MATHEMATICS - 041

Space for writing Question Number

SECTION-A

(1) (B) $1-p^2-qr=0$

(2) (B) $-I$

(3) (D) $\text{cosec}^{-1}: R-(-1,1) \longrightarrow \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(4) (C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$

(5) (A) Adj A is singular

(6) (D) -6

(7) (B) $-1, 3$

(8) (A) -1

(9) (C) 5

(10) (B) $\frac{1}{\sqrt{2}} \log|\sec x + \tan x| + C$

(11) (A) $-\log 5$

(12) (B) 3 sq. units

(13) (A) order = 3, Degree = 3

(14) (A) $3e^{y} = e^{3x} + C$

(15) (B) 300

(16) (D) ± 7

(17) (C) $\frac{2}{5}$

(18) (D) $\frac{3}{5}$

(19) (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(20) (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

SECTION-B

(21) $\sin[\tan^{-1}(\tan\frac{3\pi}{4})]$

$= \sin[\tan^{-1}(-1)] \quad [\because \tan\frac{3\pi}{4} = -1]$

$= \sin[-\tan^{-1}1] \quad [\because \tan^{-1}(-x) = -\tan^{-1}x$

$= \sin[-\pi/4] \quad \tan\pi/4 = 1$

$\tan^{-1}(1) = \pi/4]$

$= \frac{-1}{\sqrt{2}}$ //

$\therefore \sin[\tan^{-1}(\tan\frac{3\pi}{4})] = \frac{-1}{\sqrt{2}}$.

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(22) (b) Given,

$y = P\cos ux + Q\sin ux$, Differentiating w.r.t 'x'

$$\frac{dy}{dx} = P\frac{d(\cos ux)}{dx} + Q\frac{d(\sin ux)}{dx}$$

$$\frac{dy}{dx} = Pu(-\sin ux) + Qu(\cos ux)$$

$$= Qu\cos ux - Pu\sin ux$$

Again differentiate w.r.t 'x'

$$\frac{d^2y}{dx^2} = Qu\frac{d(\cos ux)}{dx} - Pu\frac{d(\sin ux)}{dx}$$

$$= Qu(-u\sin ux) - Pu(u\cos ux)$$

$$= -Qu^2\sin ux - Pu^2\cos ux$$

Now, $LHS = \frac{d^2y}{dx^2} + u^2y$

$$= -Qu^2\sin ux - Pu^2\cos ux + u^2(P\cos ux + Q\sin ux)$$

$$= -Qu^2\sin ux - Pu^2\cos ux + Pu^2\cos ux + Qu^2\sin ux$$

$$= 0 = R.H.S$$

$\therefore \boxed{\frac{d^2y}{dx^2} + u^2y = 0}$

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(Q3) Given, $f(x) = \frac{x-5}{x+1}$ $x \neq -1$

Differentiate w.r.t x:

$$f'(x) = \frac{d}{dx}\left(\frac{x-5}{x+1}\right)$$

$$= \frac{(x+1)\frac{d}{dx}(x-5) - (x-5)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right]$$

$$= \frac{(x+1)1 - (x-5)}{(x+1)^2}$$

$$f'(x) = \frac{4}{(x+1)^2}$$

$$\Rightarrow \frac{4}{(x+1)^2} > 0 \quad \forall\ x \in R - \{-1\}$$

$$\therefore f'(x) > 0 \quad \forall\ x \in R - \{-1\}$$

$\therefore f(x)$ is an increasing function on its complete domain i.e. $R - \{-1\}$

It always increases.

(24) (b) Given, A, B, C → 3 non-collinear points,

$\overline{AB} = \hat{i} + 2\hat{j} - \hat{k}$

$\overline{AC} = 2\hat{i} - 3\hat{j}$

W.K.T Area of $\triangle ABC = \frac{1}{2} |\overline{AB} \times \overline{AC}|$

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -3 & 0 \end{vmatrix}$$

$$= \hat{i}(0-3) - \hat{j}(0+2) + \hat{k}(-3-4)$$

$$= -3\hat{i} - 2\hat{j} - 7\hat{k}$$

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2} |-3\hat{i} - 2\hat{j} - 7\hat{k}|$$

$$= \frac{1}{2} \sqrt{3^2 + 2^2 + 7^2} = \frac{1}{2} \sqrt{9+4+49}$$

$$= \frac{1}{2} \sqrt{62}$$

Area of $\triangle ABC = \frac{\sqrt{62}}{2}$ sq. units

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(26) Given lines:

$$\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6} \quad , \quad \frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{2}$$

$$\Rightarrow \frac{x-2}{3} = \frac{y+5}{2} = \frac{z-1}{6} \quad , \quad \frac{x-1}{1} = \frac{y}{2} = \frac{z-6}{2}$$

$\Rightarrow$ vectors parallel to the lines are:

$b_1 = 3\hat{i}+2\hat{j}+6\hat{k}$, $b_2 = \hat{i}+2\hat{j}+2\hat{k}$

W.K.T, Now, $\cos\theta = \dfrac{b_1 \cdot b_2}{|b_1||b_2|}$

$$= \frac{(3\hat{i}+2\hat{j}+6\hat{k}) \cdot (\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{3^2+2^2+6^2} \cdot \sqrt{1^2+2^2+2^2}}$$

$$= \frac{3+4+12}{\sqrt{49} \cdot \sqrt{9}}$$

$$= \frac{19}{7 \times 3} = \frac{19}{21}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

