Class-XII

Mathematics(041)

SECTION - A

Question : 1

$\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

$\vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

projection of $(\vec{b}+\vec{c})$ on $\vec{a}$ $= \dfrac{\vec{a}\cdot(\vec{b}+\vec{c})}{|\vec{a}|}$

$$= \frac{(2\hat{i} - 2\hat{j} + \hat{k}) \cdot (3\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{2^2 + (-2)^2 + 1^2}}$$

$$= \frac{(2\times 3) + (-2\times 1) + (1\times 2)}{\sqrt{9}}$$

$$= \frac{6 - 2 + 2}{\sqrt{9}}$$

$= \frac{6}{3}$

$= 2$ units

Answer: Projection of $(\vec{b}+\vec{c})$ on $\vec{a}$ is 2 units.

Question: 2

$$\log\left(\frac{dy}{dx}\right) = ax + by$$

$$\Rightarrow \frac{dy}{dx} = e^{ax+by}$$

$$\Rightarrow \frac{dy}{dx} = e^{ax} \cdot e^{by} \qquad \left[e^{a+b} = e^{a} \cdot e^{b}\right]$$

$$\Rightarrow \quad \frac{dy}{e^{by}} = e^{ax}dx$$

$$\Rightarrow \quad e^{-by}\,dy = e^{ax}dx$$

On integrating both sides.

$$\int e^{-by}\,dy = \int e^{ax}dx$$

$$\Rightarrow \quad \frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + c$$

$$\frac{e^{ax}}{a} + \frac{e^{-by}}{b} = c' \qquad [c' = -c]$$

where c & c' are constants.

Answer: $\boxed{\dfrac{e^{ax}}{a} + \dfrac{e^{-by}}{b} = e}$

Question : 3

No. of spades = 13

Total no. of cards = 52

x denote number of spades

$$P(x=0) = P(\text{not spade}) \times P(\text{not spade})$$

$$= \frac{39}{52} \times \frac{39}{52}$$

$$= \frac{9}{16}$$

$$P(X=1) = P(\text{spade}) \times P(\text{not spade}) + P(\text{not spade}) \times P(\text{spade})$$

$$= \frac{13}{52} \times \frac{39}{52} + \frac{39}{52} \times \frac{13}{52}$$

$$= \frac{6}{16}$$

$$P(X=2) = P(\text{spade}) \times P(\text{spade})$$

$$= \frac{13}{52} \times \frac{13}{52}$$

$$= \frac{1}{16}.$$

Answer :

No. of spades (X)	Probability
$X = 0$	$9/16$
$X = 1$	$6/16 = 3/8$
$X = 2$	$1/16$

Question : 4
(option-2)

P (A hits) $= \frac{1}{3}$ (P(A))

P (B hits) $= \frac{2}{5}$ (P(B))

P (A doesn't hit) $= 1 - \frac{1}{3}$ [P(A) + P(A') = 1]

$P(A') = \frac{2}{3}$

~~P (B doesn't hit) = 1 - 2/5~~

$P(B') = \frac{3}{5}$

As these are independent events = P (not hiting)

$= P(A') \cdot P(B')$

$[P(A \cap B) = P(A) \cdot P(B)]$

~~Recoangore~~

P(target is hit) = 1 - P(target not hit)

$= 1 - P(A') \times P(B')$

$= 1 - \frac{2}{3} \times \frac{3}{5}$

$= 1 - \frac{2}{5}$

$= \frac{3}{5}$

Answer: Probability of target being hit is $\frac{3}{5}$

Question : 5

Plane P $\Rightarrow$ $x-y+z+\lambda=0$.

point Q = $(1,1,1) = (x_1, y_1, z_1)$

Dist. of point from plane = $\left|\dfrac{x_1-y_1+z_1+\lambda}{\sqrt{(1)^2+(-1)^2+(1)^2}}\right|$

$$\frac{5}{\sqrt{3}} = \left|\frac{1-1+1+\lambda}{\sqrt{3}}\right|$$

$$\frac{5}{\sqrt{3}} = \left|\frac{1+\lambda}{\sqrt{3}}\right|$$

$\dfrac{5}{\sqrt{3}} = \dfrac{1+\lambda}{\sqrt{3}}$ OR $\dfrac{5}{\sqrt{3}} = -\left(\dfrac{1+\lambda}{\sqrt{3}}\right)$

