

# Statistics

## **Unit-1: Probability and Distribution Theory**

Trial, sample point, sample space, definitions of equally likely, mutually exclusive and exhaustive events, definition of probability, classical and relative frequency approach to probability, axiomatic approach to probability and its properties, total and compound probability theorems, conditional probability, independence of events, Bayes' theorem and its applications.

Random Variable, Concept of discrete random variable, probability mass function and distribution function, joint probability mass function of several discrete random variables, marginal and conditional probability mass functions, Continuous random variable, Probability density function, distribution function, joint density function of two continuous variables, marginal and conditional probability density functions, independence of random variables. Mathematical expectation, moments, factorial moments, moment generating function of a random variable, their properties and uses, probability generating function, Chebyshev's inequality and its applications, basic ideas of convergence in probability and convergence in distribution, Markov's inequality, Bernoulli distribution, binomial distribution, Poisson distribution, geometric distribution, negative binomial distribution, hyper geometric distribution, multinomial distribution, uniform distribution, normal distribution and its relationship with the binomial and Poisson distribution, Cauchy distribution, bivariate normal distribution and its marginal and conditional distributions.

## **Unit-2: Statistical Methods and Statistical Quality Control and Time Series**

Concept of Statistical population and sample, classification of data, quantitative and qualitative data, ordinal and nominal data, time series and cross-sectional data, multivariate data, Construction of tables (with one or more factors), diagrammatic and graphical representation of grouped data, frequency and cumulative frequency distribution and their applications, histogram, frequency polygon, ogives, stem and leaf charts, box plot, Concept of central tendency and its measures, partition values, dispersion and relative dispersion, moments, Sheppard's correction for moments (without derivation), skewness, kurtosis and their measures. Bivariate data, Scatter diagram, correlation coefficient and its properties, coefficient of determination, correlation ratio, interclass correlation, and concept of error in regression, principle of least square, fitting of linear regression and related results, rank correlation, Partial and multiple Correlation in three variables, their measures and related results. Theory of attributes, Independence and Association of attributes, various measures of association for two way classified data.

Statistical process and product control, Quality of a product, need for quality control, basic concept of process control, process capability and product control, general theory of control charts, causes of variation in quality, control limits, sub grouping summary of out of control criteria. Charts for attributes, p chart, np chart, c-chart, Charts for variables,  $(\bar{x}, R)$ ,  $(\bar{x}, \sigma)$  charts, Principle of acceptance sampling-problem of lot acceptance, stipulation of good and bad lots, producers and consumers risks, single and double sampling plans, their OC functions, concepts of AQL, LTPD, AOQL, average amount of inspection and ASN function. Measurement of Fertility, Crude birth rate, general fertility rate, age specific birth rate, total fertility rate, gross reproduction rate, net reproduction rate, logistic model for population projection, Time Series Analysis, Economic time series, different components, illustration, additive and multiplicative models, determination of trend, seasonal and cyclical fluctuations.

### **Unit-3: Statistical Inference, Demography and Index Number**

Sampling distribution of a statistic, Derivation of  $\chi^2$ , t, F and Z distributions, Beta Gamma and Laplace densities, Point estimation, properties of estimators, mean square and minimum mean square error estimator, unbiasedness and minimum variance unbiased estimator, Cramer-Rao lower bound, amount of information, consistency of estimators and sufficient conditions for consistency, relative efficiency of an estimator, asymptotic efficiency, sufficiency, factorization theorem (without proof), concept of complete sufficient statistics, Rao-Blackwell theorem. Completeness and sufficiency, Lehman-Scheffe theorem, one parameter exponential family and its completeness, Cramer-Rao inequality, Best linear unbiased estimator, maximum likelihood, minimum chi-square, least square with examples, BAN and CAN estimators, point estimates of measures of location, dispersion, regression, correlation and other useful parameters. Concepts of confidence interval and confidence coefficient, confidence intervals for the parameters of univariate, normal, two independent normal distributions and exponential distributions. Statistical hypotheses, critical region, size and power of a test, most powerful test, randomized and non-randomized test, Neyman-Pearson lemma and its applications, uniformly most powerful unbiased test, power likelihood ratio test and its applications, functions of UMP with simple illustration. Applications of  $\chi^2$ , t, F and z distributions in tests of significance, Likelihood ratio test, Unbiased test, Neyman-Pearson Lemma for randomized tests, randomized test for binomial and Poisson distribution,  $\chi^2$  test of goodness of fit, Test of equality of several variances, Significance test for correlation coefficient.

Measurement of Mortality and Life Table: Crude death rate, Standardized death rates, Age-specific death rates, Infant Mortality rate, Death rate by cause, Complete and abridged life table and its main features, Uses of life table. Index Numbers: Price relatives and quantity or volume relatives, Link and chain relatives composition of index numbers; Laspeyre's, Paasche's, Marshall-Edgeworth's and Fisher's index numbers; chain base index number, tests for index number, cost of living index number.

### **Unit-4: Sampling Theory and Design of Experiments**

Concept of population and sample, need for sampling, complete enumeration versus sampling, Basic concepts in sampling, sampling and Non-sampling errors. Simple random sampling with and without replacement, estimation of population mean, population proportion and their standard errors, Stratified random sampling, proportional and optimum allocation, comparison with simple random sampling for fixed sample size. Post stratification, Double sampling with post stratification, Ratio, product and regression methods of estimation, estimation of population mean, evaluation of bias and variance to the first order of approximation, comparison with simple random sampling, Systematic sampling, Cluster sampling with equal clusters, two stage sampling.

One-way ANOVA, two-way ANOVA with single observation per cell and equal number of observations per cell, Randomization, Replication, Local Control, Completely Randomized Design (CRD), Randomized Block Design (RBD), Latin square design (LSD), Missing plot technique, General block design and its information matrix (C). Criteria for connectedness, balanced and orthogonality, Intra-block analysis, incomplete block design, Balanced

Incomplete Block Design (IBD), General factorial experiments, factorial effects, study of  $2^n$  and  $3^n$  factorial experiments in randomized blocks, complete and partial confounding, construction of confounded factorial experiments, split plot experiment.

### **Unit-5: Linear Estimation and Multivariate Analysis**

General linear model, assumptions, estimation of parameters by least squares, estimable functions, error and estimation space, Gauss-Markov theorem, Distribution of quadratic form and its application in analysis of variance model, Estimable linear hypothesis, generalized F and t tests.

Multivariate normal distribution (MND), normal generating function and Characteristic function, marginal and conditional distributions, multiple and partial correlation coefficients for MND, Maximum likelihood estimators of the mean vector and covariance matrix, Distribution of sample mean vector, null distribution of sample correlation coefficient, sample multiple and partial correlation coefficients and their null sampling distributions, distribution of sample regression coefficient, Null distribution and non-null distribution of Hotelling's  $T^2$  statistic, Application in tests for mean vector of one and more multivariate normal populations and for equality of the components of a mean vector in a multivariate normal population and their applications, Mahalanobis'  $D^2$ , Wishart distribution and its properties, Classification and discrimination procedures for discrimination between two multivariate normal distributions, populations-sample discriminant function and test associated with discriminant functions, probabilities of misclassification and their estimation, classification into more than two multivariate normal populations,.