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Senior Secondary School Certificate Examination

September 2021

Marking Scheme — Mathematics 65 (B)

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2. "Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/ Website etc may invite action under IPC."
3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.
4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
5. Evaluators will mark (✓) wherever answer is correct. For wrong answer 'X" be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
10. A full scale of marks __________(example 0-80 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
12. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrongquestion wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
14. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65 (B)
EXPECTED ANSWER/VALUE POINTS
PART A

(Section – I)

Question numbers 1 to 16 carry 1 mark each. **16 × 1 = 16**

1. A relation R on N × N is defined as (a, b) R (c, d) iff a + d = b + c. Show that R is a reflexive relation.

Ans. As a + b = b + a $\frac{1}{2}$

$\therefore$ (a, b) R (a, b) $\frac{1}{2}$

$\therefore$ R is reflexive relation

2. (a) If a function f: $\mathbb{R} \rightarrow \mathbb{R}$ is defined as $f(x) = 3 + x^2$, show that f is not a one-one function.

Ans. f (–1) = f (1) = 4 $\frac{1}{2}$

but $-1 \neq 1$ $\frac{1}{2}$

$\therefore$ f is not a one-one function

OR

(b) Show that a relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b) : b = a + 1} is not transitive.

Ans. (1, 2) ∈ R, (2, 3) ∈ R but (1, 3) ∉ R 1

$\therefore$ R is not transitive

3. (a) Find the principal value of $\tan^{-1}\left(\tan\frac{4\pi}{3}\right)$.

Ans. $\frac{\pi}{3}$ 1

OR

(b) Write the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

Ans. $\frac{3\pi}{4}$ 1

4. Find $\frac{dy}{dx}$, if 3x + 4y = sin y.

Ans. Differetiating both sides w.r.t x

$$3+4\frac{dy}{dx}=\cos y\frac{dy}{dx}$$ $\frac{1}{2}$

$$\therefore \frac{dy}{dx}=\frac{3}{\cos y-4}$$ $\frac{1}{2}$

5. If $y = x^x$, find $\frac{dy}{dx}$.

Ans. Taking log on both sides to get log y = x log x $\frac{1}{2}$

Differentiating both sides to get

$$\frac{dy}{dx}=x^x(1+\log x)$$ $\frac{1}{2}$

6. If x = 4t and y = $\frac{4}{t}$, find $\frac{dy}{dx}$ at t = $\frac{1}{2}$.

Ans. $\frac{dx}{dt}=4, \frac{dy}{dt}=-\frac{4}{t^2}$ $\frac{1}{2}$

$$\therefore \frac{dy}{dx}=\frac{-1}{t^2} \quad \therefore \left(\frac{dy}{dx}\right)_{t=\frac{1}{2}}=-4$$ $\frac{1}{2}$

7. Find a point on the curve $y = x^3 - 11x + 5$ at which the equation of the tangent is y = x – 11.

Ans. $\frac{dy}{dx}=3x^2-11$ $\frac{1}{2}$

$3x^2 - 11 = 1$ gives x = 2, –2

y = – 9, 19

Rquired Point is (2, –9) $\frac{1}{2}$

8. Find:

$$\int\frac{5-4\sin x}{\cos^2 x}dx$$

Ans. $I = \int [5\sec^2 x - 4\sec x \tan x]\, dx$

$= 5\tan x - 4\sec x + C$ $\frac{1}{2} + \frac{1}{2}$

9. (a) Write the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 1 = 0$

Ans. Order : 2 $\frac{1}{2}$

Degree : 2 $\frac{1}{2}$

OR

(b) Solve the differential equation $\frac{dy}{dx} + y = 3, (y \neq 3).$

Ans. Given differential equation can be written as

$\frac{dy}{y-3} = -dx$ $\frac{1}{2}$

Integrating both sides, we get

$\log |y - 3| = -x + C$ $\frac{1}{2}$

10. Find the integrating factor of the differential equation $x\frac{dy}{dx} + y = 2x^2.$

Ans. Given differential equation can be written as

$\frac{dy}{dx} + \frac{1}{x}y = 2x$ $\frac{1}{2}$

Integrating factor = $e^{\int \frac{1}{x} dx} = e^{\log x} = x$ $\frac{1}{2}$

11. (a) Find the unit vector in the direction of the sum of the vectors $\vec{a} = \hat{i} + \hat{j} - 6\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 4\hat{k}.$