$5 = 1+\lambda$ or $5 = -(1+\lambda)$

$\lambda = 4$ or $\lambda = -6$

$\boxed{\text{Answer : } \lambda = 4, -6}$

Question : 6.

$$\int \frac{dx}{x^2 - 6x + 13}$$

$$= \int \frac{dx}{x^2 - 6x + 9 + 4}$$

$$= \int \frac{dx}{(x-3)^2 + 2^2} \quad \left[x^2 - 2ax + a^2 = (x-a)^2 \right]$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + c$$

[where c is constant]

$$\left[\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} \right]$$

$$\boxed{\text{Answer: } \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C}$$

Section : B

$\boxed{\text{Question : 7}}$

(choice-2)

$|\vec{a}| = 3$

$|\vec{b}| = \cancel{4}\ 5$

$|\vec{c}| = 4$

$\vec{a} + \vec{b} + \vec{c} = 0$.

$(\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = \vec{0}.\vec{0}$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\dfrac{(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)}{2}$

$= -\dfrac{(3^2 + 5^2 + 4^2)}{2}$

$$= -\left(\frac{50}{2}\right) = -25$$

$\boxed{\text{Answer: } \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -25}$

$\boxed{\text{Question : 8}}$

$$I = \int_{-1}^{2} |x^3 - x| \, dx$$

$$I = \int_{-1}^{2} |x(x-1)(x+1)|$$

It changes sign at 0, 1, -1. So, we have to break limits here

$$I = \int_{-1}^{0} |x^3 - x| \, dx + \int_{0}^{1} |x^3 - x| \, dx + \int_{1}^{2} |x^3 - x| \, dx$$

$$\left[\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \right]$$

$a < c < b$

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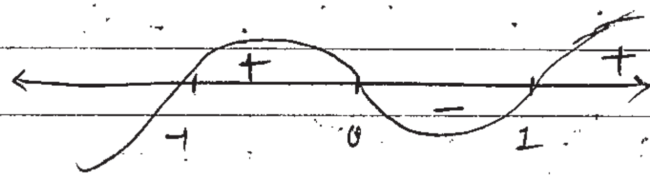


In $(-1,0)$ x^3-x is +ve

In $(0,1)$ x^3-x is −ve

In $(1,2)$ x^3-x is +ve.

$$I = \int_{-1}^{0} (x^3-x)\,dx + \int_{0}^{1} -(x^3-x)\,dx + \int_{1}^{2} (x^3-x)\,dx$$

$$I = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} - \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$

$$I = \left(0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right) - \left(\left(\frac{1}{4} - \frac{1}{2}\right) - 0\right) + \left(\left(\frac{16}{4} - \frac{4}{2}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)\right)$$

$$I = \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4}$$

$$I = 2 + \frac{3}{4}$$

$I = \frac{11}{4}$

$\boxed{\text{Answer : } \int_{-1}^{2} |x^3 - x|\,dx = \frac{11}{4}}$

$\boxed{\text{Question : 9.}}$

Two lines are coplanar if they are parallel or intersecting.

$\frac{1-x}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{2y-2}{-4} = \frac{z-1}{1}$

converting to standard form,

$\frac{x-1}{-2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{1}$

Clearly lines are not parallel as dir^n ratios are not proportional.

Now we can check whether they are intersecting or not.

$x_1 = \cancel{d}\ 1$ $\qquad$ $x_2 = \cancel{d}\ 4$

$y_1 = \cancel{e}\ 3$ $\qquad$ $y_2 = \cancel{e}\ 1$

$z_1 = 0$ $\qquad$ $z_2 = \cancel{d}\ 1$

$a_1 = -2$ $\qquad$ $a_2 = 3$

$b_1 = \cancel{d}\ 4$ $\qquad$ $b_2 = -2$

$c_1 = -1$ $\qquad$ $c_2 = \cancel{e}\ 1$

For lines to be intersecting, shortest distance should be zero (0).
For this, the following determinant should be zero (0).