$\therefore$ Angle b/w the pair of lines is $\cos^{-1}\left(\frac{19}{21}\right)$

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SECTION-C

(25) Let volume of the balloon be V, radius of the balloon be r and surface Area be S

Given, $\frac{dV}{dt} \propto S$

$\Rightarrow \frac{dV}{dt} = -KS \quad (K>0)$ (K is a positive constant)
(-' as Volume decreases and S>0)

$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = -K(4\pi r^2) \quad \left(V=\frac{4}{3}\pi r^3\right.$
$\left. S = 4\pi r^2 \text{ for sphere}\right)$

$\Rightarrow \frac{4}{3}\pi(3r^2)\frac{dr}{dt} = -K(4\pi r^2)$

$\Rightarrow \boxed{\frac{dr}{dt} = -K} \quad (\because K>0)$

∴ Radius of the balloon decreases at a constant rate.,,

(25) (b) $\int_0^2 \frac{1}{\sqrt{x^2+2x+3}}\,dx$

$= \int_0^2 \frac{1}{\sqrt{(x^2+2x+1)+2}}\,dx$

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$$= \int_0^2 \frac{1}{\sqrt{(x+1)^2+2}}\,dx$$

Substitute $t = x+1 \Rightarrow dt = dx$ } changing limits of the integral
$x \to 0,\ t \to 1$
$x \to 2,\ t \to 3$

$$= \int_1^3 \frac{1}{\sqrt{t^2+2}}\,dt$$

$$= \left[\log\left|t+\sqrt{t^2+2}\right|\right]_1^3 \qquad \left[\because \int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left|x+\sqrt{x^2+a^2}\right|\right]$$

$$= \left(\log\left|3+\sqrt{3^2+2}\right| - \log\left|1+\sqrt{1^2+2}\right|\right)$$

$$= \log\left|(3+\sqrt{11})\right| - \log\left|1+\sqrt{3}\right|$$

$$= \log\left|\frac{3+\sqrt{11}}{1+\sqrt{3}}\right| \qquad \left[\because \log a - \log b = \log\left(\frac{a}{b}\right)\right]$$

$$\therefore \int_0^2 \frac{1}{\sqrt{x^2+2x+3}} = \log\left|\frac{3+\sqrt{11}}{1+\sqrt{3}}\right|.$$

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(28) Given,

$$(x+xy^3)\,dy = y\,dx$$

$$\frac{dx}{dy} = \frac{x+xy^3}{y}$$

$$\frac{dx}{dy} = \frac{x}{y} + xy^2$$

$$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = xy^2$$

This • is of form $\frac{dx}{dy} + Px = Q$

$$\Rightarrow P = (-1/y) \quad Q = xy^2$$

Integrating factor (I.F) $= e^{\int -1/y\,dy}$

$$= e^{-\ln y}$$

$$= \frac{1}{y}$$

∴ Solution for the D.E is given by,

$$\left(\frac{1}{y}\right) x = \int \frac{1}{y} \times xy^2 + C$$

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$$\Rightarrow \frac{x}{y} = \int 2y + c$$

$$\frac{x}{y} = 2\frac{y^2}{2} + c$$

$$\frac{x}{y} = y^2 + c$$

$$\Rightarrow \boxed{x = y^3 + cy}$$

(20) (a) Given, Tharee toys A, B, C is same line.

∴ A, B, C are collinear points

$\Rightarrow \overline{AB}$ and $\overline{AC}$ are collinear vectors.

$A(55\hat{i} - 2\hat{j})$

$B(5\hat{i} + 8\hat{j})$

$C(a\hat{i} - 52\hat{j})$

$$\overline{AB} = (5-55)\hat{i} + (8-(-2))\hat{j} = -50\hat{i} + 10\hat{j}$$

$$\overline{AC} = (a-55)\hat{i} + (52-(-2))\hat{j} = (a-55)\hat{i} - 50\hat{j}$$

W.K.T components of 2 collinear vectors are proportional;

$$\therefore \frac{-50}{a-55} = \frac{10}{-50} \Rightarrow \frac{50}{a-55} = \frac{1}{5}$$

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$250 = a - 55$

$\Rightarrow a = 305$

$\therefore$ value of a is 305

Verification.

$\overline{BC}$ & $\overline{BA}$ also are collinear.

$\overline{BC} \rightarrow (305 - 5)\hat{i} + (-52 - 8)\hat{j} = 300\hat{i} - 60\hat{j}$

$\overline{BA} \rightarrow 50\hat{i} - 10\hat{j}$

$\frac{300}{50} = \frac{-60}{-10}$ ✓

$\therefore$ Value of $a = 305$ is

(OR)

(b) Given, a, b, c are unit vectors,

$|a| = |b| = |c| = 1$

Now, $|a-b|^2 = (a-b) \cdot (a-b)$

$= |a|^2 + |b|^2 - 2a \cdot b$ — (i)

similarly $(b-c)^2 = |b|^2 + |c|^2 - 2b \cdot c$ — (ii)

$(c-a)^2 = |c|^2 + |a|^2 - 2c \cdot a$ — (iii)

