

Marking Scheme
Strictly Confidential
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Senior Secondary Supplementary School Examination, 2026 (XII)
SUBJECT - Mathematics (041) (Q.P. CODE /Set No - 65(B)/7)

General Instructions: -

1.	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2.	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, evaluation done and several other aspects. Its leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC.”
3.	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In Class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4.	The Marking scheme carries only suggested value points for the answers. These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5.	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6.	Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
7.	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8.	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9.	If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question” .
10.	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11.	A full scale of marks _____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12.	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.

13.	Ensure that you do not make the following common types of errors committed by the Examiner in the past :- • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totaling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totaling on the title page. • Wrong totaling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
14.	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15.	Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16.	The Examiners should acquaint themselves with the guidelines given in the “ Guidelines for Spot Evaluation ” before starting the actual evaluation.
17.	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18.	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
PHYSICS CLASS XII (Subject Code: 062)
(PAPER CODE: 65(B)/7)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	If $\begin{vmatrix} 3 & x & -4 \\ 5 & -6 & 0 \\ -7 & 8 & 1 \end{vmatrix} = 0$, then the value of x is : (A) 2 (B) - 2 (C) $-\frac{26}{5}$ (D) $\frac{26}{5}$		
Ans	(B) - 2		1
2.	If $A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \\ 1 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, then the order of A is : (A) 1×3 (B) 1×2 (C) 3×1 (D) 1×1		
Ans	(D) 1×1		1
3.	The matrix $B = \begin{bmatrix} 0 & 0 & 7 \\ 0 & 7 & 0 \\ -7 & 0 & 0 \end{bmatrix}$ is a : (A) skew symmetric matrix (B) square matrix (C) identity matrix (D) diagonal matrix		
Ans	(B) square matrix		1

4.	The function $f(x) = \frac{3-x}{3x^2-x^3}$ is : (A) discontinuous at only one point (B) discontinuous at exactly two points (C) discontinuous at exactly three points (D) continuous at every point		
Ans.	(B) discontinuous at exactly two points		1
5.	If $y = e^{\sin \sqrt{x}}$, then $\frac{dy}{dx}$ is equal to : (A) $e^{\sin \sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$ (B) $e^{\cos \sqrt{x}}$ (C) $e^{\sin \sqrt{x}} \cdot \frac{\cos \sqrt{x}}{2\sqrt{x}}$ (D) $\frac{e^{\cos \sqrt{x}}}{2\sqrt{x}}$		
Ans.	(C) $e^{\sin \sqrt{x}} \cdot \frac{\cos \sqrt{x}}{2\sqrt{x}}$		1
6.	$\int x \sin^3 x^2 \cos x^2 dx$ is equal to : (A) $\frac{1}{8} \sin^4 x^2 + C$ (B) $\frac{1}{4} \sin^4 x^2 + C$ (C) $\frac{1}{2} \sin^4 x^2 + C$ (D) $\frac{1}{8} \sin^4 x + C$		
Ans.	(A) $\frac{1}{8} \sin^4 x^2 + C$		1
7.	The solution of the differential equation $\frac{dy}{dx} = e^{x-y}$ is (A) $e^y + e^x = C$ (B) $e^{-y} + e^x = C$ (C) $e^y - e^x = C$ (D) $e^x - e^{-y} = C$		
Ans.	(C) $e^y - e^x = C$		1

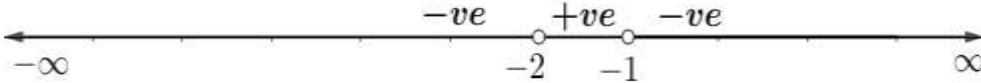
8.	The integrating factor of the differential equation $x dy + (2y - x^3) dx = 0$ is : (A) x (B) $\frac{1}{x}$ (C) x^2 (D) $\frac{1}{x^2}$		
Ans.	(C) x^2		1
9.	If A(3, 5, 1) and B(1, 1, 5) are the end points of the vector $\vec{AB}$, then the direction cosines of $\vec{AB}$ are : (A) $-2, -4, 4$ (B) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$ (C) $-\frac{1}{2}, -1, 1$ (D) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$		
Ans.	(B) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$		1
10.	The unit vector perpendicular to both the vectors $\vec{a} = (-\hat{i} + \hat{j} + \hat{k})$ and $\vec{b} = (-\hat{i} - \hat{j} - \hat{k})$ is : (A) $-2\hat{j} + 2\hat{k}$ (B) $2\hat{j} - 2\hat{k}$ (C) $-\frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$ (D) $-\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$		
Ans.	(D) $-\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$		1
11.	If the line $\frac{1-x}{-3} = \frac{2y-2}{8} = \frac{z+6}{5}$ is parallel to the line $\vec{r} = (2\hat{i} - 3\hat{j} + 5\hat{k}) + \lambda(-6\hat{i} + \lambda\hat{j} - 10\hat{k})$, then the value of λ is : (A) 8 (B) -8 (C) 4 (D) -4		
Ans.	(B) -8		1