CBSE HTP

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} -4-(-1) & (-1)-(-3) & (-1)-0 \\ -2 & -4 & -1 \\ 3 & -2 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 2 & -1 \\ -2 & -4 & -1 \\ 3 & -2 & -1 \end{vmatrix}$$

$$= -3\left((-4)(-1) - (-2)(-1)\right) - 2\left((-2)(-1) - (3)(-1)\right) + -1\left((-2)(-2) - (3)(-4)\right)$$

$$= -3(4-2) - 2(2+3) - 1($$

next page

$$\Delta = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 4-1 & 1-3 & 1-0 \\ -2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix}$$

$$\Delta = 3(4-2) - (-2)(-2+3) + 1(4-12)$$

$$\Delta = 3 \times 2 + 2 \times 1 + 1 \times (-8)$$

$$\Delta = 6 + 2 - 8$$

$$\Delta = 0.$$

Thus, these lines are intersecting.

$\boxed{\text{Hence proved, they are coplanar.}}$

Question: 10

(choice-2)

$$x\frac{dy}{dx} = y(\log y - \log x + 1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right) \quad \left[\log a - \log b = \log\left(\frac{a}{b}\right)\right]$$

on putting $x = \lambda x$, $y = \lambda y$.

$$f(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x}\left(\log\left(\frac{\lambda y}{\lambda x}\right) + 1\right)$$

$$f(\lambda x, \lambda y) = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$$

$$= f(x, y).$$

Thus, this equation is homogenous equation.

Let $\frac{y}{x} = t$ or $y = tx$

on differentiating with respect to x

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$$

$$t + x\frac{dt}{dx} = t(\log t + 1) \quad \left[\because \frac{y}{x} = t\right]$$

$$t + x\frac{dt}{dx} = t\log t + t$$

$$x\frac{dt}{dx} = t\log t$$

$$\frac{dt}{t\log t} = \frac{dx}{x}$$

on integrating both sides.

$$\int \frac{dt}{t\log t} = \int \frac{dx}{x}$$

$$\int \frac{dt}{t\log t} = \log_e x + c$$

let $\log t = u$

on differentiating,

$$\frac{1}{t}\,dt = du$$

$$\int \frac{du}{u} = \ln x + c \qquad [\,\log_e x = \ln x\,]$$

$\ln u = \ln x + c$ [c is integration constant]

$\ln(\ln t) = \ln x + c$ [$u = \log_e t = \ln t$].