Adding (i), (ii), (iii):

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$$|a-b|^2 + |b-c|^2 + |c-a|^2 = 2(|a|^2+|b|^2+|c|^2) - 2(a\cdot b + b\cdot c + c\cdot a)$$
$$= 2(1+1+1) - 2(a\cdot b + b\cdot c + c\cdot a)$$
$$= 6 - 2(a\cdot b + b\cdot c + c\cdot a) \quad \text{— (iv)}$$

And $a\cdot b + b\cdot c + c\cdot a \le |a||b| + |b||c| + |c||a|$
$$\le 3$$

~~$\therefore (a\cdot b + b\cdot c +$~~

consider $|a+b+c|^2 = (a+b+c)\cdot(a+b+c)$
$$= |a|^2 + |b|^2 + |c|^2 + 2(a\cdot b + b\cdot c + c\cdot a)$$

As $|a+b+c|^2 \ge 0$

$$|a|^2 + |b|^2 + |c|^2 + 2(a\cdot b + b\cdot c + c\cdot a) \ge 0$$
$$3 + 2(a\cdot b + b\cdot c + c\cdot a) \ge 0$$
$$\Rightarrow -2(a\cdot b + b\cdot c + c\cdot a) \le 3 \quad \text{— (v)}$$

From (v) and (iv) we have:

$$|a-b|^2 + |b-c|^2 + |c-a|^2 \le 6 + 3$$
$$\le 9$$

$\therefore$ $\boxed{|a-b|^2 + |b-c|^2 + |c-a|^2 \le 9}$

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(31) (a) Given, $A = \{1,2,5\}$ $B = \{3,5\}$ $C = \{2,3,4,5\}$

And, $S = \{1,2,3,4,5,6\}$

(i) $P(A/C)$ & $P(C/A)$

$P(A) = \frac{n(A)}{n(S)}$ $P(C) = \frac{n(C)}{n(S)}$

$= \frac{3}{6} = \frac{1}{2}$ * $= \frac{4}{6} = \frac{2}{3}$ *

$A \cap C = \{2,5\} \Rightarrow P(A \cap C) = \frac{n(A \cap C)}{n(S)}$

$= \frac{2}{6} = \frac{1}{3}$

By K-T:

$\Rightarrow P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{(1/3)}{(2/3)}$

$\therefore P(A/C) = 1/2$

$\Rightarrow P(C/A) = \frac{P(A \cap C)}{P(A)} = \frac{(1/3)}{(1/2)}$

$\therefore P(C/A) = \frac{2}{3}$

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(ii) ~~$A \cap B \cap C = 5$~~ $P(A \cap B/C)$ and $P(A \cup B/C)$:

$A \cap B \cap C = \{5\}$

$\therefore P(A \cap B \cap C) = \frac{1}{6}$

W.K.T, $P(A \cap B/C) = \frac{P(A \cap B \cap C)}{P(C)}$ $\left[\because P(C) = 2/3\right]$

$= \frac{(1/6)}{(2/3)}$

$\therefore P(A \cap B/C) = \frac{1}{4}$

And, $P(A \cup B/C) = P(A/C) + P(B/C) - P(A \cap B/C)$

$B \cap C = \{3, 5\}$

$P(B/C) = \frac{(2/6)}{2/3} = \frac{1}{2}$

$\rightarrow P(A \cup B/C) = (\frac{1}{2}) + (\frac{1}{2}) - \frac{1}{4}$

$\therefore P(A \cup B/C) = \frac{3}{4}$

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SECTION-D

(35) Let cost of one apple be ₹ x
cost of one orange be ₹ y
cost of one banana be ₹ z

Now, given that:

$$4x+3y+2z=60$$
$$2x+4y+6z=90$$
$$6x+2y+3z=70$$

This can also be represented as:

$$\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \quad AX = B$$

where $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$

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To find A^{-1}:

Co-factors of elements of A:

$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$

$A_{11} = (-1)^{1+1}\begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix}$ $A_{13} = (-1)^{1+3}(4-24)$

$= 0$ $= -20$

$A_{12} = (-1)^{1+2}(6-36)$

$= 30$ $A_{22} = (-1)^{2+2}(12-12)$

$A_{21} = (-1)^{2+1}(9-4)$ $= 0$

$= -5$

$A_{23} = (-1)^{2+3}(8-8) = 10$

$A_{31} = (-1)^{3+1}(18-8) = 10$

$A_{32} = (-1)^{3+2}(24-4) = -20$

$A_{33} = (-1)^{3+3}(16-6) = 10$

$\therefore \text{Adj } A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$

$\therefore |A| = 0.4 + 30.3 + (-20).2 = 50$ $\left[\because |A| = A_{11}.a_{11} + A_{12}.a_{12} + A_{13}.a_{13}\right]$