12.	The value of λ for which the lines $\frac{x-3}{3\lambda} = \frac{y+2}{1} = \frac{3-z}{7}$ and $\frac{4-x}{3} = \frac{y-5}{2\lambda} = \frac{z-1}{2}$ are perpendicular to each other, is : (A) -2 (B) 2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$		
Ans.	(A) -2		1
13.	The projection of $\vec{a} = (2\hat{i} - \hat{j} + \hat{k})$ on $\vec{b} = (\hat{i} - 2\hat{j} + \hat{k})$ is : (A) $\frac{3}{\sqrt{6}}$ (B) $-\frac{5}{\sqrt{6}}$ (C) $\frac{2}{\sqrt{6}}$ (D) $\frac{5}{\sqrt{6}}$		
Ans.	(D) $\frac{5}{\sqrt{6}}$		1
14.	In an LPP, if the objective function $Z = px + y$ is minimum at a corner point (3, 2) of the feasible region and the minimum value is 14, then the value of p is : (A) 4 (B) $\frac{16}{3}$ (C) $\frac{11}{2}$ (D) $\frac{14}{3}$		
Ans.	(A) 4		1
15.	In an LPP, the optimal value of the objective function is attained at the points : (A) on x-axis (B) on y-axis (C) on corners of the feasible region (D) inside the feasible region		
Ans.	(C) on corners of the feasible region		1

16.	A coin is tossed 3 times. The probability of getting at least one head is : (A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{7}{8}$ (D) $\frac{1}{2}$		
Ans.	(C) $\frac{7}{8}$		1
17.	A die is thrown once and a card is selected at random from a pack of 52 playing cards. The probability of getting a number greater than 4 and a card of Hearts is : (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{7}{12}$ (D) $\frac{1}{12}$		
Ans.	(D) $\frac{1}{12}$		1
18.	If $P(A) = 0.5$, $P(B) = 0.7$ and $P\left(\frac{B}{A}\right) = 0.6$, then $P(A \cup B)$ is equal to : (A) 0.78 (B) 0.9 (C) 0.48 (D) 0.96		
Ans.	(B) 0.9		1
19.	<i>Assertion (A) :</i> If A is a skew symmetric matrix of order 3, then $ A = 0$. <i>Reason (R) :</i> If A is a square matrix of order 2, then $ A = A' $.		
Ans.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).		1
20.	<i>Assertion (A) :</i> If a line makes angles 45° , 90° and 45° with the positive x, y and z-axis respectively, then direction cosines of the line are $\langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle$. <i>Reason (R) :</i> If a line makes angles α , β , γ with positive axes, then $\cos \alpha$, $\cos \beta$, $\cos \gamma$ are its direction cosines.		
Ans.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1

	SECTION B	Steps	Marks
	Q. Numbers 21 to 25 are very short answer questions of 2 marks each.		
21. (a)	Find the product $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$.		2
Sol.	$\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$		$\frac{1}{2}$ Marks for every two correct entries
	OR		
21 (b)	If $A' = \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 3 & 2 \end{bmatrix}$, then find $A' - B'$.		
Sol	$A' - B' = \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 1 & 3 \\ 2 & 2 \end{bmatrix}$ $= \begin{bmatrix} 1 & 5 \\ 2 & 3 \\ -1 & 3 \end{bmatrix}$	I II	1 1
22.	The sides of an equilateral triangle are increasing at the rate of 3 cm/s. Find the rate at which the area of triangle increases, when the length of each side is 10 cm.		
Sol:	Let side of the equilateral triangle = x Then area of the triangle = $A = \frac{\sqrt{3}}{4} x^2$ According to Question, $\frac{dx}{dt} = 3$ cm/s Now $\frac{dA}{dt} = \frac{\sqrt{3}}{2} x \frac{dx}{dt}$ So, $\left. \frac{dA}{dt} \right _{x=10 \text{ cm}} = 15\sqrt{3} \text{ cm}^2/\text{s}$	I II III	1 $\frac{1}{2}$ $\frac{1}{2}$
23.	Find : $\int \sec^3 (3x + 2) \tan (3x + 2) dx$		