$\ln(\ln(y/x)) = \ln x + c$

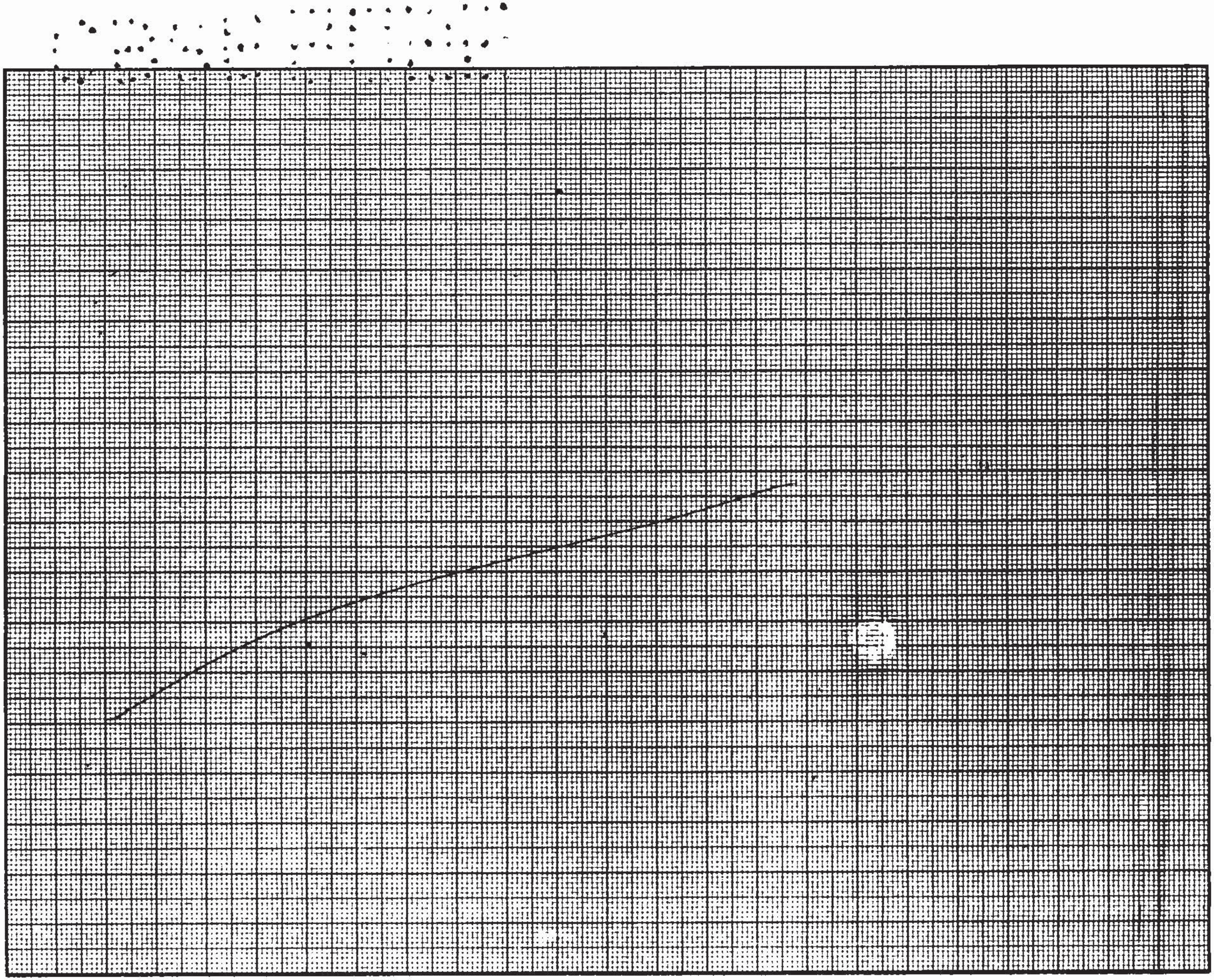
$\ln(\ln(y/x)) - \ln x = c$

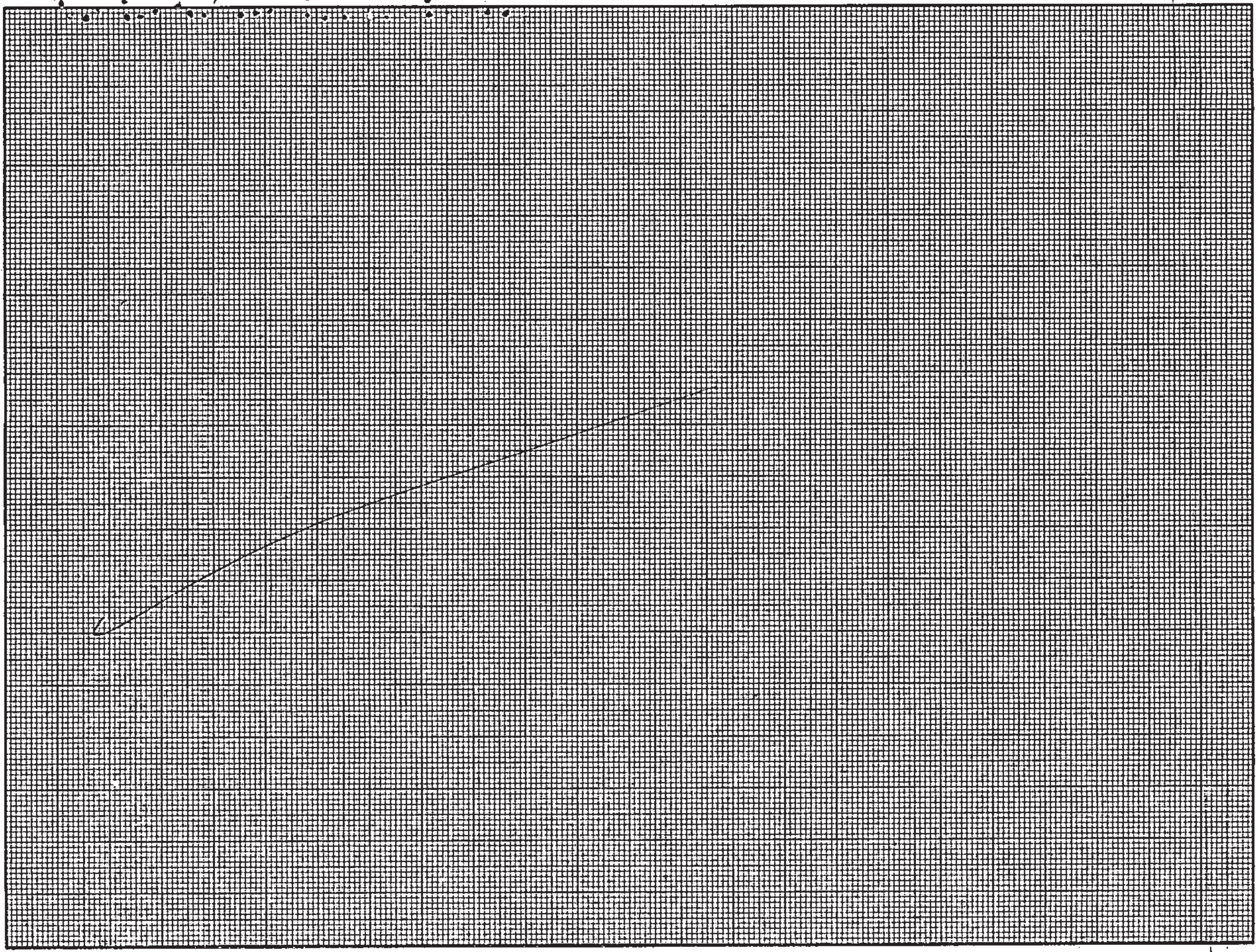
$$\ln\left(\frac{\ln(y/x)}{x}\right) = c$$

