MATHEMATIC BASIC - 241

Section - A :

1) A) 7

2) B) intersecting at (2,3)

3) A) 1

4) C) -4

5) B) a^2

6) D) 147 π sq. cm

7) C) 1

8) C) no real roots

9) B) $\sqrt{80}$

$a = 2^5 \times 3^3$

$b = 2^3 \times 3^2 \times 5$

$c = 3^3 \times 5^n$

$LCM = 2^5 \times 3^3 \times 5^7$

$a - 9x = 0$

$d = a_2 - a_1$

$= -2 - 2$ $\quad -10+6$

$= -4$ $\quad -4$

$a^2 \sin^2\theta + a^2 \cos^2\theta$

$a^2 (\sin^2\theta + \cos^2\theta)$

a^2

$3\pi r^2$

$3 \times \pi \times 49$

$b^2 - 4ac$

$= 5 - 4(2)(1)$

$= 5 - 8$

$= -3$

$\sqrt{5}, 2\sqrt{5}, 3\sqrt{5}$

10) C) $\frac{4}{13}$

11) C) $x = \frac{ay}{a+b}$

12) B) $\frac{1}{2\sqrt{2}}$

13) B) $\frac{1}{2}$

14) A) $\sqrt{8} \times \sqrt{32}$

15) B) $10\sqrt{2}$

16) C) 30

S S
12 17
20 37
8 45
5 50

25
20-30 = MC
30

$\frac{a}{a+b} = \frac{x}{y}$

$\frac{ay}{a+b} = x$

$\cos\theta = \frac{1}{3}$

$\sin^2\theta = 1 - \frac{1}{9}$

$= \frac{8}{9}$

$\sin\theta = \frac{2\sqrt{2}}{3}$

$\cot\theta = \frac{1}{3} \times \frac{3}{2\sqrt{2}}$

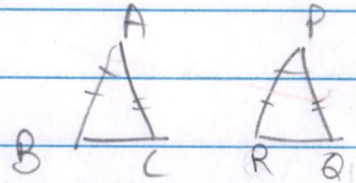
$\frac{2k^2}{k} = 1$

$k = \frac{1}{2}$

$2\sqrt{2} \times 4\sqrt{2}$

$100 + 100 = AB^2$
$200 = AB^2$
$10\sqrt{2} = AB$

17) A) $\angle A = \angle P$



18) D) 145°

19) A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

20) C) Assertion (A) is true, but Reason (R) is false.

Section-B :

21) a) A(a, b) is equidistant from B(2, 4) and C(-2, -3)

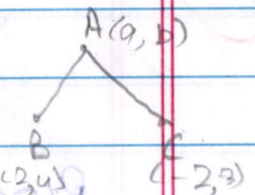
$\therefore AB = AC$

Squaring both sides,

$\Rightarrow AB^2 = AC^2$

$\Rightarrow (a-2)^2 + (b-4)^2 = (a+2)^2 + (b+3)^2$

[Applying distance formula]



$\Rightarrow a^2 + 4 - 4a + b^2 + 16 - 8b = a^2 + 4 + 4a + b^2 + 9 + 6a + 6b$

$[(a+b)^2 = a^2 + b^2 + 2ab]$ $[(a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow 16 - 4a - 8b = 4a + 9 + 6b$

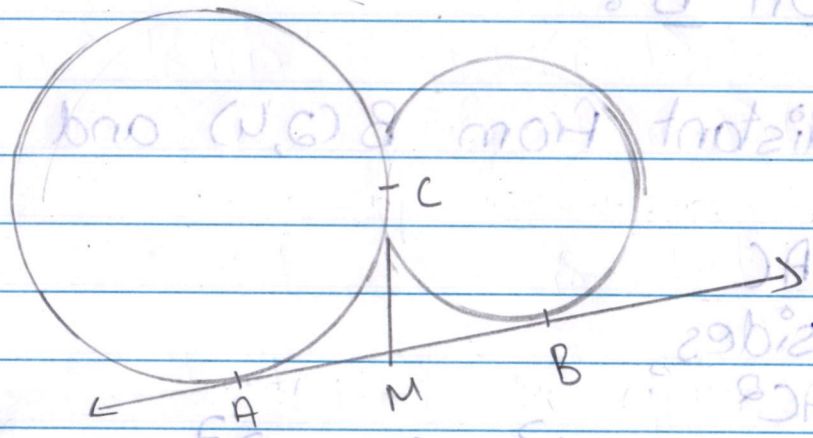
$\Rightarrow 16 - 9 = 4a + 4a + 6b + 8b$

$\Rightarrow \boxed{7 = 8a + 14b}$

$\therefore 7 = 8a + 14b$ is the required relation

Space for writing Question Number

22)



Given: $\overleftrightarrow{AB}$ is a common tangent and CM

To prove: AM = MB

Proof:

We know that,

① – AM = MC [Tangents drawn from same external point]

② and MC = MB (Theorem 10.2)

From ① & ②,

AM = MB = MC

∴ AM = MB

∴ Proved

M is mid-point of AB.