Sol.	Let $\sec(3x + 2) = t \Rightarrow 3 \sec(3x + 2) \tan(3x + 2) dx = dt$ So $\int \sec^3(3x + 2) \tan(3x + 2) dx = \frac{1}{3} \int t^2 dt$ $= \frac{1}{9} t^3 + C = \frac{1}{9} \sec^3(3x + 2) + C$	I II III	$\frac{1}{2}$ $\frac{1}{2}$ 1
24. (a)	If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $ \vec{a} = 3$, $ \vec{b} = 5$ and $ \vec{c} = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.		
Sol.	Given $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ $\Rightarrow \vec{a} + \vec{b} = -\vec{c}$ $\Rightarrow (\vec{a} + \vec{b})^2 = (-\vec{c})^2$ $\Rightarrow \vec{a} ^2 + \vec{b} ^2 + 2\vec{a} \cdot \vec{b} = \vec{c} ^2$ $\Rightarrow 9 + 25 + 30 \cos \theta = 49$, where θ is the angle between $\vec{a}$ and $\vec{b}$ $\Rightarrow \cos \theta = \frac{15}{30} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$	I II III IV	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
OR			
24. (b)	Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}$.		
Sol	Here $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & 4 \end{vmatrix} = -5\hat{i} - 2\hat{j} + 3\hat{k}$ Area of the parallelogram $= \vec{a} \times \vec{b} = \sqrt{38}$	I II	1 1
25.	Find the vector equation of a line, passing through the point with position vector $\hat{i} - 2\hat{j} - 3\hat{k}$ and parallel to the line joining the points with position vectors $\hat{i} - \hat{j} + 4\hat{k}$ and $2\hat{i} + \hat{j} + 2\hat{k}$. Also, write its cartesian equation.		
Sol.	Vector equation is $\vec{r} = \hat{i} - 2\hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$ Cartesian equation is $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z+3}{-2}$	I II	1 1
SECTION C			
Q. Numbers 26 to 31 are short answer questions of 3 marks each.			
26.	Evaluate : $\cos^{-1}\left(\cos \frac{2\pi}{3}\right) + \sin^{-1}\left(\sin \frac{2\pi}{3}\right) + \tan^{-1}\left(\tan \frac{2\pi}{3}\right)$.		

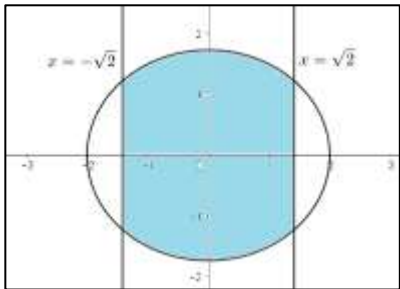
Sol	$\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ $= \frac{2\pi}{3} + \frac{\pi}{3} - \frac{\pi}{3}$ $= \frac{2\pi}{3}$	I II	$\frac{1}{2} + 1 + 1$ $\frac{1}{2}$
27.	Find the intervals in which the function $f(x) = 1 - 12x - 9x^2 - 2x^3$ is strictly increasing or decreasing.		
Sol.	Given function $f(x) = 1 - 12x - 9x^2 - 2x^3$ For critical points $f'(x) = -12 - 18x - 6x^2 = 0$ $\Rightarrow -6(x^2 + 3x + 2) = 0$ $\Rightarrow x = -2, -1$ So, intervals to be considered are $(-\infty, -2), (-2, -1), (-1, \infty)$ or $(-\infty, -2], [-2, -1], [-1, \infty)$  Thus $f(x)$ is strictly increasing in $(-2, -1)$ or $[-2, -1]$ And $f(x)$ is strictly decreasing in $(-\infty, -2) \cup (-1, \infty)$ or $(-\infty, -2] \cup [-1, \infty)$	I II III IV V	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$
	OR		
27 (b)	Find the maximum and minimum values of the function $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$.		
Sol	Given function $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ For critical points $f'(x) = 12x^3 - 24x^2 + 24x - 48 = 0$ $\Rightarrow 12(x^3 - 2x^2 + 2x - 4) = 0 \Rightarrow 12(x - 2)(x^2 + 2) = 0$ $\Rightarrow x = 2 \text{ as } x^2 + 2 = 0 \text{ is not possible for real } x \in [0, 3].$ Now $f(0) = 25, f(2) = -39, f(3) = 16$ Thus, maximum value of $f(x)$ is 25 and minimum value of $f(x)$ is -39	I II III IV	1 $\frac{1}{2}$ 1 $\frac{1}{2}$
28 (a)	Find : $\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx$		
Sol.	Let $e^x = t \Rightarrow e^x dx = dt$ Then $\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx = \int \frac{1}{\sqrt{3^2 - (t+2)^2}} dt$ $= \sin^{-1}\left(\frac{t+2}{3}\right) + C$	I II III	$\frac{1}{2}$ 1 1