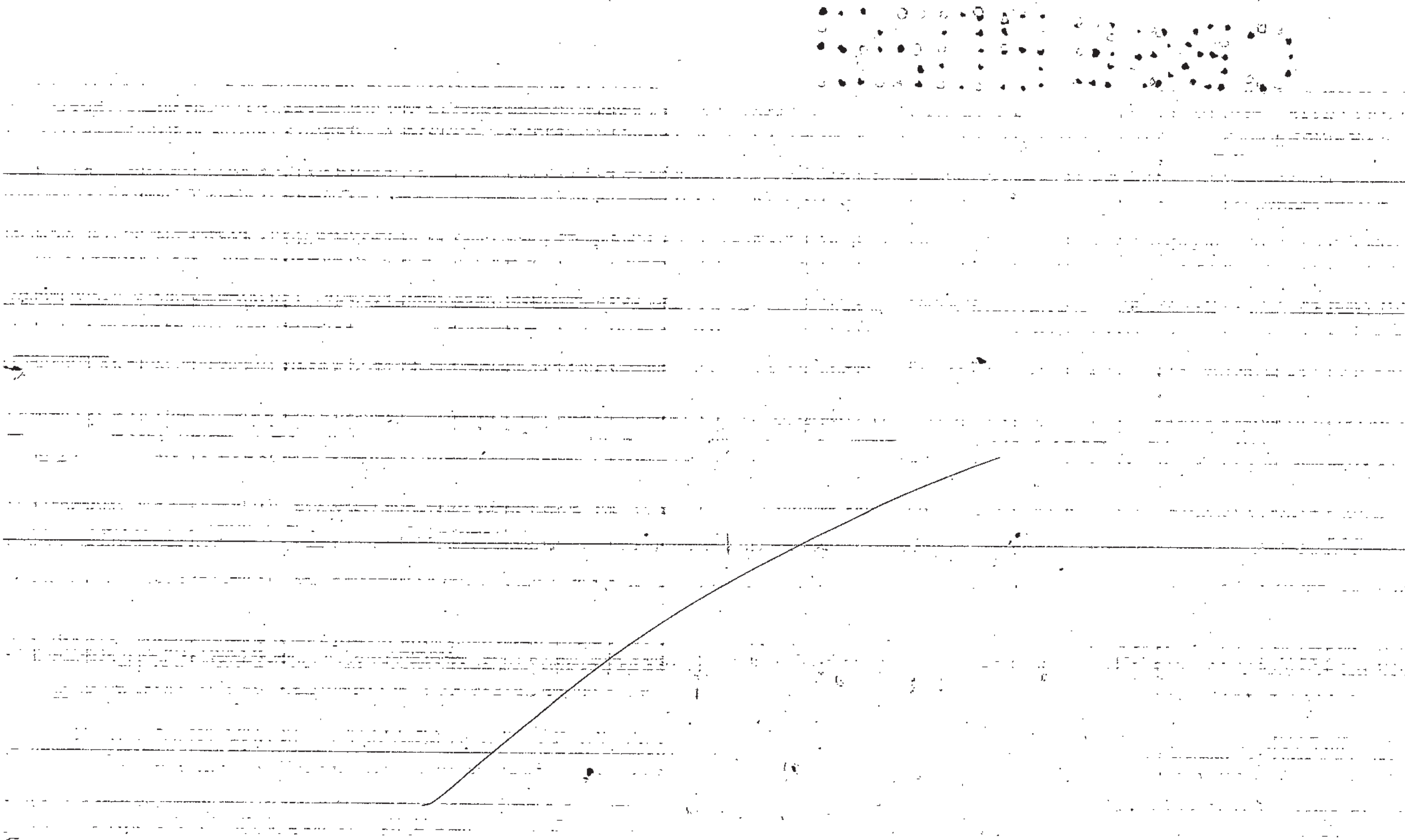
[$\log a - \log b = \log(a/b)$]