23) b) $a_5 = 3$

$\Rightarrow a + 4d = 3$ $[a_n = a + (n-1)d]$ - ①

$a_{12} = 17$

$\Rightarrow a + 11d = 17$ $[a_n = a + (n-1)d]$ - ②

Subtracting ② & ①,

$a + 11d - a - 4d = 17 - 3$

$\Rightarrow 7d = 14$

$\Rightarrow d = 2$

$a + 4d = 3$ (①)

$\Rightarrow a = 3 - 4d$

$\Rightarrow a = 3 - 8$

$\Rightarrow a = -5$

n^{th} term $= a + (n-1)d$

$= -5 + (n-1)(2)$

$= -5 + 2n - 2$

$= \boxed{2n - 7}$

$\therefore$ n^{th} term of given AP is $2n-7$

24

Class	f_i	x_i	u_i	$f_i u_i$
6-10	4	8	-2	-8
10-14	6	12	-1	-6
14-18	k	[16]	0	0
18-22	5	20	1	5
22-26	3	24	2	6
	$\Sigma f_i = 18+k$			$\Sigma f_i u_i = -3$

$(a = 16)$

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$15.6 = 16 + \frac{-3}{18+k} \times 4$$

$$\Rightarrow 15.6 = 16 - \frac{12}{18+k}$$

$$\Rightarrow -0.4 = -\frac{12}{18+k}$$

$$\Rightarrow -7.2 - 0.4k = -12$$

$$\Rightarrow -0.4k = -4.8 - 4.8$$

$$\Rightarrow k = -4.8 + 4.8 \; 12$$

$$+0.4$$

$$\boxed{k = 12}$$

$\therefore$ Value of k is 12.

Space for writing Question Number

25) $\cos\theta = \frac{4}{5}$ (given)

$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$

$\because \tan^2\theta = \sec^2\theta - 1 \quad [\sec^2\theta - \tan^2\theta = 1]$

$\Rightarrow \tan^2\theta = \frac{25}{16} - 1$

$\Rightarrow \tan^2\theta = \frac{9}{16}$

$\Rightarrow \tan\theta = \frac{3}{4}$ [taking square root]
(side length is always positive]

$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = \frac{\frac{5}{4} - \frac{3}{4}}{\frac{5}{4} + \frac{3}{4}}$

$= \frac{2}{4} \times \frac{4}{8}$

$= \boxed{\frac{1}{4}}$

Space for writing Question Number

Section - C :

26) b) To prove : $\sqrt{5}$ is an irrational number.

Proof :

Let $\sqrt{5}$ be a rational number

$\therefore \sqrt{5} = \frac{a}{b}$ where a, b are co-prime integers and $b \neq 0$.

Squaring both sides,

$\Rightarrow 5 = \frac{a^2}{b^2}$

$\Rightarrow 5b^2 = a^2$ — ①

$\Rightarrow b^2 = \frac{a^2}{5}$

5 divides a^2

$\therefore$ 5 divides a [Theorem 1.2]

Let $a = 5c$ for some value c.

Putting $a = 5c$ in (1),

$\Rightarrow 5b^2 = (5c)^2$

$\Rightarrow 5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

$\Rightarrow \frac{b^2}{5} = c^2$

5 divides b^2

$\therefore$ 5 divides b [Theorem 1.2]

This shows that 5 is a common factor of both a and b.
But it contradicts the fact that a and b are co-prime.
This contradiction has arisen due to our incorrect assumption that $\sqrt{5}$ is rational
$\therefore \sqrt{5}$ is an irrational number.

∴ Proved.

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27) b) As per question,

units digit = $x-2$
[Let tens digit be x]

Number = $10x + x - 2$
= $11x - 2$

Number by reversing digits = $10(x-2) + x$
= $10x - 20 + x$
= $11x - 20$

Their sum = 154
∴ $11x - 2 + 11x - 20 = 154$
⇒ $22x - 22 = 154$
⇒ $2x - 2 = 14$
⇒ $x - 1 = 7$

$\Rightarrow x = 8$

$\therefore$ Number $= 11x - 2$

$= 11(8) - 2$

$= 88 - 2$

$= \boxed{86}$

$\therefore$ The number is 86.

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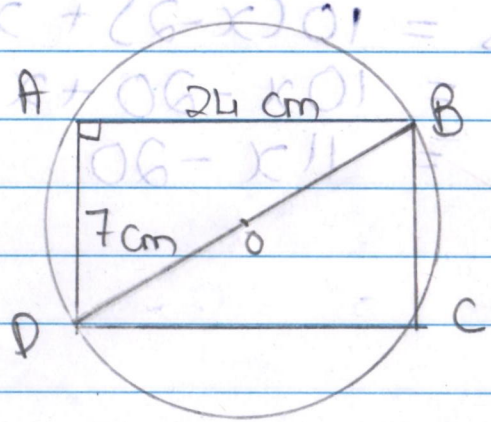
28)



Given: $\square ABCD$ is a rectangle inscribed in circle with centre O.

$AB = 24$ cm

$AD = 7$ cm

To find: radius of circle (OD) = ?

Construction: Join BD

Place for writing Question Number

Solution:

We know that,

$\angle A = \angle B = \angle C = \angle D = 90^\circ$ [angles in rectangle]

$\therefore$ In ΔABD,

~~$(AB)^2 + (BD)^2 = (AD)^2$~~

$(AB)^2 + (AD)^2 = (BD)^2$ [Pythagorus theorem]

$\Rightarrow (24)^2 + (7)^2 = BD^2$

$\Rightarrow 576 + 49 = BD^2$

$\Rightarrow 625\ cm^2 = BD^2$

Taking square root,

$\Rightarrow 25\ cm = BD$ [-25 $\rightarrow$ not possible as side cannot be negative]

Also, diagonal of rectangle = diameter of circle [as it is inscribed]

$\therefore BD = d$

$\therefore 25\ cm = d$

$\Rightarrow r = \frac{d}{2}$

$\therefore r = \frac{25}{2} = \boxed{12.5 \text{ cm}}$

$\therefore$ Radius of circle is 12.5 cm.