	$= \sin^{-1}\left(\frac{e^x+2}{3}\right) + C$	IV	$\frac{1}{2}$
	OR		
28 (b)	Find : $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$		
	Let $x^2 = t$ Then $\frac{x^2}{(x^2+4)(x^2+9)} = \frac{t}{(t+4)(t+9)}$ Let $\frac{t}{(t+4)(t+9)} = \frac{A}{t+4} + \frac{B}{t+9} \Rightarrow t = A(t+9) + B(t+4)$ Comparing like terms and simplifying, $A = -\frac{4}{5}, B = \frac{9}{5}$ Thus $\int \frac{x^2}{(x^2+4)(x^2+9)} dx = -\frac{4}{5} \int \frac{1}{x^2+4} dx + \frac{9}{5} \int \frac{1}{x^2+9} dx$ $= -\frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + \frac{3}{5} \tan^{-1}\left(\frac{x}{3}\right) + C$	I II III	1 1 1
29.	Evaluate : $\int_1^3 (x-1 + x-2 + x-3) dx$		
Sol.	$\int_1^3 (x-1 + x-2 + x-3) dx$ $= \int_1^3 (x-1) dx + \int_1^2 (2-x) dx + \int_2^3 (x-2) dx + \int_3^3 (3-x) dx$ $= \left[\frac{x^2}{2} - x\right]_1^3 + 2x - \frac{x^2}{2} \Big _1^2 + \frac{x^2}{2} - 2x \Big _2^3 + 3x - \frac{x^2}{2} \Big _3^3$ $= 2 + \frac{1}{2} + \frac{1}{2} + 2 = 5$	I II III	$1\frac{1}{2}$ 1 $\frac{1}{2}$
30. (a)	Solve the differential equation $(3xy + y^2) dx + (x^2 + xy) dy = 0.$		
Sol.	Given differential equation is $(3xy + y^2)dx + (x^2 + xy)dy = 0$ $\Rightarrow \frac{dy}{dx} = \frac{-(3xy+y^2)}{(x^2+xy)}$ $\Rightarrow \frac{dy}{dx} = \frac{-\left(3\frac{y}{x} + \left(\frac{y}{x}\right)^2\right)}{\left(1+\frac{y}{x}\right)} \quad \dots(i)$ Let $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$ Then equation (i) becomes $v + x \frac{dv}{dx} = \frac{-(3v+v^2)}{(1+v)}$	I II	$\frac{1}{2}$ $\frac{1}{2}$