Ans. $\vec{a} + \vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$ $\frac{1}{2}$

Required unit vector is $\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{2}{3}\hat{k}$ $\frac{1}{2}$

OR

(b) Find the direction cosines of the vector joining the points A (–1, –2, 1) and B (1, 2, –3) directed from A to B.

Ans. $\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 4\hat{k}$ $\frac{1}{2}$

Direction cosines of $\overrightarrow{AB}$ are $\frac{2}{6}, \frac{4}{6}, \frac{-4}{6}$ or $\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}$ $\frac{1}{2}$

12. Find the values of and λ and μ,

if $(3\hat{i} + 6\hat{j} + 15\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$.

Ans. $\frac{3}{1} = \frac{6}{\lambda} = \frac{15}{\mu}$ $\frac{1}{2}$

$\lambda = 2, \mu = 5$ $\frac{1}{2}$

13. (a) Find the direction cosines of a line which makes equal angles with the coordinate axes.

OR

Ans. Let direction cosines be *l*, *l*, *l*

$3l^2 = 1$ $\frac{1}{2}$

direction cosines are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ or $\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$ $\frac{1}{2}$

(b) Find the distance of the point (2, 5, –3) from the plane

$\vec{r} \cdot (3\hat{i} + 2\hat{j} + 6\hat{k}) = 4$.

Ans. Required distance = $\frac{|2.(3) + 5.(2) + (-3).6 - 4|}{\sqrt{3^2 + 2^2 + 6^2}}$ $\frac{1}{2}$

$= \frac{6}{7}$ $\frac{1}{2}$

14. The corner points of the feasible region for an LPP are (0, 3), (1, 1) and (3, 0). If the objective function is z = ax + by, a, b > 0, then find the condition on a and b so that the minimum of z occurs at (3, 0) and (1, 1).

Ans. $3a + b \cdot 0 = a \cdot 1 + b \cdot 1$ $\frac{1}{2}$

$2a = b$ $\frac{1}{2}$

15. A die is rolled. Consider the events E = {1, 3, 5} and F = {2, 3}.

Find P(E | F) + P(F | E).

Ans. $P(E \mid F) = \frac{1}{2}, P(F \mid E) = \frac{1}{3}$ $\frac{1}{2}$

$P(E \mid F) + P(F \mid E) = \frac{5}{6}$ $\frac{1}{2}$

16. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the probability of getting one king and one non-king.

Ans. P (one king & one non-king) = $\frac{4}{52} \times \frac{48}{51} \times 2$ $\frac{1}{2}$

$= \frac{32}{221}$ $\frac{1}{2}$

(Section II)

17. A manufacturer produces three products P, Q and R, which he sells in two markets A and B. Annual sales are indicated below:

Market		**Products**	
	P	**Q**	**R**
A	5000	3000	6000
B	7000	9000	5000

The unit sales price of P, Q and R are ₹ 3, ₹ 2 and ₹ 1 respectively.

Based on the above information, answer any four of the following five questions: $4 \times 1 = 4$

(i) Which of the following matrix equations represents his total revenue (R_1) in the market A?

(A) $[5000 \quad 3000 \quad 6000][3 \quad 2 \quad 1] = [R_1]$

(B) $[5000 \quad 3000 \quad 6000]\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = [R_1]$

(C) $[5000 \quad 3000 \quad 6000]\begin{bmatrix}3\\2\\1\end{bmatrix}=[R_1]$

(D) $\begin{bmatrix}5000\\3000\\6000\end{bmatrix}[3 \quad 2 \quad 1]=[R_1]$

Ans. (C) $[5000 \quad 3000 \quad 6000]\begin{bmatrix}3\\2\\1\end{bmatrix}=[R_1]$ 1