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$$\Rightarrow \vec{A^{-1}} = \frac{1}{|A|} \text{Adj } A$$

$$A^{-1} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Now, $AX = B$

$A^{-1}AX = A^{-1}B$ Left multiplying with A^{-1}

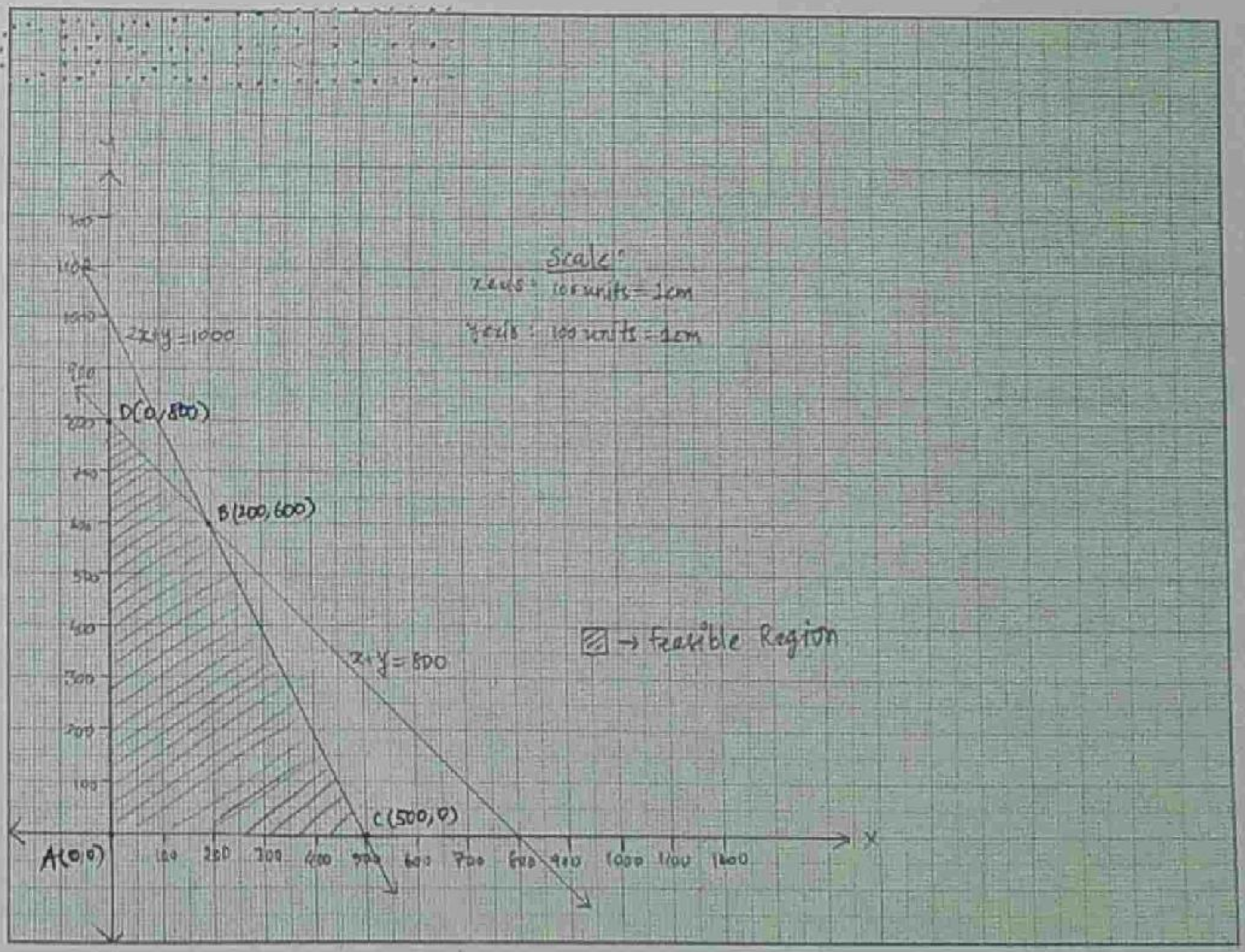
$\Rightarrow X = A^{-1}B$ [$\because AA^{-1} = I$, $XI = X$]

