-5\sqrt{2}}{2}$	$x + \sqrt{2} = 0$ $x = -\sqrt{2}$

(10) Required nos (3-digit no. divisible by 9)

$= 108, 117, \ldots\ldots, 999$

Here $a = 108$, $d = 9$, $a_n = 999$

~~999~~

$a_n = a + (n-1)\,d$

$999 = 108 + (n-1)9$

$999 - 108 = (n-1)9$

$891 = (n-1)9$

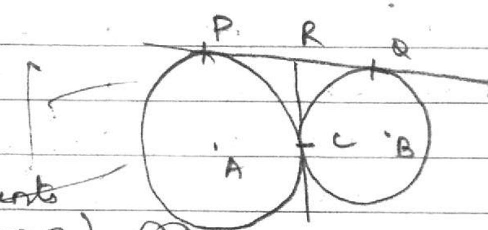
$(n-1) = \frac{891}{9}$

$(n-1) = 99$

$n = 100$

so, no. of all 3 digit nos. divisible by 9 = 100

(11) Given :- CR & PQ are common tangents to circles with centre A & B



To Prove :- CR bisects PQ

Proof :- PR = RC (tangents from R) — (i)

RC = QR (tangents from R) — (ii)

from (i) & (ii)

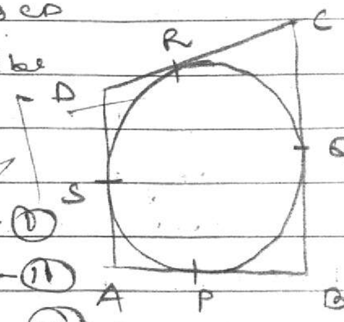
PR = RC = QR

=> PR = QR

i.e. CR bisects PQ as R is mid point of PQ

Hence proved

12. Given:- A quadrilateral ABCD is drawn to circumscribe a circle.



To prove:- $AB + CD = AD + BC$

Proof:- $AP = AS$ (tangents from A) - (I)

$BP = BQ$ (tangents from B) - (II)

$DR = DS$ (tangents from D) - (III)

$CR = CQ$ (tangents from C) - (IV)

Adding (I), (II), (III) & (IV)

$$AP + BP + DR + CR = AS + BQ + DS + CQ$$

$$(AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ)$$

$$AB + CD = AD + BC$$ proved//

13. Total no. of outcomes $= 2 \times 2 = 4$

Let A : getting atleast one head

$A = \{HT, TH, HH\}$

activities and F

$P(A) = \frac{3}{4}$

(14.) Angle covered by minute hand in 60 mins = 360'

Angle covered by minute hand in 1 min = $\frac{360}{60}$

Angle covered by minute hand in 5 min = $\frac{\cancel{360}^{6}}{\cancel{60}} \times 5$

$= 30'$

length of minute hand (r) = 14 cm

Area swept by minute hand in 5 mins

$= \frac{\theta}{360} \pi r^2$

$= \frac{\cancel{30}}{\cancel{\cancel{360}}_{\cancel{12}\ \cancel{6}\ 3}} \times \frac{22}{\cancel{7}} \times \cancel{14}^{2} \times \cancel{14}\ 7$

$= \frac{154}{3}$ cm^2

so, Area swept in 5 mins $= \frac{154}{3}$ cm^2

Section - A

1. (A) -1

2. (B) 5 cm

3. (C) 21 cm

4. (C) $\frac{1}{2}$

5. (C) $\frac{15}{2}$ m

6. (C) $\frac{1}{9}$

7. (B) $\frac{2}{0}$

8. (B) 1:1