	$\Rightarrow x \frac{dv}{dx} = \frac{-(3v+v^2)}{(1+v)} - v = \frac{-2(2v+v^2)}{(1+v)}$ $\Rightarrow \frac{v+1}{v^2+2v} dv = -\frac{2}{x} dx$ Integrating both sides $\Rightarrow \frac{1}{2} \int \frac{2v+2}{v^2+2v} dv = -2 \int \frac{1}{x} dx$ $\Rightarrow \frac{1}{2} \log v^2 + 2v = -2 \log x + \log C$ $\Rightarrow \log v^2 + 2v = -4 \log x + 2 \log C$ $\Rightarrow v^2 + 2v = \frac{C^2}{x^4}$ $\Rightarrow \left(\frac{y}{x}\right)^2 + 2\left(\frac{y}{x}\right) = \frac{C^2}{x^4} \text{ or } x^2(y^2 + 2xy) = K, \text{ where } K = C^2$	III	$\frac{1}{2}$
		IV	1
		V	$\frac{1}{2}$
OR			
30. (b)	Find the particular solution of the differential equation $x(1+y^2)dx - y(1+x^2)dy = 0$, given that $y(1) = 0$.		
Sol.	Given equation is $x(1+y^2)dx - y(1+x^2)dy = 0$ $\Rightarrow \frac{y}{(1+y^2)} dy = \frac{x}{(1+x^2)} dx$ Integrating both sides $\frac{1}{2} \log(1+y^2) = \frac{1}{2} \log(1+x^2) + \log C$ $\Rightarrow (1+y^2) = C^2(1+x^2)$ Now as $y(1) = 0$ $\text{So } 1 = C^2(1+1) \Rightarrow C^2 = \frac{1}{2}$ Thus, particular solution is $2(1+y^2) = (1+x^2)$	I	$\frac{1}{2}$
		II	1
		III	$\frac{1}{2}$
		IV	1
31.	Determine the maximum and the minimum values of $Z = 4x + 3y$, if the feasible region for an LPP is a bounded region with corner points, A(25, 0), B(16, 16) and C(0, 24).		
Sol.	Here $Z_A = 100$, $Z_B = 112$, $Z_C = 72$ So, maximum value of $Z = 112$ at $B(16,16)$ and minimum value of $Z = 72$ at $C(0,24)$	I	1
		II	1
		III	1
SECTION D			
Q. Numbers 32 to 35 are long answer questions of 5 marks each.			
32 (a)	Consider the set $A = N \times N$ as the set of all ordered pairs of natural numbers. Let R be a relation in A defined by $(a, b) R (c, d)$, iff $ad = bc$, $\forall a, b, c, d \in N$. Prove that R is an equivalence relation.		
Sol.	Given R be a relation in $A = N \times N$ defined by		

	$(a, b) R (c, d)$, iff $ad = bc$, $\forall a, b, c, d \in N$. To Show: R is an Equivalence Relation. Reflexive: For two numbers $a, b \in N$, $ab = ba$ $\Rightarrow (a, b)R(a, b) \quad \forall (a, b) \in N$ So, R is Reflexive. Symmetric: Let $a, b, c, d \in N$; $(a, b)R(c, d)$ $\Rightarrow ad = bc$ $\Rightarrow bc = ad$ $\Rightarrow cb = da$ $\Rightarrow (c, d)R(a, b)$ So, R is symmetric. Transitive: Let $a, b, c, d, e, f \in N$; $(a, b)R(c, d)$ and $(c, d)R(e, f)$ Then $ad = bc$ and $cf = de$ Consider $af = \left(\frac{bc}{d}\right)\left(\frac{de}{c}\right) = be$ i.e., $af = be$ $\Rightarrow (a, b)R(e, f)$ So, R is transitive. Thus, R is Reflexive, Symmetric and Transitive. Hence, R is an Equivalence Relation.	I	1
		II	1 ½
		III	2
		IV	½
	OR		
32 (b)	Let $A = \mathbb{R} - \{2\}$ and $B = \mathbb{R} - \{1\}$. If $f : A \rightarrow B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, then show that f is one-one and onto.		
Sol	Given $f : A \rightarrow B$ is defined as $f(x) = \frac{x-1}{x-2}$. One-One: For $a, b \in A$ let $f(a) = f(b)$ $\Rightarrow \frac{a-1}{a-2} = \frac{b-1}{b-2}$ $\Rightarrow (a-1)(b-2) = (a-2)(b-1)$ $\Rightarrow ab - 2a - b + 2 = ab - a - 2b + 2$ $\Rightarrow -2a + a = -2b + b$ $\Rightarrow -a = -b$ $\Rightarrow a = b$ So f is one-one. Onto: Let $f(x) = y$ $\Rightarrow \frac{x-1}{x-2} = y$ (clearly $x \neq 2$)	I	1
		II	1
		III	½
		IV	½