(ii) The total revenue (R_1) in market A is

(A) ₹ 27,000

(B) ₹ 30,000

(C) ₹ 29,000

(D) ₹ 33,000

Ans. (A) ₹ 27,000 1

(iii) The total revenue (R_2) in market B is

(A) ₹ 42,000

(B) ₹ 44,000

(C) ₹ 46,000

(D) ₹ 40,000

Ans. (B) ₹ 44,000 1

(iv) If the unit costs of the above products are ₹ 2.5, ₹ 1 and 50 paise respectively, the matrix representing the total cost C of all the products produced is

(A) $\begin{bmatrix}5000 & 3000 & 6000\\7000 & 9000 & 5000\end{bmatrix}[2.5 \quad 1 \quad 0.5]=[C]$

(B) $\begin{bmatrix}5000 & 3000 & 6000\\7000 & 9000 & 5000\end{bmatrix}\begin{bmatrix}2.5\\1\\0.5\end{bmatrix}=[C]$

(C) $[12000 \quad 12000 \quad 11000]\begin{bmatrix} 2.5 \\ 1 \\ 0.5 \end{bmatrix} = [C]$

(D) $[12000 \quad 12000 \quad 11000][2.5 \quad 1 \quad 0.5] = [C]$

Ans. (C) $[12000 \quad 12000 \quad 11000]\begin{bmatrix} 2.5 \\ 1 \\ 0.5 \end{bmatrix} = [C]$ 1

(v) The gross profit from both the markets is

(A) ₹ 22,500

(B) ₹ 23,000

(C) ₹ 25,000

(D) ₹ 23,500

Ans. (D) ₹ 23,500 1

18. A quantity that has magnitude as well as direction is called a vector. The quantities like velocity and displacement are the vector quantities.

A girl, at a point O, walks 6 km towards east to reach at a point A. She then walks 4 km in a direction 30° west of north and stops at a point B. Thus the vector $\overrightarrow{OB}$ represents the displacement of the girl from initial point O to the terminal point B.

Let the direction towards east be represented by positive x-axis and the direction towards north be represented by positive y-axis, with O as origin.

Based on the above information, answer any four of the following five questions: $4 \times 1 = 4$

(i) Vector $\overrightarrow{OA}$ is given by

(A) $6\hat{i} + 4\hat{j}$

(B) $6\hat{i} - 4\hat{j}$

(C) $6\hat{i}$

(D) $-6\hat{i}$

Ans. (C) $6\hat{i}$ 1

(ii) Vector $\overrightarrow{AB}$ is given by

(A) $-4\cos 60^\circ\,\hat{i} + 4\sin 60^\circ\,\hat{j}$

(B) $4\cos 60^\circ\,\hat{i} + 4\sin 60^\circ\,\hat{j}$

(C) $-4\cos 30^\circ\,\hat{i} + 4\sin 30^\circ\,\hat{j}$

(D) $4\cos 30^\circ\,\hat{i} + 4\sin 30^\circ\,\hat{j}$

Ans. (A) $-4\cos 60^\circ\,\hat{i} + 4\sin 60^\circ\,\hat{j}$ 1

(iii) Vector $\overrightarrow{OB}$ is equal to

(A) $\overrightarrow{OA} + \overrightarrow{AB}$

(B) $\overrightarrow{OA} + \overrightarrow{BA}$

(C) $\overrightarrow{AB} + \overrightarrow{AO}$

(D) $\overrightarrow{BA} + \overrightarrow{AO}$

Ans. (A) $\overrightarrow{OA} + \overrightarrow{AB}$ 1

(iv) The displacement $\overrightarrow{OB}$, in terms of its components, is

(A) $-4\hat{i} + 2\sqrt{3}\,\hat{j}$

(B) $4\hat{i} + 2\sqrt{3}\,\hat{j}$

(C) $8\hat{i} + 2\sqrt{3}\,\hat{j}$

(D) $8\hat{i} - 2\sqrt{3}\,\hat{j}$

Ans. (B) $4\hat{i} + 2\sqrt{3}\,\hat{j}$ 1

(v) The distance OB (in km) is

(A) $2\sqrt{7}$

(B) $2\sqrt{19}$

(C) 10

(D) 2

Ans. (A) $2\sqrt{7}$ 1

PART B

(Section III)