Answer: $\boxed{\ln\left(\frac{\ln(y/x)}{x}\right) = c}$

[where $\ln x = \log_e x$]







Question : 11

(choice - 1) .

$$I = \int \frac{x^2}{(x^2+1)\,(3x^2+4)}\,dx$$

Assuming $x^2 = y$ for simplicity.

$$\frac{y}{(y+1)(3y+4)} = \frac{A}{y+1} + \frac{B}{3y+4}$$

$$y = A(3y+4) + B(y+1)$$

on equating coefficients ;

$$3A + B = 1$$

$$4A + B = 0$$

on solving we get $A = -1,\ B = 4$

$$I = \int \frac{-1}{x^2+1} + \int \frac{4}{3x^2+4}$$

$$I = \frac{-1}{1}\tan^{-1}\left(\frac{x}{1}\right) + \frac{4}{3}\int \frac{1}{x^2+4/3}$$

$$I = -\tan^{-1}x + \frac{4}{3}\int \frac{1}{x^2+\left(\sqrt{4/3}\right)^2}$$

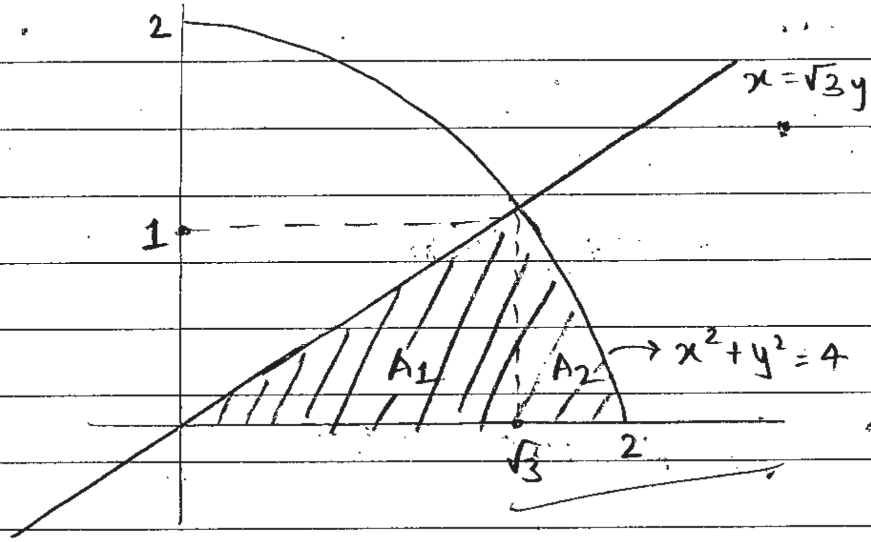
$$\left[\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$I = -\tan^{-1}x + \frac{4/3}{\sqrt{4/3}}\tan^{-1}\left(\frac{x}{\sqrt{4/3}}\right) + C$$

$$I = -\tan^{-1}x + \sqrt{\frac{4}{3}}\tan^{-1}\left(\frac{x\sqrt{3}}{2}\right) + C$$