	$\Rightarrow x - 1 = y(x - 2)$ $\Rightarrow x - 1 = xy - 2y$ $\Rightarrow x - xy = 1 - 2y$ $\Rightarrow x(1 - y) = 1 - 2y$ $\Rightarrow x = \frac{1-2y}{1-y}$, a well-defined real number (as $y \in B = R - \{1\}$) So, Range = $R - \{1\} = Co - domain$ Thus f is onto. Hence f is both one-one & onto.	IV V VI	1 $\frac{1}{2}$ $\frac{1}{2}$
33.	If $x = \sin t$ and $y = \sin pt$, then prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2y = 0.$		
Sol.	Given $x = \sin t$ and $y = \sin pt$ $\Rightarrow \frac{dx}{dt} = \cos t$ and $\frac{dy}{dt} = p \cos pt$ So, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{p \cos pt}{\cos t}$ $\Rightarrow \cos t \frac{dy}{dx} = p \cos pt$ Differentiating both sides w.r.t. x , we get $\cos t \frac{d^2y}{dx^2} + (-\sin t) \frac{dt}{dx} \frac{dy}{dx} = p^2(-\sin pt) \frac{dt}{dx}$ $\Rightarrow \cos t \frac{d^2y}{dx^2} + (-\sin t) \frac{1}{\cos t} \frac{dy}{dx} = p^2(-\sin pt) \frac{1}{\cos t}$ $\Rightarrow \cos^2 t \frac{d^2y}{dx^2} + (-\sin t) \frac{dy}{dx} = p^2(-\sin pt)$ $\Rightarrow (1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2y = 0$ Alternative Way: Given $x = \sin t$ and $y = \sin pt$ $\Rightarrow \frac{dx}{dt} = \cos t$ and $\frac{dy}{dt} = p \cos pt$ So, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{p \cos pt}{\cos t}$ $\Rightarrow \frac{dy}{dx} = \frac{p\sqrt{1-\sin^2 pt}}{\sqrt{1-\sin^2 t}} = \frac{p\sqrt{1-y^2}}{\sqrt{1-x^2}}$ $\Rightarrow \sqrt{1-x^2} \frac{dy}{dx} = p\sqrt{1-y^2}$ Differentiating both sides w.r.t. x , we get $\sqrt{1-x^2} \frac{d^2y}{dx^2} + \frac{1}{2\sqrt{1-x^2}} (-2x) \frac{dy}{dx} = p \frac{1}{2\sqrt{1-y^2}} (-2y) \frac{dy}{dx}$ $\Rightarrow \sqrt{1-x^2} \frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}} \frac{dy}{dx} + \frac{py}{\sqrt{1-y^2}} \times \frac{p\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$ $\Rightarrow (1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2y = 0$	I II III IV V VI I II III IV V VI	1 1 $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 $\frac{1}{2}$ 1 $\frac{1}{2}$

34.	Using integration, find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ and the lines $x = -\sqrt{2}$ and $x = \sqrt{2}$.		
Sol.	 $\frac{x^2}{4} + \frac{y^2}{3} = 1 \Rightarrow y = \frac{\sqrt{3}}{2} \sqrt{4 - x^2} = \frac{\sqrt{3}}{2} \sqrt{2^2 - x^2}$ Required Area = $4 \int_0^{\sqrt{2}} \left(\frac{\sqrt{3}}{2} \sqrt{2^2 - x^2} \right) dx$ $= 2\sqrt{3} \int_0^{\sqrt{2}} \sqrt{2^2 - x^2} dx$ $= 2\sqrt{3} \left[\frac{x}{2} \sqrt{2^2 - x^2} + \frac{2^2}{2} \sin^{-1} \left(\frac{x}{2} \right) \right]_0^{\sqrt{2}}$ $= 2\sqrt{3} \left[\frac{\sqrt{2}}{2} \sqrt{2^2 - (\sqrt{2})^2} + 2 \sin^{-1} \left(\frac{\sqrt{2}}{2} \right) - 0 \right]$ $= 2\sqrt{3} \left[1 + 2 \sin^{-1} \left(\frac{1}{\sqrt{2}} \right) \right]$ $= \sqrt{3}(2 + \pi)$	I II III IV V	1 1 ½ 1 1 ½
35. (a)	A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn randomly and are found to be both Spades. Find the probability of the lost card being a Spade.		
Sol.	Let the events be E_1: Lost card is of Spade E_2: Lost card is not of Spade A: Two drawn cards are of Spade $P(E_1) = \frac{1}{4}, P(E_2) = \frac{3}{4},$ $P(A E_1) = \frac{12}{51} \times \frac{11}{50}, P(A E_2) = \frac{13}{51} \times \frac{12}{50}$ Probability that lost card is of spade is $= P(E_1 A) = \frac{P(E_1)P(A E_1)}{P(E_1)P(A E_1) + P(E_2)P(A E_2)}$ $= \frac{\frac{1}{4} \times \frac{12}{51} \times \frac{11}{50}}{\frac{1}{4} \times \frac{12}{51} \times \frac{11}{50} + \frac{3}{4} \times \frac{13}{51} \times \frac{12}{50}} = \frac{132}{600} = \frac{11}{50}$	I II III IV V	1 1 1 ½ 1+½
OR			