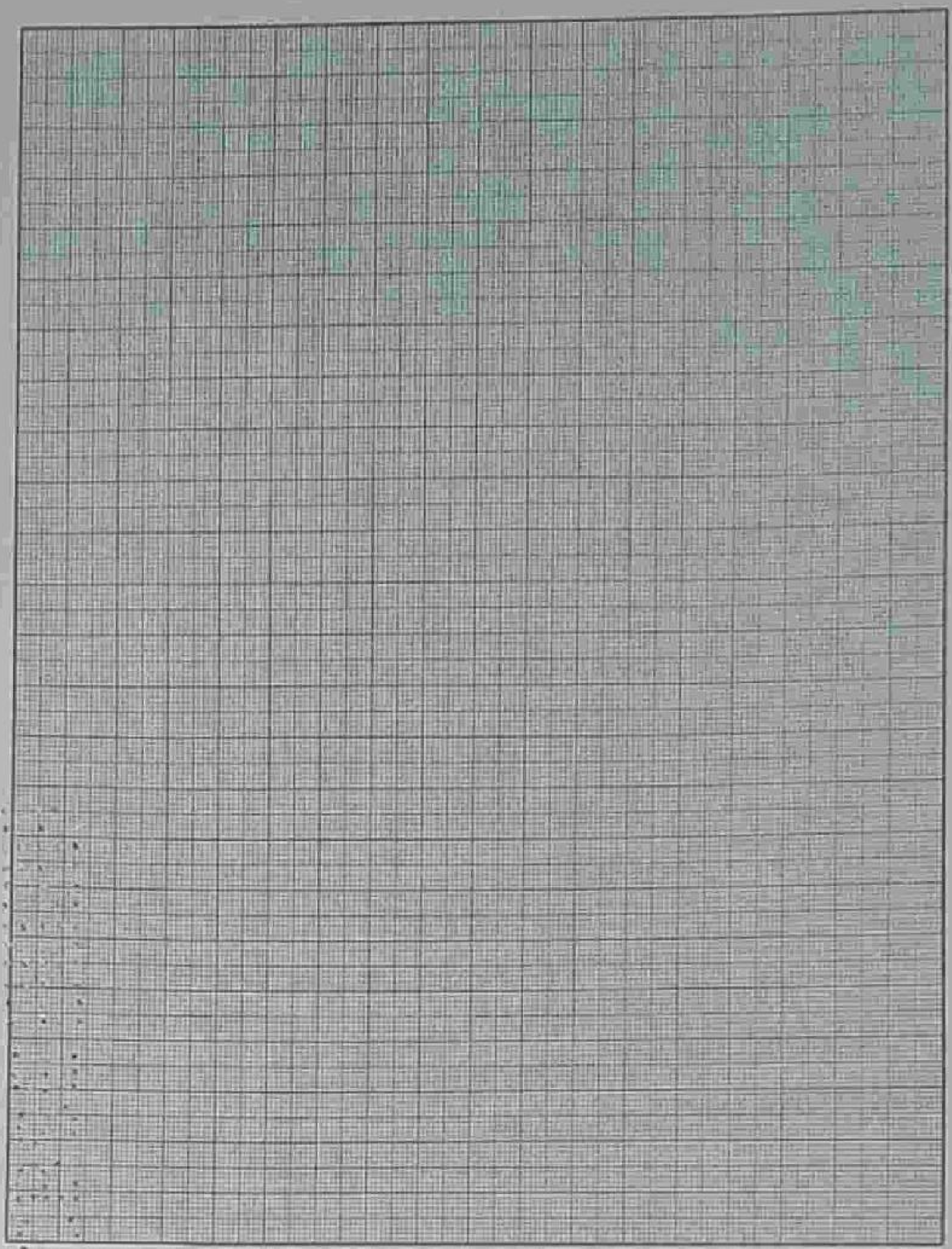
$$= \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$= \frac{1}{50}\begin{bmatrix} 0+(-5)90+(10)(70) \\ (30)60+0+(-20)70 \\ (-20)60+900+700 \end{bmatrix}$$

$$= \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \rightarrow (x,y,z) = (5,8,8)$$

$\therefore$ cost of one apple (x) = 5₹, one orange (y) = 8₹ and one banana (z) = 8₹ //


Scale:
x axis: 100 units = 1cm
y axis: 100 units = 1cm
2x+y=1000
D(0,800)
B(200,600)
x+y=800
C(500,0)
A(0,0)
→ Feasible Region
X
X'
Y'



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Ques

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SECTION - C

(29) Given objective function:

$$Z = \frac{2x}{5} + \frac{3y}{10}$$

Constraints:

$2x + y \leq 1000$

$x + y \leq 800$

$x, y \geq 0$

Origin test:

$2x + y \leq 1000$	$x + y \leq 800$
$0 + 0 \leq 1000$ ✓	$0 + 0 \leq 800$ ✓
∴ Half plane contains (0,0)	∴ Half-Plane contains (0,0)

Intersection points:

$2x + y = 1000$	$2x + y = 1000$	$x + y = 800$
$x + y = 800$ (-)	$x = 0$ & $y = 0$	$x = 0 \Rightarrow y = 800$
$\Rightarrow x = 200$	$\Rightarrow x = 0 \Rightarrow y = 1000$	$y = 0 \Rightarrow x = 800$
$y = 600$	$y = 0 \Rightarrow x = 500$	∴ (0, 800)
∴ (200, 600)	∴ (0, 1000) (500, 0)	(800, 0)

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Corner Points	$Z = \frac{2x}{5} + \frac{3y}{10}$
A (0,0)	$Z = 0$
B (200,600)	$Z = \frac{400}{5} + \frac{1800}{10} = 260$ → Maximum
C (500,0)	$Z = \frac{1000}{5} = 200$
D (0,500)	$Z = 240$

There, Maximum of Z occurs at B(200,600)
and maximum value of Z is 260.
(260)

SECTION - D

(33) (a)

Given, $y\sqrt{x^2+1} = \log\sqrt{x^2+1} - a$

Differentiating w.r.t 'x' on both sides.

$$\frac{d}{dx}(y\sqrt{x^2+1}) = \frac{d}{dx}(\log\sqrt{x^2+1}) - \frac{d}{dx}(a)$$