Question numbers 19 to 28 carry 2 marks each. **10 × 2 = 20**

19. Find whether the function f: $\mathbb{Z} \to \mathbb{Z}$ given by $f(x) = x^3$ is one-one and onto.

Ans. Let $x_1, x_2 \in \mathbb{Z}$ such that $f(x_1) = f(x_2)$ $\frac{1}{2}$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one 1

$2 \in \mathbb{Z}$ has no pre image in $\mathbb{Z}$ as $x = \sqrt[3]{2} \notin \mathbb{R}$

$\therefore$ f is not onto $\frac{1}{2}$

20. If x = a sec θ, y = b tan θ, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{6}$.

Ans. $\frac{dx}{d\theta} = a \sec\theta \tan\theta$ $\frac{1}{2}$

$\frac{dy}{d\theta} = b \sec^2\theta$ $\frac{1}{2}$

$\frac{dy}{dx} = \frac{b}{a} \frac{\sec\theta}{\tan\theta}$ $\frac{1}{2}$

$\left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{6}} = \frac{2b}{a}$ $\frac{1}{2}$

21. (a) Show that the function f given by $f(x) = x^3 - 3x^2 + 5x$, $x \in \mathbb{R}$ is strictly increasing on $\mathbb{R}$.

Ans. $f'(x) = 3x^2 - 6x + 5$ 1

$= 3\,(x - 1)^2 + 2$ $\frac{1}{2}$

Clearly $f'(x) > 0 \ \forall \ x \in R$

$\therefore$ f(x) is strictly increasing on $\mathbb{R}$ $\frac{1}{2}$

OR

(b) Prove that the function given by $f(x) = x^3 + x^2 + x + 1$ does not have maxima or minima.

Ans. $f'(x) = 3x^2 + 2x + 1$ 1

$= 2x^2 + (x + 1)^2$ or $3\left(x+\frac{1}{3}\right)^2+\frac{2}{3}$ $\frac{1}{2}$

$f'(x) > 0$

$\therefore$ f(x) is an increasing function

hence f(x) does not have maxima or minima $\frac{1}{2}$

22. For the matrix $A = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, verify that $(A + A')$ is a symmetric matrix.

Ans. $A' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$ $\frac{1}{2}$

$A + A' = \begin{bmatrix} -8 & 2 & -4 \\ 2 & 4 & 3 \\ -4 & 3 & 2 \end{bmatrix} = B$ say 1

$B' = \begin{bmatrix} -8 & 2 & 4 \\ 2 & 4 & 3 \\ -4 & 3 & 2 \end{bmatrix} = B$ $\frac{1}{2}$

$\therefore$ $B = A + A'$ is a symmetric matrix

23. Examine whether the given system of equations has a unique solution or not.

$3x - y - 2z = 2;\ 2y - z = 1;\ 3x - 5y = 3$

Ans. Given system is

$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ $\frac{1}{2}$

Let $A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

$|A| = 0$ 1

$\therefore$ Given system of equations does not have a unique solution. $\frac{1}{2}$

24. (a) Solve the system of linear equations, $5x + 2y = 4$, $7x + 3y = 5$ by matrix method.

Ans. Given system can be written as $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ $\frac{1}{2}$

Let $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\therefore AX = B$

$|A| = 1$ $\frac{1}{2}$

$\therefore X = A^{-1}B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ $\frac{1}{2}$

$= \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ $\frac{1}{2}$

$x = 2$, $y = -3$

OR

(b) For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the number a such that $A^2 + aA + I = 0$.