35 (b)	Three coins are tossed simultaneously. Consider the event A to be getting 'three heads or three tails', B 'at most two heads' and C 'at least two heads'. Of the pairs (A, C) and (B, C), find which are independent and which are dependent.		
Sol.	Here Sample Space = $\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$ Given events are A: Three heads or three tails B: At most two heads C: At least two heads Here $P(A) = \frac{2}{8} = \frac{1}{4}, P(B) = \frac{7}{8}, P(C) = \frac{4}{8} = \frac{1}{2}$ Also $P(A \cap C) = \frac{1}{8}$ and $P(B \cap C) = \frac{3}{8}$ Here $P(A).P(C) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} = P(A \cap C)$ So, A and C are independent events. $P(B).P(C) = \frac{7}{8} \times \frac{1}{2} = \frac{7}{16} \neq \frac{3}{8} = P(B \cap C)$ So, B and C are not independent events.	I II III IV V	$\frac{1}{2}$ $1 \frac{1}{2}$ 1 1 1
SECTION E This section (Q. 36 to 38) has 3 case-study based questions of 4 marks each.			
36.	Two motorcycle dealers, P and Q deal in two types of motorcycles, say A and B. The sales figures for the years 2022 and 2023 showed that dealer P sold 160 motorcycles of type A and 70 of type B in 2022 and 300 motorcycles of type A and 150 of type B in 2023. Dealer Q sold 100 motorcycles of type A and 40 of type B in 2022 and 200 motorcycles of type A and 60 of type B in 2023. Based on the above information, answer the following questions : (i) Write the matrix representing the sales of the two dealers in 2022 : $\begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} P \\ Q \end{matrix} & \begin{bmatrix} - & - \\ - & - \end{bmatrix} \end{matrix}$ 1 (ii) Write the matrix representing the sales of the two dealers in 2023 : $\begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} P \\ Q \end{matrix} & \begin{bmatrix} - & - \\ - & - \end{bmatrix} \end{matrix}$ 1		

	(iii) (a) If dealers P and Q each earned a profit of ₹ 30,000 on one motorcycle A and ₹ 20,000 on one motorcycle B in the year 2022, then find the total profit earned by each, using the matrix multiplication. 2 OR (iii) (b) If dealers P and Q each earned a profit of ₹ 40,000 on one motorcycle A and ₹ 30,000 on one motorcycle B in the year 2023, then using the matrix multiplication find the total profit of each in the year 2023. 2		
Sol.	(i) Matrix representing sales of two dealers in 2022 = $S_1 = \begin{bmatrix} 160 & 70 \\ 100 & 40 \end{bmatrix}$ (ii) Matrix representing sales of two dealers in 2023 = $S_2 = \begin{bmatrix} 300 & 150 \\ 200 & 60 \end{bmatrix}$	I I	1 1
	(iii) (a) Profit matrix = $P_1 = \begin{bmatrix} 30000 \\ 20000 \end{bmatrix}$ So, profit earned by both in 2022 = $S_1 P_1 = \begin{bmatrix} 160 & 70 \\ 100 & 40 \end{bmatrix} \begin{bmatrix} 30000 \\ 20000 \end{bmatrix} = \begin{bmatrix} 6200000 \\ 3800000 \end{bmatrix}$ Profit earned by dealer P in 2022 = ₹ 62,00,000 Profit earned by dealer Q in 2022 = ₹ 38,00,000 OR (b) Profit matrix = $P_2 = \begin{bmatrix} 40000 \\ 30000 \end{bmatrix}$ So, profit earned by both in 2023 = $S_2 P_2 = \begin{bmatrix} 300 & 150 \\ 200 & 60 \end{bmatrix} \begin{bmatrix} 40000 \\ 30000 \end{bmatrix} = \begin{bmatrix} 16500000 \\ 9800000 \end{bmatrix}$ Profit earned by dealer P in 2023 = ₹ 1,65,00,000 Profit earned by dealer Q in 2023 = ₹ 98,00,000	I II I II	1 1 1 1