29) A(−7, 5) B(8, −3)

i) Length of AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
[Distance formula]

$= \sqrt{(8+7)^2 + (-3-5)^2}$
$= \sqrt{(15)^2 + (-8)^2}$
$= \sqrt{225 + 64}$
$= \sqrt{289}$
$= \boxed{17 \text{ units}}$

ii) $\frac{m_1}{m_2} = \frac{3}{2}$

Applying section formula,

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$= \frac{(3)(8) + (2)(-7)}{3+2}$$

$$= \frac{24 + -14}{5}$$

$$= \frac{10}{5}$$

$$x = 2$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$= \frac{(3)(-3) + (2)(5)}{3+2}$$

$$= \frac{-9+10}{5}$$

$$y = \frac{1}{5}$$

∴ Co-ordinates of point $P(2, \frac{1}{5})$ and length of AB is 17 units.

30) $\sin(A-B) = \frac{1}{2}$

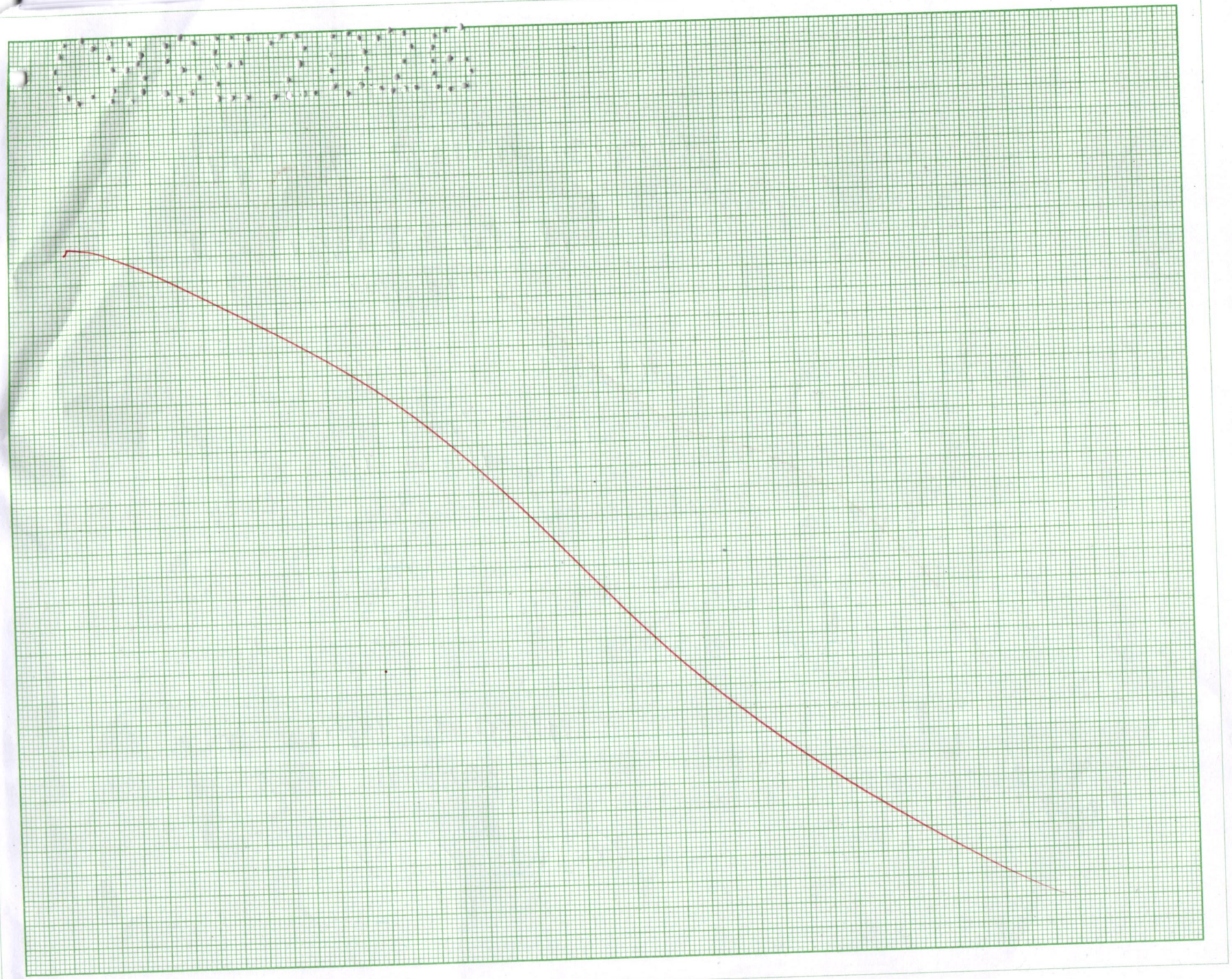
$\cos(A+B) = \frac{1}{2}$ {given}

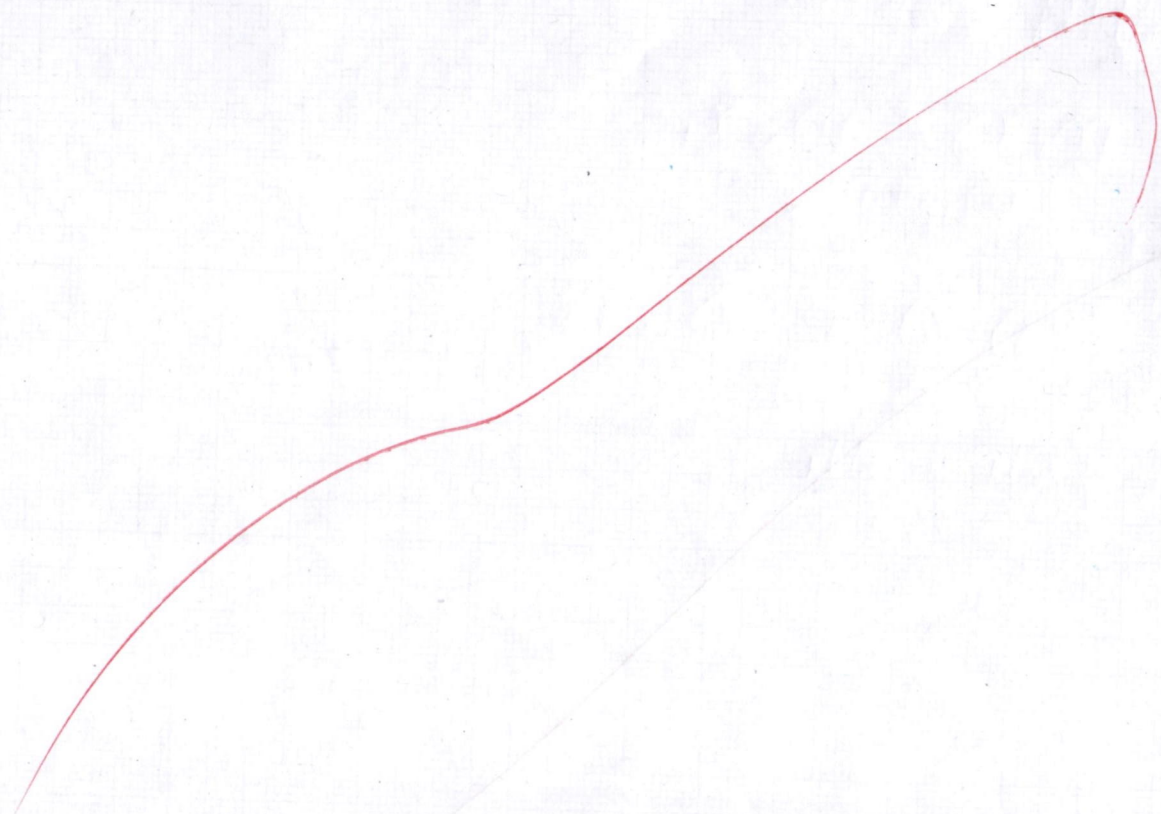
We know that,

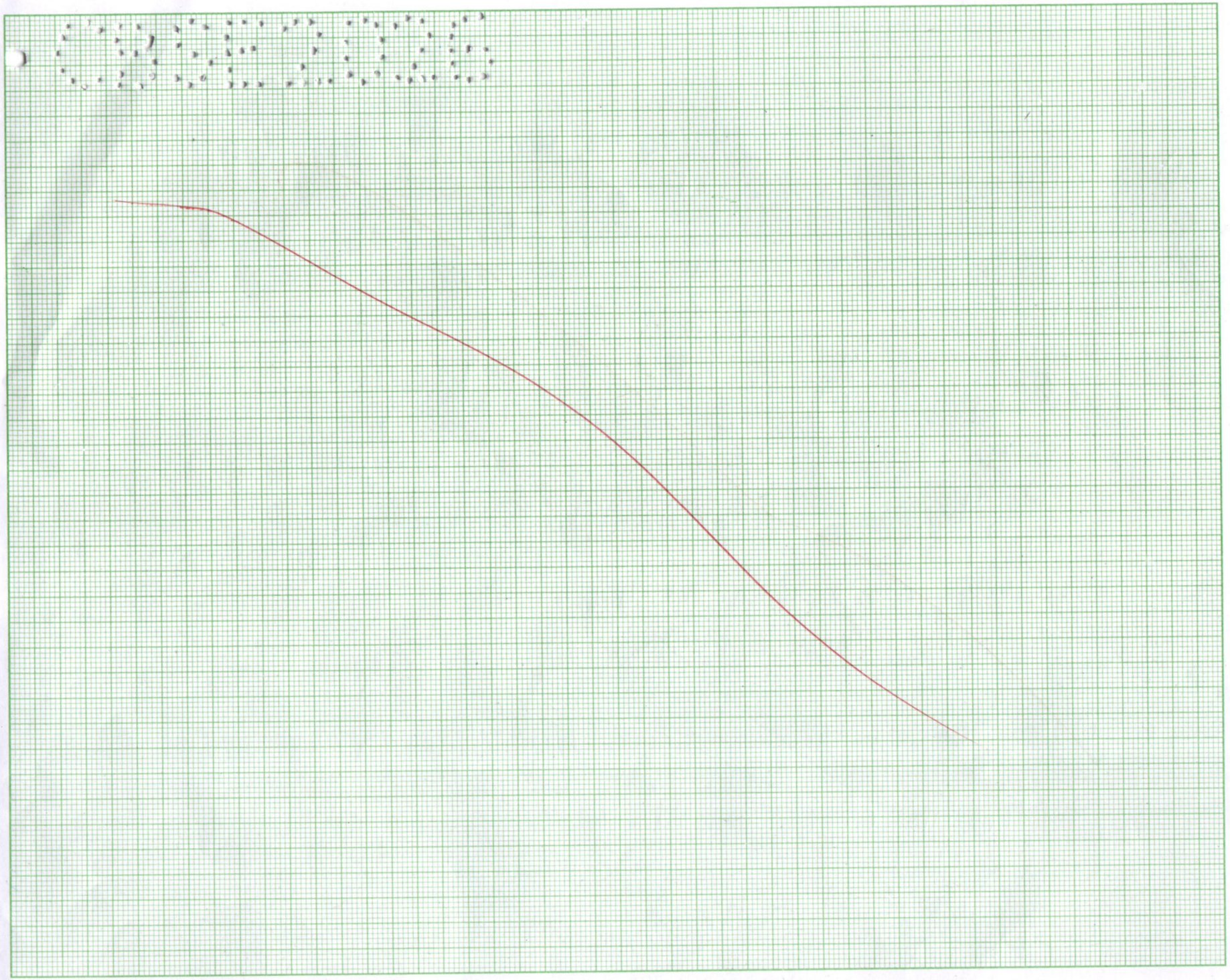
$\sin 30° = \frac{1}{2}$ and $\cos 60° = \frac{1}{2}$