Ans. $A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$ 1

$A^2 + aA + I = 0 \Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$ $\frac{1}{2}$

on solving we get a = –4 $\frac{1}{2}$

25. Find:

$$\int \frac{1}{x + x \log x} dx$$

Ans. Let I = $\int \frac{1}{(1+\log x)} \frac{1}{x} dx$

Let 1+ log x = t $\frac{1}{2}$

$\Rightarrow \frac{1}{x} dx = dt$ $\frac{1}{2}$

I = $\int \frac{1}{t} dt$

I = log |t| + C $\frac{1}{2}$

= log |1 + log x| + C $\frac{1}{2}$

26. (a) Find the vector and Cartesian equations of the line that passes through the points (5, –1, –4) and (5, –1, 7).

Ans. Vector equation is $\vec{r} = 5\hat{i} - \hat{j} - 4\hat{k} + \lambda(11\hat{k})$ 1

Cartesian equation is $\frac{x-5}{0} = \frac{y+1}{0} = \frac{z+4}{11}$ 1

OR

(b) Find the equation of a plane passing through the points (3, 4, 5), (3, 5, 5) and (0, 5, 3).

Ans. Equation of plane is given by

$$\begin{vmatrix} x-3 & y-4 & z-5 \\ 0 & 1 & 0 \\ -3 & 1 & -2 \end{vmatrix} = 0$$ 1

which gives –2(x – 3) + 3 (z – 5) = 0 1

or –2x + 3z – 9 = 0 or $\vec{r}.(-2\hat{i} + 3\hat{k}) - 9 = 0$

27. For an LPP define

(i) Feasible region

Ans. Feasible region: The common region determined by all the constraints of a Linear Programming Problem. 1

(ii) Optimal solution

Ans. Optimal solution: Any point in the feasible region that optinaizes the objective function. 1

28. For an LPP, the corner points of the bounded feasible region are (9, 0), (4, 3), (2, 5) and (0, 8). Find the minimum value of $z = 4x + 3y$.

Ans.

Corner point:	Value of $z = 4x + 3y$	
(9, 0)	36	$\frac{1}{2}$
(4, 3)	25	$\frac{1}{2}$
(2, 5)	23 $\rightarrow$ minimum value	$\frac{1}{2}$
(0, 8)	24	$\frac{1}{2}$

(Section IV)

Question numbers 29 to 35 carry 3 marks each. **7 × 3 = 21**

29. Prove that:

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$$

Ans. Given expression is

$$\cot^{-1}\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}} \qquad 1\frac{1}{2}$$

$$=\cot^{-1}\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}=\cot^{-1}\cot\frac{x}{2} \qquad 1$$

$$=\frac{x}{2} \qquad \frac{1}{2}$$

30. (a) If $e^y(x + 1) = 1$, show that $\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$.

Ans. $e^y = \dfrac{1}{x+1} \Rightarrow e^y dy = \dfrac{-1}{(x+1)^2}$ 1

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-1}{x+1}$ 1

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{+1}{(x+1)^2} = \left(\dfrac{dy}{dx}\right)^2$ Hence Proved. 1

OR

(b) If $y^x = x^y$, find $\dfrac{dy}{dx}$.

Ans. Taking log on both sides

$x \log y = y \log x$ $\frac{1}{2}$

Differentiating both sides w.r.t. x

$\dfrac{x}{y}\dfrac{dy}{dx} + \log y = \dfrac{y}{x} + \log x \dfrac{dy}{dx}$ 1+1

$\therefore \dfrac{dy}{dx} = \dfrac{\dfrac{y}{x} - \log y}{\dfrac{x}{y} - \log x}$ or $\dfrac{y(y - x\log y)}{x(x - y\log x)}$ $\frac{1}{2}$