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$$\Rightarrow y\frac{d}{dx}(\sqrt{x^2+1}) + \sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}}\cdot\frac{d}{dx}(\sqrt{x^2+1}) - 1$$

$$\left[\because \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}\right.$$

$$\text{and } \frac{d}{dx}(\log x) = \frac{1}{x}$$

$$\left.\frac{d}{dx}(f(g(x)) = f'(g(x))\cdot\frac{d}{dx}g(x)\right.$$

$$\left.\text{chain rule}\right]$$

$$\Rightarrow y\cdot\frac{1}{2\sqrt{x^2+1}}\frac{d}{dx}(x^2+1) + \sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}}\cdot\frac{1}{2\sqrt{x^2+1}}\cdot\frac{d}{dx}(x^2+1) - 1$$

$$\frac{y}{2\sqrt{x^2+1}}\cancel{2x} + \sqrt{x^2+1}\left(\frac{dy}{dx}\right) = \frac{1}{2(x^2+1)}\times\cancel{2x} - 1$$

$$\frac{xy}{\sqrt{x^2+1}} + \sqrt{x^2+1}\frac{dy}{dx} = \frac{x}{x^2+1} - 1$$

$$\frac{1}{\sqrt{x^2+1}}\left(xy + (x^2+1)\frac{dy}{dx}\right) = \frac{x}{x^2+1} - 1$$

(35) (a)

Given $y\sqrt{x^2+1} = \log(\sqrt{x^2+1}-x)$

Differentiate w.r.t. x on both sides,

$$\frac{d}{dx}(y\sqrt{x^2+1}) = \frac{d}{dx}(\sqrt{x^2+1}-x)\log(\sqrt{x^2+1}-x))$$

W.K.T $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$ and $\frac{d}{dx}(f(g(x)) = f'(g(x)).\frac{d}{dx}(g(x))$ (chain rule)

$\Rightarrow$ ∴

$$\frac{y}{dx}\frac{d(\sqrt{x^2+1})}{dx} + \sqrt{x^2+1}\left(\frac{dy}{dx}\right) = \frac{1}{\sqrt{x^2+1}-x}\cdot\frac{d}{dx}(\sqrt{x^2+1}-x)$$

$$\Rightarrow \quad y\cdot\frac{1}{2\sqrt{x^2+1}}\cdot\frac{d}{dx}(x^2+1) + \sqrt{x^2+1}\left(\frac{dy}{dx}\right) = \frac{1}{\sqrt{x^2+1}-x}\cdot\left[\frac{1}{2\sqrt{x^2+1}}\frac{d}{dx}(x^2+1) - 1\right]$$

$$= \frac{y(2x)}{2\sqrt{x^2+1}} + \frac{dy}{dx}(\sqrt{x^2+1}) = \frac{1}{\sqrt{x^2+1}-x}\left[\frac{2x}{2\sqrt{x^2+1}} - 1\right]$$

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$$\frac{xy}{\sqrt{x^2+1}} + \sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}-x}\left[\frac{x}{\sqrt{x^2+1}} - 1\right]$$

$$\frac{xy + (x^2+1)(dy/dx)}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}-x}\left[\frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}}\right]$$

$$\Rightarrow \frac{xy + (x^2+1)(dy/dx)}{\sqrt{1+x^2}} = \frac{-1}{\sqrt{x^2+1}}$$

$$\Rightarrow xy + (x^2+1)\frac{dy}{dx} + 1 = 0$$

$$\Rightarrow (x^2+1)\frac{dy}{dx} + xy + 1 = 0 \quad //$$

Hence, proved.

(34) Given curve can be written as:

$100x^2 + 25y^2 = 2500$

$$\Rightarrow \frac{x^2}{25} + \frac{y^2}{100} = 1$$

which is standard equation of an ellipse $[b > a]$

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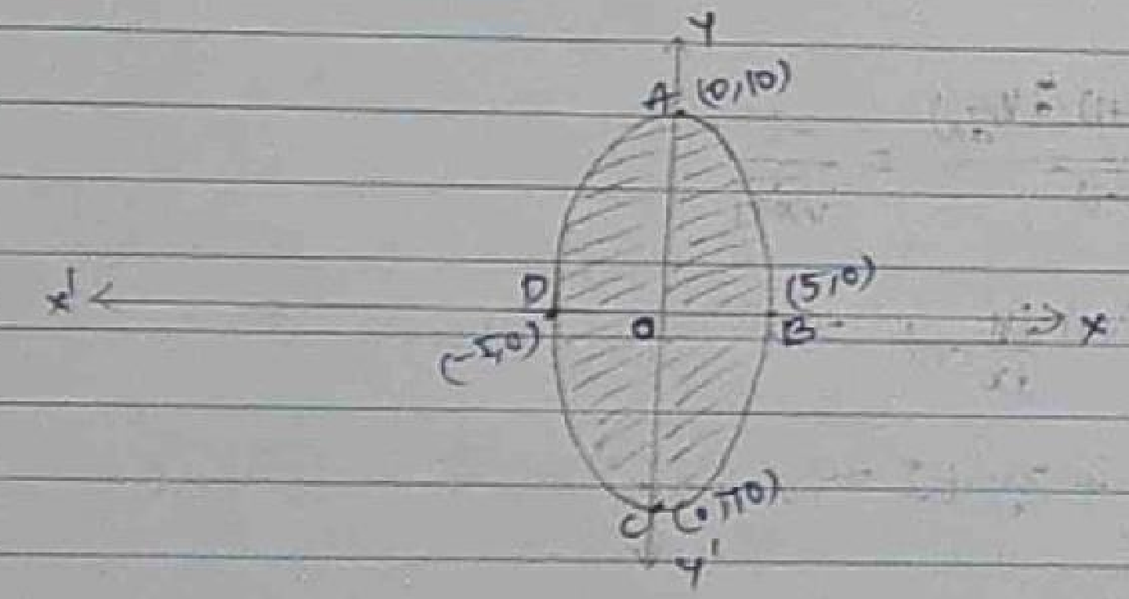
Comparing with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a = 5$ & $b = 10$

∴ The curve would look like:



Clearly curve is symmetrical along x and y axes.