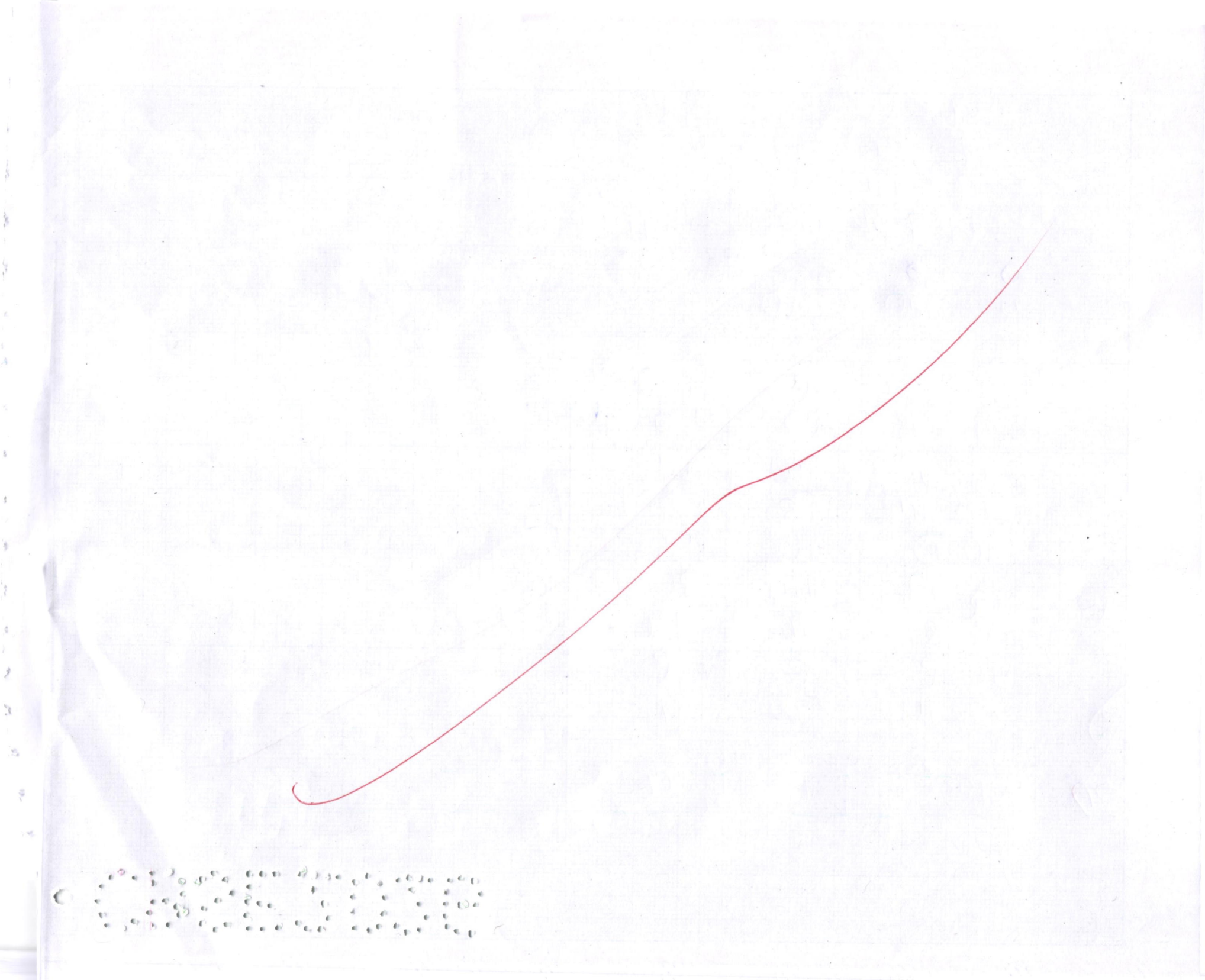
∴ By comparing,

$A-B = \frac{1}{2}\ 30° \quad -①$









and $A+B = 60^\circ$ - (2)

Adding (1) & (2),

$A - B + A + B = 30^\circ + 60^\circ$

$\Rightarrow 2A = 90^\circ$

$\Rightarrow \boxed{1A = 45^\circ}$

$A + B = 60^\circ$ ((2))

$\therefore \boxed{B} = 60^\circ - A = 60 - 45^\circ = \boxed{15^\circ}$

$\therefore \sin A = \sin 45^\circ = \boxed{\frac{1}{\sqrt{2}}}$

$\therefore$ Answer : Value of A and B is 45° and 15°, $\sin A = \frac{1}{\sqrt{2}}$.

