Mathematics

Real Analysis:
Real number system as an ordered field with least upper bound property. Sequences, limit of a sequence, Cauchy sequence, completeness of real line. Convergence, absolute and conditional convergence of series of real terms, rearrangement of series. Continuity and uniform continuity of functions, properties of continuous functions on compact sets. Riemann integral, improper integrals, Fundamental theorems of integral calculus. Uniform convergence, continuity, differentiability and integrability for sequences and series of functions. Partial derivatives of functions of several (two or three) variables, maxima and minima.

Linear Algebra:
Vector spaces, subspaces, linear dependence, basis, dimension, Linear transformation. Algebra of matrices, rank and determinant of matrices, system of linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms.

Complex Analysis:
Extended Complex Plane, Stereographic projection of complex numbers, continuity and differentiability of complex functions, Analytic functions, Cauchy-Riemann equations, harmonic functions. Mappings by elementary functions: Translation, Rotation, Magnification and Inversion. Conformal Mappings, Mobius transformations. Fixed points, Cross-ratio, Inverse Points and critical mappings.

Numerical Analysis:
Numerical methods: Solution of algebraic and transcendental equations of one variable by bisection method, Fixed point iteration method, Regula-Falsi method, Secant method and Newton-Raphson methods, their order of convergence and convergence analysis, solution of system of linear equations by Gaussian Elimination and Gauss-Jorden (direct), Gauss-Seidel (iterative) methods. Newton's (forward and backward) and interpolation, Lagrange's interpolation. Numerical integration: Trapezoidal rule, Simpson's rule, Gaussian quadrature formula. Numerical solution of ordinary differential equations: Euler and Runga Kutta methods (2nd order and 4th order).

Ordinary Differential Equations (ODEs):
Formulation of differential equations, Equations of first order and first degree, Exact equation, integrating factor. Orthogonal trajectory, Equations of first order but not of first degree, Clairaut's equation, singular solution, Applications of first order differential equations to Newton's law of cooling, exponential growth and decay problems.
Second and higher order linear equations with constant coefficients, complementary functions, particular integral and general solution,
Second order linear equations with variable coefficients, Euler-Cauchy equation, determination of complete solution when one solution is known using method of variation of parameters, Method of undetermined coefficients, Application of second order differential equation: Simple pendulum problem.

Partial Differential Equations (PDEs):
Family of surfaces in three dimensions and formulation of partial differential equations, Solution of quasi linear partial differential equations of the first order, Cauchy's method of characteristics, Method of separation of variables. Linear partial differential equations of the second order with constant coefficients, canonical form. Equation of a vibrating string, heat conduction, Laplace equation and their solutions. Existence and uniqueness of the solutions of vibrating string problem and heat conduction problem.

Topology and metric space:
Metric spaces, compactness, Connectedness, Topological spaces, closed sets, closure, Dense set, Neighbourhood. Interior, exterior, boundary, accumulation points and derived sets, Cantor's theorem. Bases and sub-bases. First and Second Countable spaces, Separable spaces, Separation axioms, compactness, continuous functions and compact sets, connected spaces.

Group and Ring:
Groups, subgroups, cyclic groups, Group of symmetries, cosets, Lagrange's Theorem, normal subgroups, quotient groups, homomorphism of groups, basic isomorphism theorems, permutation groups, Cayley's theorem.
Rings, subrings and ideals, homomorphisms of rings. Integral domains, principal ideal domains,
Euclidean domains and unique factorization domains; Fields, quotient fields.

Number Theory and Trigonometry:
Permutations, Combinations, Pigeon-Hole principle, Inclusion-Exclusion principle, derangements. Fundamental theorem of Arithmetic, divisibility in Z, Congruences, Chinese Remainder Theorem, Euler's Ø- function, Primitive roots. De Moivre's Theorem and its Applications. Expansion of trigonometrical functions. Direct circular and hyperbolic functions and their properties. Inverse circular and hyperbolic functions and their properties. Logarithm of a complex quantity. Gregory's series. Summation of Trigonometry series.

Special Functions and Integral Transforms:
Series solution of differential equations– Power series method, Beta and Gamma functions. Bessel equation and its solution, Bessel functions and their properties, Orthogonality of Bessel functions. Legendre and Hermite differentials equations and their solutions, Legendre and Hermite functions and their properties, Orthogonality of Legendre and Hermite polynomials. Rodrigues' Formula for Legendre & Hermite Polynomials, Laplace Integral Representation of Legendre polynomial. Laplace Transforms: Existence theorem for Laplace transforms, Linearity property, Shifting theorems, Laplace transforms of derivatives and integrals, Differentiation and integration of Laplace transforms, Convolution theorem, Inverse Laplace transforms, convolution theorem, Inverse Laplace transforms of derivatives and integrals, solution of ordinary differential equations using Laplace transform. Fourier transforms: Linearity property, Shifting, Modulation, Convolution Theorem, Fourier Transform of Derivatives, Relations between Fourier transform and Laplace transform, Parseval's identity for Fourier transforms, solution of differential Equations using Fourier Transforms.

Solid Geometry:
General equation of second degree. Tracing of conics. Polar equation of a conic, tangent and normal to the conic. Sphere: Plane section of a sphere, Sphere through a given circle, Intersection of two spheres, radical plane of two spheres, Co-axial system of spheres. Cones: Right circular cone, enveloping cone and reciprocal cone. Cylinder: Right circular cylinder and enveloping cylinder. Central Conicoids: Equation of tangent plane, Director sphere, Normal to the conicoids. Polar plane of a point. Enveloping cone of a coincoid. Enveloping cylinder of a conicoid. Paraboloids: Circular section, Plane sections of conicoids. Generating lines. Confocal coincoid. Reduction of second degree equations.

Dynamics and statics:
Rectilinear motion, simple harmonic motion, motion in a plane, projectiles, Constrained motion. Work and energy, conservation of energy, Kepler's laws, orbits under central forces, General motion under central forces, Planetary orbits.
Equilibrium of a system of particles, Work and potential energy, friction, Common catenary. Principle of virtual work, Stability of equilibrium, equilibrium of forces in three dimensions, Potential energy and conservative field, Mass centres, Centers of gravity.

Vector calculus:
Differentiation of Vectors, Del operator, Gradient, divergent, Curl and directional derivative, their identities and related theorems. Integration of Vectors. Line, Surface and Volume integration of vectors. Gauss Divergence, Stokes and Green theorem.

Linear Programming:
Standard form of the LPP, basic feasible and Optimal solution, graphical method and simplex method of solution, Artificial variables, Introduction to duality, Formulation of the dual problem
Transportation problem, finding initial basic feasible solution using Northwest-corner method, Least-cost method, and Vogel approximation method, Hungarian method of solving assignment problem.

Graph Theory:
Graphs, Diagraphs, Networks and subgraphs, Vertex degree, Paths and cycles, Regular and bipartite graphs, four cube problem, social networks, Exploring and traveling, Eulerian and Hamiltonian graphs, Applications to dominoes, Diagram tracing puzzles, Knight's tour problem, Gray codes.