Therefore, $Ar(AOB) = Ar(BOC) = Ar(COD) = Ar(AOD)$

∴ Area of curve = 4 × Area of (AOB)

In Region AOB:

$\frac{x^2}{25} + \frac{y^2}{100} = 1 \Rightarrow 4x^2 + y^2 = 100$

$\boxed{y = +\sqrt{100 - 4x^2}}$

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$$\therefore \text{Area of } AOB = \int_0^5 \sqrt{100-4x^2}\, dx$$

$$= \int_0^5 \sqrt{4\left(\frac{100}{4}-x^2\right)}\, dx$$

$$= 2\int_0^5 \sqrt{25-x^2}\, dx \quad -(i)$$

By ~~in~~ Now consider Integral $I = \int \sqrt{25-x^2}\, dx$

By integration by parts:

$$\left[\because \int uv\, dx = u\int v\, dx - \int \frac{du}{dx}\left(\int v\, dx\right) dx\right]$$

$$I = \int 1 \cdot \sqrt{25-x^2}\, dx$$

$$= \sqrt{25-x^2}\cdot\int 1\, dx - \int \frac{d\left(\sqrt{25-x^2}\right)}{dx}\cdot\int 1\, dx\, dx$$

$$= x\sqrt{25-x^2} - \int \frac{(-x)}{\sqrt{25-x^2}}\cdot x\, dx$$

$$= x\sqrt{25-x^2} - \int \frac{-x^2}{\sqrt{25-x^2}}\, dx$$

$$= x\sqrt{25-x^2} - \left[\int \frac{25-x^2}{\sqrt{25-x^2}} - \frac{25}{\sqrt{25-x^2}}\, dx\right.$$

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$$= x\sqrt{25-x^2} - \int \sqrt{25-x^2}\,dx + \int \frac{25}{\sqrt{25-x^2}}\,dx$$

$$I = x\sqrt{25-x^2} - I + 25\int \frac{1}{\sqrt{25-x^2}}\,dx$$

$$2I = x\sqrt{25-x^2} + 25\sin^{-1}\left(\frac{x}{5}\right)$$

$$\Rightarrow I = \frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)$$

Now coming back to (i)

$$\text{Area of } AOB = 2\int_0^5 \sqrt{25-x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$$

$$= 2\left[\frac{5}{2}(0) + \frac{25}{2}\left(\frac{\pi}{2}\right) - (0+0)\right]$$

$$= 2 \cdot \frac{25\pi}{4} = \frac{25\pi}{2}$$

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$\therefore$ Area of the curve (ellipse) $= 4 \times (25\pi)$

$\therefore$ Area of ellipse $= 50\pi$ sq. units

Check:

Area of Ellipse
$= \pi ab$
$= \pi \times 5 \times 10$
$= 50\pi$ ✓

(35) (a)

Let point (0,2,3) be point A.

And foot of perpendicular on L be point B

Line:

$$\frac{-x+3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$$

$$\Rightarrow \frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$$

In vector form:

$$\bar{r} = -3\hat{i} + \hat{j} - 4\hat{k} + \lambda(5\hat{i} + 2\hat{j} + 3\hat{k}) \quad \Rightarrow \bar{b} = 5\hat{i} + 2\hat{j} + 3\hat{k}$$

So, Point B will be of form:

$$(-3+5\lambda, 1+2\lambda, -4+3\lambda)$$

As B is foot of perpendicular, $\Rightarrow \overline{AB} \perp^{ar}$ Line ($\bar{b}$)

$\therefore \overline{AB} \cdot \bar{b} = 0$

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$\overline{AB} = (-3+5\lambda-0)\hat{i} + (1+2\lambda-2)\hat{j} + (-4+3\lambda-3)\hat{k}$

$= (-3+5\lambda)\hat{i} + (2\lambda-1)\hat{j} + (3\lambda-7)\hat{k}$

Now $\overline{AB} \cdot \vec{5} = 0$

$\Rightarrow (-3+5\lambda)\hat{i} + (2\lambda-1)\hat{j} + (3\lambda-7)\hat{k}) \cdot (5\hat{i}+2\hat{j}+3\hat{k}) = 0$

$-15+25\lambda+4\lambda-2+9\lambda-21 = 0$

$38\lambda = 38$

$\boxed{\lambda = 1}$

$\therefore$ Point B $= (-3+5(1), 1+2(1), -4+3(1))$

$= (2, 3, -1)$

$\therefore$ Foot of perpendicular of $(0, 2, 3)$ is $(2, 3, -1)$

Length of perpendicular $|AB| = \sqrt{(2-0)^2 + (3-2)^2 + (-1-3)^2}$

$= \sqrt{4+1+16}$ $\left[AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}\right.$

$\therefore$ **Length of perpendicular $= \sqrt{21}$ units**

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SECTION-E

(36)

(i) $2^{3\times4}$ [No. of relations $= 2^{mn}$]

$= 2^{12}$

$= 4096$

(ii) No, It is not an injective function.