31)

Class	f	cf
2-4	1	1
4-6	5	6
6-8	11	17
8-10	14	31
10-12	16	47
12-14	13	60
14-16	14	74
16-18	60	134
18-20	6	140

$\frac{n}{2} = \frac{140}{2} = 70$

∴ Median class = 14-16

$l = 14$

$f = 14$ $\quad h = 2$

$cf = 60$

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$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 14 + \frac{70 - 60}{47} \times 2$$

$$= 14 + \frac{10}{7}$$

$$= 14 + 1.428$$

$$= \boxed{15.428 \text{ students}} \text{ (approx)}$$

Modal class → 16-18

$l = 16$

$f_1 = 60$ $\quad h = 2$

$f_0 = 14$

$f_2 = 6$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 16 + \frac{60-14}{120-14-6} \times 2$$

$$= 16 + \frac{46}{100} \times 2$$

$$= 16 + \frac{92}{100}$$

$$= 16 + 0.92$$

$$= \boxed{16.92 \text{ students}}$$

∴ Median and mode of given data are 15.428 and 16.92 students respectively.

Section-D:

32) b) $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$

$\Rightarrow \frac{x+5-(x-3)}{x^2+5x-3x-15} = \frac{1}{6}$

$\Rightarrow \frac{x+5-x+3}{x^2+2x-15} = \frac{1}{6}$

$\Rightarrow \frac{8}{x^2+2x-15} = \frac{1}{6}$

$\Rightarrow 48 = x^2+2x-15$

$\Rightarrow 0 = 1x^2+2x-63$

↳ Standard form $[ax^2+bx+c=0]$

$\Rightarrow 0 = x^2+9x-7x-63$

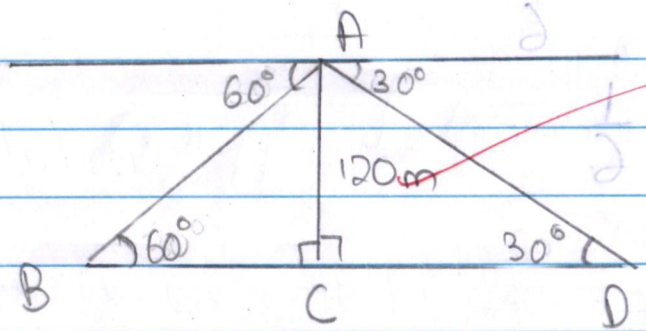
$\Rightarrow 0 = x(x+9)-7(x+9)$

$\Rightarrow 0 = (x-7)(x+9)$

$\therefore$ $\boxed{x = 7}$ or $\boxed{x = -9}$
(by comparing)

$\therefore$ Roots of the equation are 7, -9 and standard form is $x^2 + 2x - 63 = 0$

33)



AC → height of building (120 m)

BD → Distance between the two persons

In $\triangle ACD$,

$\tan 30° = \frac{AC}{CD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{120}{CD}$

$\Rightarrow CD = 120\sqrt{3}$ m

In ΔACB,

$\tan 60^\circ = \frac{AC}{BC}$

$\Rightarrow \sqrt{3} = \frac{120}{BC}$

$\Rightarrow BC = \frac{120}{\sqrt{3}}$ m

$= \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ [Rationalising]

$= \frac{120\sqrt{3}}{3}$

$= 40\sqrt{3}$ m

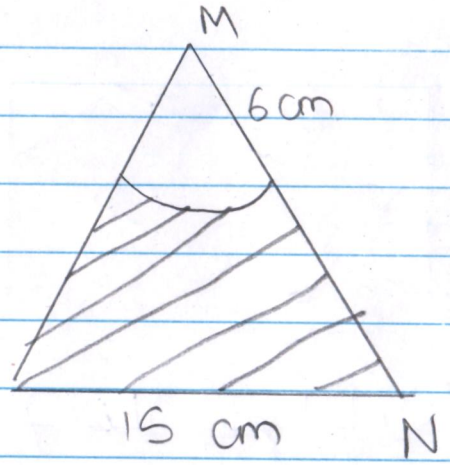
$BD = BC + CD$ [from figure]

$\Rightarrow BD = 40\sqrt{3} + 120\sqrt{3}$ m

$\Rightarrow \boxed{BD = 160\sqrt{3} \text{ m}}$

∴ Distance between the 2 persons is $160\sqrt{3}$ m.

34)



Given : ΔMPN is equilateral with side 15 cm

$r = 6$ cm, $\sqrt{3} = 1.732$

To find: Area of shaded region

Solution :

Area of $\Delta MPN = \frac{\sqrt{3}}{4}(a)^2$ [it is equilateral]

$= \frac{\sqrt{3}}{4} \times 225$

$= \frac{225\sqrt{3}}{4}$ cm^2 $= \frac{389.7}{4}$ cm^2

Area of sector drawn from $M = \frac{\theta}{360} \times \pi r^2$

$\theta = 60^\circ$ [all angles are 60° in an equilateral triangle]

$\therefore$ area of sector $= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6$

$= \frac{132}{7}$

$= 18.85 \text{ cm}^2$ (approx.)