31. Find:

$\int x \tan^{-1} x \, dx$

Ans. Let I = $\int x \tan^{-1} x \, dx$

= $\dfrac{\pi}{2}$ 1

$= (\tan^{-1}x)\dfrac{x^2}{2} - \dfrac{1}{2}\int \dfrac{x^2}{1+x^2} dx$ 1

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int 1 - \frac{1}{1+x^2}\,dx$$ $\frac{1}{2}$

$$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x + C$$ $\frac{1}{2}$

32. (a) Evaluate:

$$\int_0^{\pi/4} \log(1+\tan x)\,dx$$

Ans. Let $I = \int_0^{\pi/4} \log(1+\tan x)\,dx$ ....(i)

replacing x by $\frac{\pi}{4} - x$, we get

$$I = \int_0^{\pi/4} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)\,dx$$ 1

$$= \int_0^{\pi/4} \log\frac{2}{1+\tan x}\,dx$$ ...(ii) 1

(i) + (ii) implies

$$2I = \int_0^{\pi/4} \log 2\,dx$$

$$I = \frac{1}{2}.\log 2.\left[x\Big|_0^{\pi/4}\right]$$ $\frac{1}{2}$

$$= \frac{\pi}{8}\log 2$$ $\frac{1}{2}$

OR

(b) Find:

$$\int \frac{dx}{x(x^2+1)}$$

Ans. Let $I = \int \frac{1}{x^3\left(1+\frac{1}{x^2}\right)}$ $\frac{1}{2}$

Let $1+\frac{1}{x^2}=t$ $\frac{1}{2}$

$\frac{-2}{x^3}dx = dt$ $\frac{1}{2}$

$I = \frac{-1}{2}\int\frac{dt}{t}$ $\frac{1}{2}$

$= \frac{-1}{2}\log|1+\frac{1}{x^2}|+C$ OR $\frac{-1}{2}\log(x^2+1)+\log|x|+C$ 1

33. Find the general solution of the differential equation

$x\frac{dy}{dx}-y+x\sin\left(\frac{y}{x}\right)=0$

Ans. Given differential equation can be written as

$\frac{dy}{dx}=\frac{y}{x}-\sin\frac{y}{x}$ ...(i) $\frac{1}{2}$

Let $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ 1

(i) becomes

$v+x\frac{dv}{dx} = v-\sin v$

$\Rightarrow \text{cosec } v \; dv = -\frac{dx}{x}$ $\frac{1}{2}$

Integrating both sides

$\int \text{cosec } v \, dv = -\int\frac{dx}{x}$

$\log|\text{cosec } v - \cot v| = -\log|x|+C$ $\frac{1}{2}$

$\log|\text{cosec}\frac{y}{x}-\cot\frac{y}{x}|+\log|x|=C$ $\frac{1}{2}$

or $x\left(\text{cosec}\frac{y}{x}-\cot\frac{y}{x}\right)=k$ where $\log k = C$

34. A bag contains 4 red and 4 black balls, another bag containsc 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag at random. Find the probability of getting a red ball.

Ans. E_1: Bag 1 is selected

E_2: Bag 2 is selected

A: Red ball is drawn

$P(E_1) = P(E_2) = \frac{1}{2}$ $\frac{1}{2}$

$P(A/E_1) = \frac{4}{8}$ $P(A/E_2) = \frac{2}{8}$ 1

$P(A) = P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2)$

$= \frac{1}{2}\cdot\frac{4}{8}+\frac{1}{2}\cdot\frac{2}{8}$ 1

$= \frac{6}{16}$ or $\frac{3}{8}$ $\frac{1}{2}$

35. A random variable X has the following probability distribution:

X:	0	1	2	3	4	5	6	7
P(X):	0	k	2k	2k	3k	k^2	$2k^2$	$(7k^2 + k)$

Determine:

(i) k

(ii) $P(X < 3)$

Ans. According to Question

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$10k^2 + 9k - 1 = 0$ 1

gives $k = \frac{1}{10}$ 1

(i) $k = \frac{1}{10}$ $\frac{1}{2}$

(ii) $P(X < 3) = 3k = \frac{3}{10}$ $\frac{1}{2}$

(Section V)

Question numbers 36 to 38 carry 5 marks each. **$3 \times 5 = 15$**

36. (a) Find the greatest and the least values of the function

$$f(x) = \sin 2x - x \text{ on } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