G_2 and G_1 are matched to same element B_2

Firstly it is not a function as b_1 has 2 images $\rightarrow B_1 + B_2$

(iii) (a) Given R is a relation from $A \rightarrow A$ given by:

$R = \{(x,y),\ x,y \in A,\ x \text{ and } y \text{ are students from same colony in the city}\}$

To check Reflexive:

Let $x \in A$,

Then x and x study in same colony of the city obviously.

$\therefore (x,x) \in R$.

$\therefore \forall x \in A \Rightarrow (x,x) \in R$

$\therefore$ R is a Reflexive Relation.

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To check symmetric:

Let $x, y \in A$ such that

$(x, y) \in R$

i.e. x and y study in same colony (from same colony)

Therefore y and x also study in same colony. $(y, x) \in R$

$\Rightarrow \forall (x, y) \in R \Leftrightarrow (y, x) \in R$ where $x, y \in A$ (from same colony)

$\therefore$ R is a symmetric Relation.

To check Transitive:

Let $x, y, z \in A$ such that

$(x, y) \in R$ & $(y, z) \in R$ (from same colony)

i.e x and y study in same colony

& y and z study in same colony (from same colony)

Thus, x and z also study in same colony (from same colony)

$\therefore (x, z) \in R$

$\rightarrow \therefore (x, y) \in R$ and $(y, z) \in R \Leftrightarrow (x, z) \in R$ where $x, y, z \in A$

Therefore R is a transitive Relation.

Hence, R is symmetric, Reflexive and Transitive.

$\Rightarrow$ Therefore R is an equivalence Relation.

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(57)

(i) 45%

$= \frac{45}{100} = 0.45$

$= \frac{9}{20}$

∴ Probability that person taking vaccine A_2 will get infected $= 1 - 0.55$

$= 0.45$

$= \frac{9}{20}$

(ii) By Total Probability Theorem:

Probability that he/she will be protected:

$P(A) = P(A_1)P(A/A_1) + P(A_2) \cdot P(A/A_2) + P(A_3) \cdot P(A/A_3)$

$= \left(\frac{1}{4}\right)\left(\frac{6}{10}\right) + \left(\frac{7}{20}\right)\left(\frac{11}{20}\right) + \left(\frac{4}{10}\right)\left(\frac{1}{2}\right)$

$= \frac{60 + 77 + 80}{400}$

$P(A) = \frac{217}{400}$

A → event that person is protected

A_1, A_2, A_3 → events that person takes vaccines A_1, A_2, A_3 respectively.

(iii) (b) Let A be the event the person is not infected.

A_1, A_2, A_3 be the events he is given vaccines A_1, A_2, A_3 respectively.

$P(A/A_1) = \frac{60}{100} = \frac{12}{20}$ $\quad P(A_1) = \frac{15}{100} = \frac{5}{20}$

$P(A/A_2) = \frac{55}{100} = \frac{11}{20}$ $\quad P(A_2) = \frac{35}{100} = \frac{7}{20}$

$P(A/A_3) = \frac{50}{100} = \frac{10}{20}$ $\quad P(A_3) = \frac{40}{100} = \frac{8}{20}$

By Baye's Theorem,

$$P(A_3/A) = \frac{P(A_3)\,P(A/A_3)}{P(A_1)\,P(A/A_1) + P(A_2)\,P(A/A_2) + P(A_3)\,P(A/A_3)}$$

$$= \frac{\frac{8}{20} \times \frac{10}{20}}{\frac{12}{20} \times \frac{5}{20} + \frac{11}{20} \times \frac{7}{20} + \frac{10}{20} \times \frac{8}{20}}$$

$$P(A_3/A) = \frac{80}{217}$$

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∴ The Probabability that the Person was ~~Infected~~ given vaccine A_3 given that he is not infected is $\frac{80}{217}$

(38)

Give volume = v height = h and radius = r

Let surface Area = K (constant)

Therefore $\pi r^2 + 2\pi rh = K$

$\Rightarrow h = \frac{K - \pi r^2}{2\pi r}$

Now volume $= \pi r^2 h$

$= \pi r^2 \left(\frac{K - \pi r^2}{2\pi r}\right)$

$V = \frac{rK}{2} - \frac{\pi r^3}{2}$

$\boxed{\frac{dV}{dr} = \frac{K}{2} - \frac{3\pi r^2}{2}}$ differentiating w.r.t 'r'.

$= \frac{\pi r^2 + 2\pi rh}{2} - \frac{3\pi r^2}{2} = \frac{e\pi rh - 2\pi r^2}{2}$

$\boxed{\frac{dV}{dr} = \pi rh - \pi r^2}$

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(ii) To maximize volume,

$$\frac{dV}{dr} = 0$$

$$\Rightarrow \pi r h - \pi r^2 = 0$$

$$\Rightarrow rh = r^2$$

$$\boxed{r = h}$$

$$\frac{dV}{dr} = \frac{K}{2} - \frac{3\pi r^2}{2}$$

$$\frac{d^2V}{dr^2} = -\frac{6\pi r}{2}$$

$$= -3\pi r$$

$$< 0 \quad (\text{As } r > 0)$$

$$\frac{d^2V}{dr^2} < 0$$

$\therefore$ At $r = h$, volume of container ~~with~~ will be maximum.

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