Area of shaded region = Area of triangle − Area of sector

$= 97.425 - 18.85 \text{ cm}^2$

$= 178.575 \text{ cm}^2$

$\therefore$ Area of shaded region is 78.575 cm^2.

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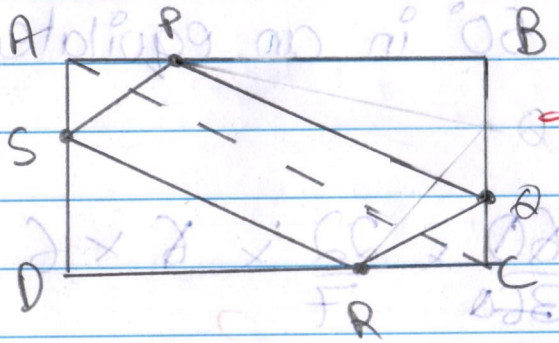
35) a)



Given: P, Q, R, S are trisection points of AB, BC, CD and DA

To prove: □PQRS is a parallelogram

Construction: Join AC.

Proof:

$\frac{AP}{PB} = \frac{CQ}{BQ} = \frac{1}{2}$ [given]

$\therefore PQ \parallel AC$ [converse BPT] — ①

Also, $\frac{AS}{DS} = \frac{CR}{DR} = \frac{1}{2}$ [given]

$\therefore SR \parallel AC$ [converse BPT] — ②

From ① & ②,

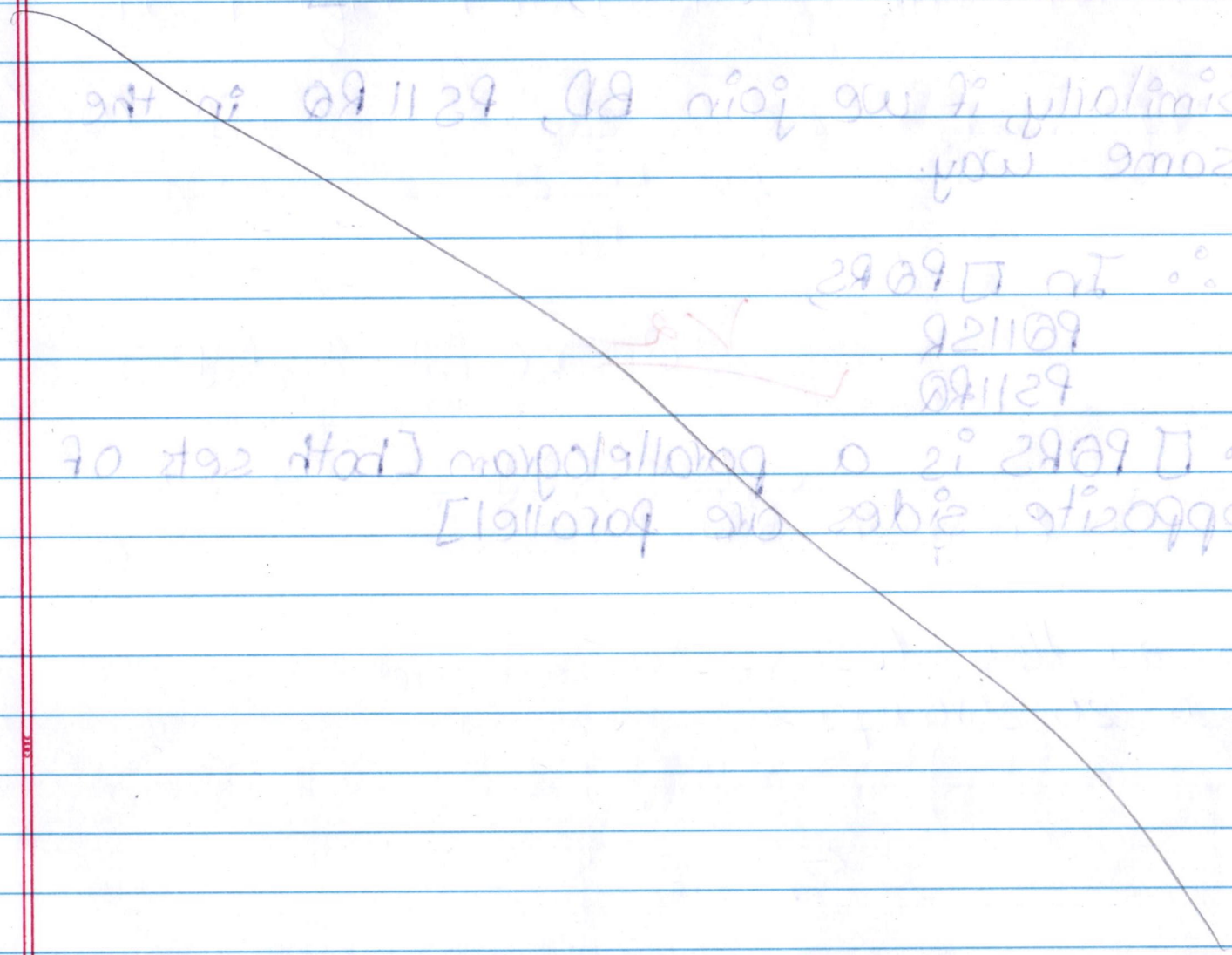
$PQ \parallel AC \parallel SR$

∴ PQ ∥ SR [$\ell \parallel m \parallel n$, then $\ell \parallel n$]

Similarly, if we join BD, PS ∥ RQ in the same way.