Answer : $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x\sqrt{3}}{2}\right) - \tan^{-1} x + C$

Question : 12



Area enclosed = $A_1 + A_2$

$A_1 = \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}\, dx$ [line $y = \sqrt{3}x$]

$A_2 = \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\, dx$ [circle $x^2 + y^2 = a^2$]

$A_1 = \left[\frac{x^2}{2\sqrt{3}} \right]^{\sqrt{3}}$

$A_1 = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ sq. units

$A_2 = \int_{\sqrt{3}}^{2} \sqrt{2^2 - x^2}\, dx$

$A_2 = \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2} \sin^{-1}\left(\frac{x}{2}\right) \right]_{\sqrt{3}}^{2}$

$$\left[\int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]$$

$$A_2 = \left[\left(0 + 2\sin^{-1}1\right) - \left(\frac{\sqrt{3}}{2} + 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$$

$$A_2 = 2\left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{2} - 2\left(\frac{\pi}{3}\right)$$

$$A_2 = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$$

Area enclosed $= A_1 + A_2$

$$= \frac{\sqrt{3}}{2} + \frac{\pi}{3} - \frac{\sqrt{3}}{2}$$

Area $= \frac{\pi}{3}$ square units.

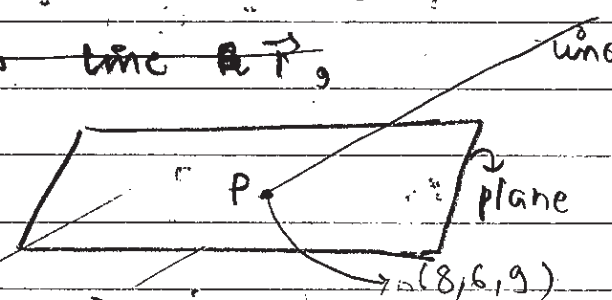
$$\boxed{\text{Answer : Area enclosed is } \frac{\pi}{3}(1.05) \text{ square units.}}$$

Question : 13

~~we~~ line $\vec{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$

~~Let P be any point on this line R~~ $\vec{r}$,

~~wor P = (4+3λ,~~ O $(1,-2,9)$



Let P be intersection of line $\vec{r}$ with the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10$.

$P = (4 + 3\lambda,\ 2 + 4\lambda,\ 7 + 2\lambda)$

coordinates from line $\vec{r}$.

P should also satisfy the eq^n of plane

$$\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=10$$

$$x-y+z=10.$$

on satisfying P in this equation.

$$4+3\lambda-(2+4\lambda)+7+2\lambda=10$$

$$9+\lambda=10$$

$$\lambda=1.$$

coordinates of P $=(4+3\lambda,\ 2+4\lambda,\ 7+2\lambda)$

$$=(7,6,9)$$

Distance of P from $(1, -2, 9)$

$$= \sqrt{(7-1)^2 + (6-(-2))^2 + (9-9)^2}$$

$$= \sqrt{6^2 + 8^2 + 0^2}$$

$$= \sqrt{64 + 36}$$

$$= \sqrt{100}$$

Distance $= 10$ units.

Answer: distance from point of intersection is 10 units.

Question : 14

E be the event that seed germinates.

(a) $P(E) = P(A_1).P(E/A_1) + P(A_2).P(E/A_2) + P(A_3).P(E/A_3)$

$P(A_1) = 4/(4+4+2) = 4/10 \quad P(A_2) = 4/10 \quad P(A_3) = 2/10.$

$A_1 \to$ event of choosing flower seed A_1

$A_2 \to$ event of choosing flower seed A_2

$A_3 \to$ event of choosing flower seed A_3.

$E/A_1 \to$ event E occurring given A_1 has occured.

similarly E/A_2 and E/A_3.

$$P(E) = \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$$

$$= \frac{180 + 240 + 70}{1000}$$

$$P(E) = \frac{490}{1000} = 0.49.$$

(b) $P(A_2/E) = \dfrac{P(A_2) \cdot P(E/A_2)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2) + P(A_3)P(E/A_3)}$

[Baye's Theorem]

$$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{490}{1000}}$$

$$= \frac{24}{49}$$

Answer: a => 0.49 or $\frac{49}{100}$

b => $\frac{24}{49}$