37.	Raghu's father wants to make a rectangular garden at the back of his house, using the back wall as one side of the garden and wire fencing for the other three sides. He has 300 m of wire for fencing. Based on the above information, answer the following questions : (i) If x denotes the length of the side of the garden perpendicular to the brick wall and y denotes the length of the side parallel to the wall, then find the relation, representing the total length of the fencing wall. 1 (ii) Write the area of garden $A(x)$ as a function of x. 1 (iii) (a) Find the value of x at which $A(x)$ is maximum. 2 OR (iii) (b) Find the maximum value of $A(x)$. 2		
Sol.	(i) Expression for Total length of the fencing = $2x + y = 300\text{ m}$ (ii) Area of the garden = $A = xy = x(300 - 2x) = 300x - 2x^2$ (iii) (a) $\frac{dA}{dx} = 0 \Rightarrow 300 - 4x = 0 \Rightarrow x = 75$ $\frac{d^2A}{dx^2} = -4 < 0$ at $x = 75$ So, area is maximum when $x = 75\text{ m}$ OR (b) $\frac{dA}{dx} = 0 \Rightarrow 300 - 4x = 0 \Rightarrow x = 75$ $\frac{d^2A}{dx^2} = -4 < 0$ at $x = 75$ So, area is maximum when $x = 75\text{ m}$ Maximum value of $A(x) = A(75) = 22500 - 11250 = 11250\text{ sq m}$	I I I II I II III	1 1 $\frac{1}{2} + \frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ 1
38.	In an exhibition, an exhibit for sale is kept in a cubical glass box, the coordinates of whose corners are : $A(1, 2, 3)$, $B(2, 2, 3)$, $C(2, 4, 3)$, $D(1, 4, 3)$ $A'(1, 2, 6)$, $B'(2, 2, 6)$, $C'(2, 4, 6)$, $D'(1, 4, 6)$ Based on the above information, answer the following questions : (i) Write the cartesian equation of diagonal AC'. 1 (ii) Write the cartesian equation of diagonal $A'C$. 1 (iii) (a) Show that AC' and $A'C$ are intersecting lines. 2 OR (iii) (b) Find the point of intersection of lines AC' and $A'C$. 2		

Sol.	(i) Cartesian equation of diagonal AC' is $\frac{x-1}{2-1} = \frac{y-2}{4-2} = \frac{z-3}{6-3} \Rightarrow \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$	I	1
	(ii) Cartesian equation of diagonal $A'C$ is $\frac{x-1}{2-1} = \frac{y-2}{4-2} = \frac{z-6}{3-6} \Rightarrow \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-6}{-3}$	I	1
	(iii) (a) To check AC' and $A'C$ are intersecting lines or not: A point on the line AC' is $(k + 1, 2k + 2, 3k + 3)$ A point on the line $A'C$ is $(m + 1, 2m + 2, -3m + 6)$ For common points $k + 1 = m + 1, 2k + 2 = 2m + 2, 3k + 3 = -3m + 6$ $\Rightarrow k = m \text{ and } k + m = 1 \Rightarrow k = m = \frac{1}{2}$ And these values satisfy all the three relations Thus lines AC' and $A'C$ are intersecting.	I	1
	OR	II	1
	(b) To check AC' and $A'C$ are intersecting lines or not: A point on the line AC' is $(k + 1, 2k + 2, 3k + 3)$ A point on the line $A'C$ is $(m + 1, 2m + 2, -3m + 6)$ For common points $k + 1 = m + 1, k + 1 = m + 1, 2k + 2 = 2m + 2, 3k + 3 = -3m + 6$ $\Rightarrow k = m \text{ and } k + m = 1 \Rightarrow k = m = \frac{1}{2}$ Point of Intersection of AC' and $A'C$ is $\left(\frac{3}{2}, 3, \frac{9}{2}\right)$	I	1
		II	1
	Alternative Way for (iii):		
	(a) Mid-point of AC' is $\left(\frac{3}{2}, 3, \frac{9}{2}\right)$ Mid-point of $A'C$ is $\left(\frac{3}{2}, 3, \frac{9}{2}\right)$ Mid-point of $AC' = \text{Mid-point of } A'C = \left(\frac{3}{2}, 3, \frac{9}{2}\right)$ Thus lines AC' and $A'C$ are intersecting.	I	1
	OR	II	1
	(b) As the box is cubical, so intersecting points of the diagonals is the mid-point of the diagonals. So Intersecting point of AC' and $A'C = \left(\frac{3}{2}, 3, \frac{9}{2}\right)$	I	1
		II	1