Ans. $f'(x) = 2\cos 2x - 1$ 1

$f'(x)\ 0 \Rightarrow \cos 2x = \frac{1}{2} = \cos\frac{\pi}{3}$

$2x = \frac{\pi}{3}, -\frac{\pi}{3}$

$x = \frac{\pi}{6}, -\frac{\pi}{6}$ 1

$f(-\pi/2) = \pi/2$ $\frac{1}{2}$

$f(-\pi/6) = \frac{-\sqrt{3}}{2} + \frac{\pi}{6}$ $\frac{1}{2}$

$f(\pi/6) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ $\frac{1}{2}$

$f(\pi/2) = -\frac{\pi}{2}$ $\frac{1}{2}$

Greatest value: $\frac{\pi}{2}$, least value: $-\frac{\pi}{2}$ $\frac{1}{2}+\frac{1}{2}$

OR

(b) Find the equations of the tangent and the normal to the curve $y^2 = 4x$ at the point (1, 2).

Ans. Differentiating $y^2 = 4x$ w..r.t. x we get

$2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$ 2

Slope of tangent at (1, 2) = 1 $\frac{1}{2}$

Slope of Normal at (1, 2) = –1 $\frac{1}{2}$

Equation of tangent: $y - 2 = 1\ (x - 1)$ or $y - x - 1 = 0$ 1

Equation of Normal: $y - 2 = -1(\ x - 1)$ or $y + x - 3 = 0$ 1

37. (a) Find the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the ordinates $x = 0$ and $x = ae$, using integration.

Ans. Required area = $\frac{2b}{a}\int_0^{ae}\sqrt{a^2-x^2}\,dx$ 2

$$=\frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{ae}$$ 1

$$=\frac{2b}{a}\left[\frac{ae}{2}\sqrt{a^2(1-e^2)}+\frac{a^2}{2}\sin^{-1}e\right]$$ 1

$$= abe\sqrt{1-e^2}+ab\sin^{-1}e$$ 1

OR

(b) Evaluate:

$$\int_0^4 |x-2|\,dx$$

Ans. $I = \int_0^4 |x-2|\,dx$

$$=\int_0^2 -(x-2)\,dx+\int_2^4 (x-2)\,dx$$ 2

$$=-\frac{(x-2)^2}{2}\Bigg|_0^2+\frac{(x-2)^2}{2}\Bigg|_2^4$$ 2

$= 2 + 2$

$= 4$ 1

38. (a) Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line

$\vec{r} = 2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k}) = 5$.

Ans. Position vector of any point on gives line is given by

$(2+3\lambda)\hat{i}+(-1+4\lambda)j+(2+2\lambda)\hat{k}$ 1

Since it lies on plane

$\therefore\ (2 + 3\lambda) \cdot 1 - (-1 + 4\lambda) + (2 + 2\lambda) \cdot 1 = 5$ 1

gives $\lambda = 0$ 1

Position vector of point of intersection are $2\hat{i}-\hat{j}+2\hat{k}$ 1

$\therefore$ Coordinates are $(2, -1, 2)$

Required Distance = $\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

$= 13$ 1

OR

(b) Find the shortest distance between the lines

$\vec{r} = 6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$ and

$\vec{r} = \ 4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$

Ans. $\overrightarrow{a_1} = 6\hat{i}+2\hat{j}+2\hat{k} \qquad \overrightarrow{a_2} = -4\hat{i}-\hat{k}$ 1

$\overrightarrow{b_1} = \hat{i}-2\hat{j}+2\hat{k} \qquad \overrightarrow{b_2} = 3\hat{i}-2\hat{j}-2\hat{k}$

$\overrightarrow{a_2} \quad \overrightarrow{a_1} = -10\hat{i}-2\hat{j}-3\hat{k}$ 1

$\overrightarrow{b_1}\times\overrightarrow{b_2} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{bmatrix} = 8\hat{i}+8\hat{j}+4\hat{k}$ $1\frac{1}{2}$

Shortest distance = $\dfrac{|(-10\hat{i}-2\hat{j}-3\hat{k})\cdot(8\hat{i}+8\hat{j}+4\hat{k})|}{\sqrt{8^2+8^2+4^2}}$ 1

$= \dfrac{108}{12} = 9$ $\frac{1}{2}$