∴ In □PQRS,
PQ ∥ SR
PS ∥ RQ

∴ □PQRS is a parallelogram [both sets of opposite sides are parallel]



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Section-E:

36) i) $p(x) = -x^2 + 6x - 5$

$\Rightarrow -x^2 + 6x - 5 = 0$

$\Rightarrow -x^2 + 5x + 1x - 5 = 0$

$\Rightarrow -1x(x-5) + 1(x-5) = 0$

$\Rightarrow (1-x)(x-5) = 0$

$\therefore \boxed{x = 1}$ or $\boxed{x = 5}$ [By comparing]

Zeroes of $p(x)$ are 1, 5.

ii) given: $x = 2m$

$p(2) = ?$

$p(2) = -(2)^2 + 6(2) - 5$

$= -4 + 12 - 5$

$= 12 - 9$

$= \boxed{3}$ m

The height of fountain is 3m when $x = 2$.

iii) a) $q(x) = -p(x)$

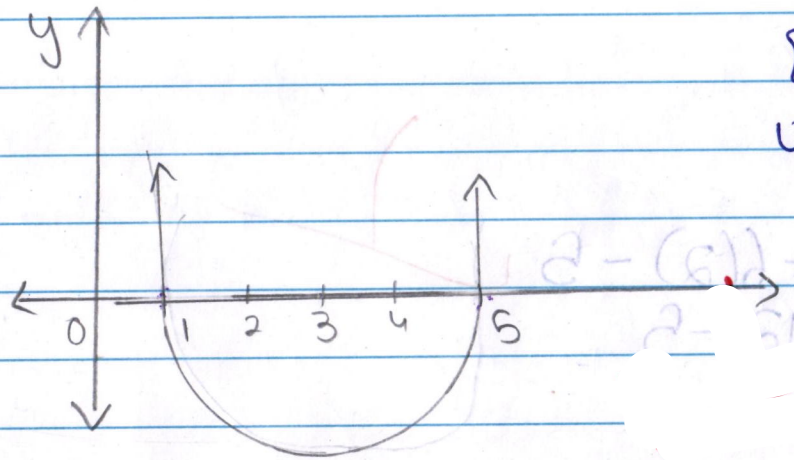
$\therefore q(x) = -[-x^2+6x-5]$

$= \boxed{+x^2-6x+5}$

$\Rightarrow q(x) = x^2-6x+5$

zeroes of $q(x) = 1, 5$

[same as $p(x)$]



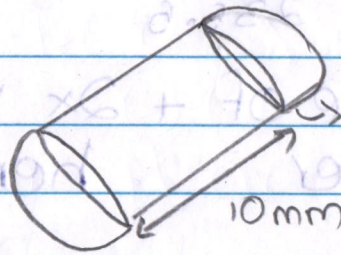
[It will be open upwards as $a > 0$]

Or

b) When water hits the ground, $p(x) = 0$

∴ At $x = 1m$ and $x = 5m$ from origin (horizontal distance), water hits the ground.

37)



$h = 10mm$ (cylinder)

$d = 5mm$

$\therefore r = \frac{5}{2} mm$

i) TSA of capsule = CSA of cylinder + 2 × CSA of hemisphere

$= 2\pi rh + 4\pi r^2$

$= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times \frac{5}{2} (10 + 5)$

$= \frac{22}{7} \times 5 \times 15$

$= \frac{1650}{7}$ mm²

$= 235.71$ mm² | 235.5 mm² (approx.)

TSA of one capsule is $\frac{235.71}{235.5}$ mm².

ii) Capacity of capsule = Volume of cylinder + 2× Volume of hemisphere

$= \pi r^2 h + \frac{4}{3}\pi r^3$

$= \pi r^2 \left(h + \frac{4}{3}r\right)$

$= \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \left(10 + \frac{4}{3} \times \frac{5}{2}\right)$

$= \frac{22}{7} \times \frac{25}{4} \times \frac{40}{3}\ 10$

$= \frac{5500}{21}$ mm³

Space for writing Question Number

$= \boxed{261.9 \text{ mm}^3}$ (approx)

Volume of capsule is 261.9 mm^3

iii) a) Cost of medecine = rate per mm^3 × volume

$= 0.05 \times 261.9$

$= \frac{5}{100} \times \frac{2619}{10}$

$= \frac{13095}{1000}$

$= \boxed{\$13.095}$

∴ It would cost $13.095 per capsule.

38) i) Total number of outcomes are $\boxed{100}$.

ii) Probability = $\frac{\text{No. of outcomes favourable to E}}{\text{Total number of outcomes}}$

Chits that have only red marking = 30 - 15 = 15.

∴ P(getting chit with only red) = $\frac{15}{100} = \boxed{\frac{3}{20}}$

iii) a) Chits that have multiple of 5 but no red marking = 20 - 15 = 5 (favourable outcomes)

Probability = $\frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

∴ P(getting chit with number but no red) = $\frac{5}{100} = \boxed{\frac{1}{20}}$